LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T / MTF131
Examination date	2007-12-21
Time	09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5

Teacher on duty: Niklas Lehto	Tel: 492085, Room E310
Examiner: Hans Weber	Tel: 492088 or 0708-592088, Room E111
The results are put up:	11 January 2008 on the notice-board, building E
The marking may be scrutinised:	after the results have been put up

Allowed aids: Fysikalia, Physics Handbook, Beta, calculator,COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

1. **Entropy**

   Considering some simple arguments regarding the specific heat C_v answer to following questions.

   a) By how large factor will the entropy of the conduction electrons of metal change as the temperature is raised from $100K$ to $400K$?

   b) By how large factor will the entropy of the electromagnetic radiation field in a cavity change as the temperature is raised from $500K$ to $1500K$?

   (3p)

2. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \sin \omega t$ and $\hat{A} = i\hbar \frac{\partial^2}{\partial t^2}$.

   (b) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (c) $\psi(z) = C_1 e^{ikz} + C_2 e^{-ikz}$ and $\hat{A} = -\hbar^2 \frac{\partial^2}{\partial z^2}$.

   (d) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.

   (e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(z) = Ce^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (3 p)

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3. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, 0) = \sin^3 kx.$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t.

(3 p)

4. **Misc.**

a) Evaluate the commutator $[y^2, p_y^2]$.

b) The ion Be^{3+} has the nuclear charge +4 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2s? Give a numerical value in electron Volts (eV)!

c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r, \theta, \phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

5. **Harmonic oscillator**

A system consists of N identical one dimensional harmonic oscillators. Evaluate the fractions of oscillators n_j/N that are in the 3 lowest (($j = 0$, 1 och 2) energy states at the characteristic temperature τ_{ch}. The energies of the oscillators are given by $\epsilon_j = (j + 1/2)\hbar\omega$ and the characteristic temperature τ_{ch} is given by $\tau_{ch} = \hbar\omega$.

(3p)

GOOD LUCK !