LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T / MTF131
Examination date Time	2008-08-30 09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088, Room E111
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room E111
The results are put up: 22 September 2008 on the notice-board, building E
The marking may be scrutinised: after the results have been put up

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Hydrogen atom**

   a) Determine for a 2p–electron the most probable distance from the nucleous of the hydrogen atom. (Do the calculations for the radial part only)

   b) Which of the following ten hydrogen states have the same energy (no external fields are applied). 3s with $m_l = 0$, 3p with $m_l = 1$, 4d with $m_l = 1$, 3p with $m_l = -1$, 5d with $m_l = 1$, 4p with $m_l = 0$, 5p with $m_l = -1$, 4p with $m_l = -1$, 5s with $m_l = 0$, 3p with $m_l = 0$.

(3p)

2. **Three–dimensional box well**

   A particle is placed in the potential (a 3 dimensional box well)

$$V(x,y,z) = \begin{cases} 0 & \text{for} \quad 0 \le x \le a \text{ and } 0 \le y \le a \text{ and } 0 \le z \le \frac{\mathbf{a}}{\mathbf{2}} \\ +\infty & \text{for} \quad x > a \ \text{ or } x < 0 \ \text{ or } y > a \ \text{ or } y < 0 \ \text{ or } z > \frac{\mathbf{a}}{\mathbf{2}} \ \text{ or } z < 0. \end{cases}$$

   (a) Calculate (solve the Schrödinger equation) the eigenfunctions ?

   (b) What are the 6 lowest eigenenergies ?

   (c) What are the degeneracies of the states associated to these 6 lowest eigenenergies ?

(3p)

3. **Quantum mechanical rotor**

A molecule has not only energy levels like a single atom but due to its distributed mass it also has rotational and vibrational energy levels. A quantum mechanical rotor (molecule) has energy rotational levels according to $\epsilon_j = j(j+1)\hbar^2/2I$ where I is the moment of inertia, each level has degeneration $g(j) = 2j+1$ where $j = 0, 1, 2, ...$.

Calculate the for the rotational degrees of freedom the contribution to the heat capacity for low temperatures ($\tau << \hbar^2/I$).

(3 p)

4. **Paramagnetic system**

A paramagnetic system consists of particles of spin 1 and magnetic moment m. Each spin can point in three directions, parallel, anti–parallel and transverse to an external magnetic field. The corresponding energies are $-mB, +mB$ and 0. Determine the change of entropy for a particle as the magnetic field changes from 0 to B_0 at constant temperature. Show that for $1 << \frac{\tau}{mB_0}$ the decrease in entropy depends on the temperature τ as $\frac{A}{\tau^2}$, determine A.

(3p)

5. **Eigenfunctions and uncertainty**

An electron confined in a quantum well has four discrete energy levels $E_1 = 0.27$ eV, $E_2 = 1.08$ eV, $E_3 = 3.65$ eV, $E_4 = 4.06$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{16}$ and $\frac{1}{16}$ respectively.

(a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta \hat{H}$.

(b) Obtain an expression for the wave function $\Psi(z)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(z)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta \hat{H}$ that you found in (a).

(3 p)

GOOD LUCK !