LULEÅ UNIVERSITY OF TECHNOLOGY
Division of Physics

Course code	F0018T/MTF131
Examination date Time	2009-12-18 09.00 - 14.00 (5 hours)

Examination in: QUANTUM MECHANICS AND STATISTICAL PHYSICS
Total number of problems: 5
Teacher on duty: Niklas Lehto Tel: 492085, 0703-337717 Room E310
Teacher on duty: Hans Weber Tel: 492088, Room B253
Examiner: Hans Weber Tel: 492088 or 0708-592088, Room B253

Allowed aids: Course litterature, Fysikalia, Physics Handbook, Beta, calculator,COLLECTION OF FORMULAE

Define notations and motivate assumptions and approximations. Present the solutions so that they are easy to follow. Maximum number of point is 15 p. 7.0 points are required to pass the examination. Grades 3: 7.0, 4: 9.5, 5: 12.0

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1. **Operators and eigenfunctions**

   Are the following functions ψ eigenfunctions of the given operators $\hat{A}$?

   (a) $\psi(t) = \sin \omega t$ and $\hat{A} = i\hbar \frac{\partial^2}{\partial t^2}$.

   (b) $\psi(z) = C(1 + z^2)$ and $\hat{A} = -i\hbar \frac{\partial}{\partial z}$.

   (c) $\psi(z) = C_1 e^{ikz} + C_2 e^{-ikz}$ and $\hat{A} = -\hbar^2 \frac{\partial^2}{\partial z^2}$.

   (d) $\psi(z) = Ce^{-3z}$ and $\hat{A} = -i\frac{\hbar}{2}\frac{\partial}{\partial z}$.

   (e) $\psi(z) = Cze^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (f) $\psi(z) = Ce^{-\frac{1}{2}z^2}$ and $\hat{A} = \frac{1}{2}(z^2 - \frac{\partial^2}{\partial z^2})$.

   (3 p)

2. **Measurement of spin**

   A spin $\frac{1}{2}$ particle is prepared to be in an eigenstate to S_z with eigenvalue $+\frac{1}{2}\hbar$. A subsequent measurement of the spin in the direction $\hat{n} = \sin(\varphi)\hat{e}_y + \cos(\varphi)\hat{e}_z$ is made. The value of φ is $\pi/4$.

   (a) What is the probability to get the value $+\hbar/2$ and $-\hbar/2$ in this new direction $\hat{n}$?

   (b) What would the result (eigenvalue and probability) be of a subsequent measurement in the z-direction of the $+\hbar/2$ state in a) ?

   (3p)

TURN PAGE!

3. **Wave functions and eigenfunctions**

Consider a free particle with mass m in one dimension. The wave function of the particle at $t = 0$ is given by

$$\psi(x, 0) = \cos^3 kx.$$

(a) Show that the state function $\psi(x, 0)$ can be written as a superposition of eigenfunctions of the free-particle Hamiltonian.

(b) Determine the energy of each plane wave in the superposition.

(c) Give the wave function $\psi(x, t)$ at an arbitrary time t.

(3 p)

4. **Binding of O_2 to hemoglobin**

A hemoglobin molecule can bind four O_2 molecules. Assume ϵ is the energy of each bound O_2, relative to O_2 at rest at infinite distance. Let λ denote the absolute activity $e^{\mu/\tau}$ of free O_2 (in solution).

(a) What is the probability that one and only one O_2 is adsorbed on a hemoglobin molecule?

(b) What is the probability that four O_2 are adsorbed on a hemoglobin molecule?

(c) Make a sketch of these probabilities as a function of λ.

(3p)

5. **Misc.**

a) Evaluate the commutator $[y^2, p_y^2]$.

b) The ion Be^{3+} has the nuclear charge +4 but only one electron. How much energy does it take to excite the electron from the ground state to the level 2s? Give a numerical value in electron Volts (eV)!

c) The wave function of a hydrogen atom in an eigenstate to the Hamilton operator is:

$$\Psi(r, \theta, \phi) = \frac{1}{81\sqrt{6\pi}}(1/a_\mu)^{3/2}(r^2/a_\mu^2)e^{-r/3a_\mu}[3\cos^2\theta - 1],$$

where a_μ is the Bohr radius (with the reduced mass). Determine the quantum numbers n, l och m_l.

(3p)

GOOD LUCK !