LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T / MTF131
Examination date Time	2008-12-19 09.00 - 14.00

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5

Teacher on duty: Niklas Lehto	Tel: 492085, 0703-337717 Room E310
Teacher on duty: Hans Weber	Tel: 492088, Room E111
Examiner: Hans Weber	Tel: 492088 or 0708-592088, Room E111
The results are put up:	21 November 2008.
The marking may be scrutinised:	after the results have been anounced

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Allowed aids: Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Reflexion and transmission at potential step**

   Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

   where $V_0 = 10.0$eV. Calculate the reflection coefficient R and the transmission coefficient T

   a) when $E = 5.0$ eV,
   b) when $E = 15.0$ eV,
   c) when $E = 10.0$ eV.

   (3p)

2. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?
   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

   (3p)

3. **Eigenfunctions and uncertainty**

An electron confined in a quantum well has four discrete energy levels $E_1 = 0.31$ eV, $E_2 = 0.97$ eV, $E_3 = 1.81$ eV, $E_4 = 3.35$ eV, $E_5 = 4.08$ eV. It is in a state in which the probabilities associated with these energies are $\frac{1}{2}$, $\frac{2}{12}$, $\frac{1}{12}$, $\frac{3}{16}$ and $\frac{1}{16}$ respectively.

(a) Find the expectation value of its energy $\langle \hat{H} \rangle$ and the corresponding uncertainty $\Delta\hat{H}$.

(b) Obtain an expression for the wave function $\Psi(z)$ describing the state of the particle in terms of its energy eigenfunctions $\psi_n(z)$ at time $t = 0$. Why is the expression not unique? Write down two different wave functions corresponding to the same values of $\langle \hat{H} \rangle$ and $\Delta\hat{H}$ that you found in (a).

(c) Asume the potential is the infinite square well of width L, and you would have calculated $\langle \hat{H} \rangle$ in some way. If one adiabatically changes L to $L/2$ by how much would $\langle \hat{H} \rangle$ change? Adiabatically means we are not inducing transitions between levels in the system.

(3 p)

4. **Hydrogen like spectra**

A space-engeneering student in Kiruna borrows a spectroscope and records the following spetrum (in visible light) of a hot star.

λ	(nm)	658.30	543.04	487.63	470.22	455.75
Intensity	(rel units)	80	30	15	200	8

λ	(nm)	435.38	421.45	411.44	403.97
Intensity	(rel units)	6	5	4	4

She reflects it looks very much like a Hydrogen Balmerseries but still it does not fit. Here friends suggest the spectrum might be from a Helium ion!

Assume it is the spectrum of a Helium ion and analyse the data. Determine for each spectral line the principal quantum numbers for the levels involved. (Note one line belongs to a different series) (3p)

5. **Binding of O_2 to hemoglobin**

A hemoglobin molecule can bind four O_2 molecules. Assume ϵ is the energy of each bound O_2, relative to O_2 at rest at infinite distance. Let λ denote the absolute activity $e^{\mu/\tau}$ of free O_2 (in solution).

(a) What is the probability that one and only one O_2 is adsorbed on a hemoglobin molecule?

(b) What is the probability that four O_2 are adsorbed on a hemoglobin molecule?

(c) Make a sketch of these probabilities as a function of λ.

(3p)

GOOD LUCK !