LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T/MTF131
Examination date Time	2009-08-29 09.00 - 14.00 (5 hours)

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Hans Weber Tel: 492088, Room B253
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room B253

Allowed aids: Course litterature, Fysikalia, Physics Handbook, Beta, calculator, COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Quantum states of Tritium and Helium**

   An electron is in the ground state of tritium ^3H. A β-decay instantaneously changes the atom into a helium ion $^3He^+$. The β particle (=high energy electron) leaves the helium ion and is no longer to be taken inte consideration. The helium ion that is left behind has one single electron bound to it.

   (a) Calculate the probability that the electron (bound to helium ion) is in the 2s-state ($n = 2$, $l = m = 0$) after the decay.

   (b) Calculate the probability that the electron is in a 2p-state ($n = 2$, $l = 1$) after the decay.

   (c) Calculate the probability that the electron is in a 1s-state ($n = 1$, $l = m = 0$) after the decay.

   (d) Is it possible for the electron to recieve the quantum numbers ($n = 1$, $l = 1$) after the decay?

   (3p)

2. **Harmonic oscillator**

   A system consists of N identical one dimensional harmonic oscillators. Evaluate the fractions of oscillators n_j/N that are in the 4 lowest (($j = 0$, 1, 2 and 3) energy states at the characteristic temperature τ_{ch}. The energies of the oscillators are given by $\epsilon_j = (j + 1/2)\hbar\omega$ and the characteristic temperature τ_{ch} is given by $\tau_{ch} = \hbar\omega$. (3p)

3. **Reflection and transmission at a potential step**

Consider an electron of energy E incident on the potential step $V(x)$,

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

where $V_0 = 5.0$ eV. Calculate the reflection coefficient R and the transmission coefficient T

a) when $E = 2.5$ eV,

b) when $E = 5.0$ eV,

c) when $E = 7.5$ eV.

(3p)

4. **Helium** ^3He

Helium ^3He has spin $= \frac{1}{2}$ and may at low temperatures to a good approximation be described as an ideal Fermi gas. At these low temperatures ^3He is in the liquid phase with a density of $\rho = 83$ kg m^{-3}.

a) Determine the Fermi temperature T_F and also the specific heat C_v of ^3He at T=0.2 K.

b) Can you still use the approximations you did in a) if the temperature where say 2-3 K? If not why? If yes why?

(3p)

5. **Quantum mechanical rotor**

A quantum mechanical rotor (molecule) has energy levels $\epsilon_j = j(j+1)\hbar^2/2I$ where I is the moment of inertia, each level has degeneration $g(j) = 2j+1$ where $j = 0, 1, 2, \ldots$. Calculate the for the rotational degrees of freedom the contribution to the heat capacity for low temperatures ($\tau << \hbar^2/I$). Is the behaviour of exponential or algebraic character at low temperatures? (3p)

GOOD LUCK !