Formelsamling för Statistisk fysik F7035T

Några beteckningar:

Temperaturen:	$\tau = k_B T$
Inre energin:	U
Volymen:	V
Antalet partiklar:	N
Kemisk potential:	μ

Mikrokanoniska ensemblen, $(U, V, N$ givna).

Antalet mikrotillstånd:	g
Entropin:	$\sigma = \ln(g)$
Sannolikheten för tillståndet s:	$P_s = 1/g$

Kanoniska ensemblen, $(\tau, V, N$ givna).

Partitions funktionen (tillståndsumman):	$Z = \sum_\epsilon e^{-\epsilon/\tau}$
Sannolikheten för tillståndet s:	$P_s = e^{-\epsilon_s/\tau}/Z$
Helmholtz fria energi:	$F = -\tau \ln(Z)$
	$F = U - \tau\sigma$

Stor kanoniska ensemblen (Gibbs ensemblen), $(\tau, V, \mu$ givna).

Partitions funktionen (tillståndsumman):	$\mathcal{Z} = \sum_{N,\epsilon} e^{(\mu N - \epsilon)/\tau}$
Sannolikheten för tillståndet s:	$P_s = e^{N\mu - \epsilon_s/\tau}/\mathcal{Z}$
Gibbs fria energi:	$G = U - \tau\sigma + pV$

Den termodynamiska identiteten: $\tau d\sigma = dU - \mu dN + pdV$

Mekaniskt arbete :	$đW$
Tillförd värme:	$đQ = \tau d\sigma = dU + đW$
Specific värme:	$C_V = \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_V = \left(\frac{\partial U}{\partial \tau}\right)_V$
	$C_p = \tau \left(\frac{\partial \sigma}{\partial \tau}\right)_p$
Carnot cykelns verkningsgrad:	$\eta_C = \left(\frac{W}{Q_h}\right)_{\text{rev}} = \frac{\tau_h - \tau_l}{\tau_h}$
Carnot kylskåpets effektivitet:	$\gamma_C = \left(\frac{Q_l}{W}\right)_{\text{rev}} = \frac{\tau_l}{\tau_h - \tau_l}$

	$\sigma(U,V,N)$	$U(\sigma,V,N)$	$F(\tau,V,N)$	$G(N,\tau,p)$
τ :	$\frac{1}{\tau}=\left(\frac{\partial\sigma}{\partial U}\right)_{V,N}$	$\tau=\left(\frac{\partial U}{\partial\sigma}\right)_{V,N}$	τ is independent variable	τ is independent variable
p :	$\frac{p}{\tau}=\left(\frac{\partial\sigma}{\partial V}\right)_{U,N}$	$p=-\left(\frac{\partial U}{\partial V}\right)_{\sigma,N}$	$p=-\left(\frac{\partial F}{\partial V}\right)_{\tau,N}$	p is independent variable
μ :	$\frac{\mu}{\tau}=-\left(\frac{\partial\sigma}{\partial N}\right)_{U,V}$	$\mu=\left(\frac{\partial U}{\partial N}\right)_{\sigma,V}$	$\mu=\left(\frac{\partial F}{\partial N}\right)_{\tau,V}$	$\mu=\left(\frac{\partial G}{\partial N}\right)_{\tau,p}$
V :				$V=\left(\frac{\partial G}{\partial p}\right)_{\tau,N}$
σ :			$\sigma=-\left(\frac{\partial F}{\partial\tau}\right)_{V,N}$	$\sigma=-\left(\frac{\partial G}{\partial\tau}\right)_{p,N}$
U :			$U=-\tau^2\left(\frac{\partial}{\partial\tau}\frac{F}{\tau}\right)_{V,N}$	

Fermi–Dirac fördelningsfunktion:

$$f(\epsilon,\mu,\tau) = \frac{1}{e^{(\epsilon-\mu)/\tau}+1}$$

Bose – Einstein fördelningsfunktion:

$$f(\epsilon,\mu,\tau) = \frac{1}{e^{(\epsilon-\mu)/\tau}-1}$$

Plancks fördelningsfunktion:

$$< s > = \frac{1}{e^{\hbar\omega/\tau}-1}$$

Plancks strålningslag:

$$u_\omega = \frac{\hbar}{\pi^2 c^3}\frac{\omega^3}{e^{\hbar\omega/\tau}-1}$$

DOS (i 3D för elektroner):

$$D(\epsilon) = \frac{dN}{d\epsilon} = \frac{3N(\epsilon)}{2\epsilon} = \frac{V}{2\pi^2}\left(\frac{2M}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{\epsilon}$$

Den Ideala gasen:

$$\begin{aligned}
n_Q &= \left(\frac{M\tau}{2\pi\hbar^2}\right)^{\frac{3}{2}} \\
\mu &= \tau \ln\left(n/n_Q\right) \\
\lambda &= e^{\mu/\tau} = n/n_Q \\
F &= N\tau\left[\ln\left(n/n_Q\right) - 1\right] \\
pV &= N\tau \\
U &= \tfrac{3}{2}\tau \\
\sigma &= N\left[\ln\left(n_Q/n\right) + \tfrac{5}{2}\right]
\end{aligned}$$

Massverkans lag: $\Pi_j n_j^{\nu_j} = K(\tau) = \Pi_j n_{Qj}^{\nu_j} e^{-\nu_j F_j(int)/\tau}$

Clausius – Clapeyron: $\frac{dp}{d\tau} = \frac{L}{\tau \Delta v}$

van der Waals: $(p + N^2 a/V^2)(V - Nb) = N\tau$

Blandnings entropi: $\sigma_M = -N[(1-x)\ln(1-x) + x\ln(x)]$

Maxwells hastighets fördelning: $P(v) = 4\pi \left(\frac{M}{2\pi\tau}\right)^{\frac{3}{2}} v^2 e^{-Mv^2/2\tau}$

Fri medelväglängd: $l = \frac{1}{n\pi d^2}$

Fick's lag: $J_n = -D\nabla n \quad ; D = \frac{1}{3}\bar{c}l$

Boltzmanns transport ekvation: $\partial f/\partial t + v \cdot \nabla_r f + \alpha \cdot \nabla_v f = (\partial f/\partial t)_{coll}$

Stirlings formel:

$$n! \approx \sqrt{2\pi n}\; n^n\; e^{-n+\frac{1}{12n}+O(\frac{1}{n^2})}$$

$$\ln(n!) \approx \frac{1}{2}\ln(2\pi) + (n+\frac{1}{2})\ln(n) - n + \frac{1}{12n} + O(\frac{1}{n^2})$$

$$\ln(n!) \approx n\ln(n) - n$$

Några integraler:

$$\int_0^\infty dx \frac{\sqrt{x}}{e^x - 1} \approx 1.306\sqrt{\pi}$$

$$\int_0^\infty dx \frac{x}{e^x - 1} = \frac{\pi^2}{12} \approx 0.82246703$$

$$\int_0^\infty dx \frac{x^{\frac{3}{2}}}{e^x - 1} = \frac{3}{4}\sqrt{\pi}\zeta(\frac{5}{2}) \approx 1.7832932$$

$$\int_0^\infty dx \frac{x^2}{e^x - 1} = 2\zeta(3) \approx 2.4041$$

$$\int_0^\infty dx \frac{x^3}{e^x - 1} = \frac{\pi^4}{15} \approx 6.4939394$$
