LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T / MTF131
Examination date	2007-11-06
Time	09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5

Teacher on duty: Hans Weber	Tel: 492088 or 0708-592088, Room E111
Examiner: Hans Weber	Tel: 492088 or 0708-592088, Room E111
The results are put up:	27 November 2007 on the notice-board, building E
The marking may be scrutinised:	after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Spin**

   A spin 1/2 particle described by the unnormalised spinor χ

$$\chi = A \begin{pmatrix} 2+5i \\ 3-i \end{pmatrix}.$$

(3p)

   (a) Evaluate the expectation values of the three Cartesian components ($< S_x >, < S_y >, < S_z >$).

   (b) For a measurement of spin along the x direction what are the possible outcomes of this experiment and their probabilities?

(3p)

2. **Thermal properties of Sodium, Na**

   Sodium is a typical metallic element. As Sodium has one s electron in its outer shell its considered to have one valence electron and hence each Sodium atom contributes with one electron to the free electron gas.

   Evaluate the lowest temperature above zero at which the electronic contribution to the specific heat equals the contribution from the phonons.

(3p)

TURN !

3. **Angular momentum and r in Hydrogen**

An electron bound in a hydrogen atom is described by the following state:

$$\psi(\boldsymbol{r}) = \psi(x, y, z) = Nxze^{-\sqrt{x^2+y^2+z^2}/3a_0},$$

where a_0 is the Bohr radius and N is a constant (normalisation).

(a) A measurement of L^2 and L_z is done on the system. Calculate the possible values and their probabilities.

(b) Calculate the expectation value of the electrons distance $< r >$ from the nucleus.

(3p)

4. **Two dimensional Square well**

A particle is placed in the potential (a 2 dimensional 2quare well)

$$V(x) = \begin{cases} 0 & \text{for} \quad -\frac{a}{2} \leq x \leq \frac{a}{2} \text{ and } -\frac{a}{2} \leq y \leq \frac{a}{2} \\ +\infty & \text{for} \quad x > \frac{a}{2},\ x < -\frac{a}{2} \text{ and } y > \frac{a}{2},\ y < -\frac{a}{2}. \end{cases}$$

(a) Calculate (solve the Schrödinger equation) the eigenfunctions !

(b) As the Hamiltonian commutes with the parity operator P, $P\Psi(x, y) = \Psi(-x, -y) = \lambda\Psi(x, y)$ where the eigenvalue λ can take two possible values ± 1.

Write down the eigenstates corresponding to the four lowest **energies** in such a way that they are also eigenfunctions of the parity operator P. What is the parity of these states?

(3p)

5. **Harmonic oscillator**

A three dimensional harmonic oscillator has energy levels according to

$$\epsilon_{n_1,n_2,n_3} = (n_1 + n_2 + n_3 + \frac{3}{2})\ \hbar\omega$$

where n_1, n_2, n_3 are integers $n_i = 0, 1, 2, 3, ...\infty$. The oscillator is coupled to a heatbath of temperature τ with which it can exchange energy.

(a) At what temperature equals the probability to find the oscillator in a state of energy $\frac{3}{2}\hbar\omega$ to find it in a state of energy $\frac{5}{2}\hbar\omega$?

(b) How large is this probability ?

(3p)

GOOD LUCK !