LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T / MTF131
Examination date Time	2008-10-30 09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5

Teacher on duty: Niklas Lehto	Tel: 492085, 0703-337717 Room E310
Teacher on duty: Hans Weber	Tel: 492088, Room E111
Examiner: Hans Weber	Tel: 492088 or 0708-592088, Room E111
The results are put up:	21 November 2008.
The marking may be scrutinised:	after the results have been anounced

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator,COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Harmonic oscillator solution**

   a) Show that $\psi(\xi) = \xi e^{\xi^2/2}$ is an eigenfunction of the linear harmonic oscillator equation (in dimensionless form):
   $$\frac{d^2\psi(\xi)}{d\xi^2} + (\lambda - \ \xi^2)\psi(\xi) = 0$$
   corresponding to the eigenvalue $\lambda = -3$. Where $\xi = \sqrt{\frac{m\omega}{\hbar}}x$ and $\lambda = \frac{2E}{\hbar\omega}$.

   b) Is this eigenfunction physical acceptable? Motivate!

   (3p)

2. **Time evolution of solution**

   A particle of mass m, which moves freely inside a one–dimensional infinite square well potential of length a, has the following initial wave function at time $t = 0$:
   $$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$
   where A is a real constant.

   a) Find A so that $\psi(x, 0)$ is normalised.

   b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle $< E >$.

   c) Find the wave function $\psi(x, 0)$ at any later time t.

   (3p)

3. **Harmonic oscillator**

A two dimensional harmonic oscillator has energy levels according to

$$\epsilon_{n_1,n_2} = (n_1 + n_2 + 1)\, \hbar\omega$$

where n_1, n_2 are integers $n_i = 0, 1, 2, 3, ...\infty$. The oscillator is coupled to a heatbath of temperature τ with which the oscillator can exchange energy.

(a) Calculate the partition function of the oscillator for any temperature.

(b) At what temperature equals the probability to find the oscillator in a state of energy $\hbar\omega$ to find it in a state of energy $2\hbar\omega$?

(c) How large is this probability ?

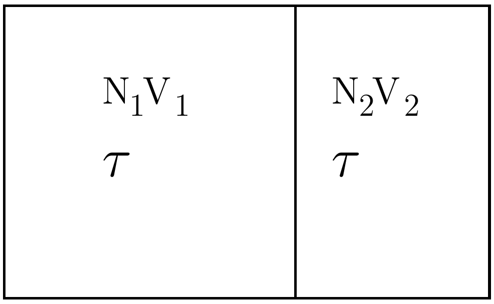
(3p)

4. **Ideal mono atomic gas**

An ideal mon atomic gas confined in a box. The box is devided into two sub parts (compartment 1 and 2) according to the figure below. For compartment one 1 we have volume $V_1 = 2V$, number of particles $N_1 = N$ and temperature τ. For compartment two 2 we have volume $V_2 = V$, number of particles $N_2 = N$ and temperature τ.

Calculate the change of entropy as the wall between compartment 1 and 2 is removed.

The temperature τ is kept constant.



(3p)

5. **Identical particles in an infinite square well**

Two non–interacting identical particles, with the same mass m are confined to a one–dimensional "box" with impenetrable walls, inside which they can move freely. The box has length a.

(a) Solve the Schrödinger equation for the single particle case and give the eigenfunctions and eigenenergies.

(b) Now take both particles into consideration as bosons (integer spin). Give the wave function of the ground state.

(c) Now take both particles into consideration as fermions (half integer spin). Give the wave function of the ground state.

(3 p)

GOOD LUCK !