Models for highway traffic and their connections to thermodynamics.

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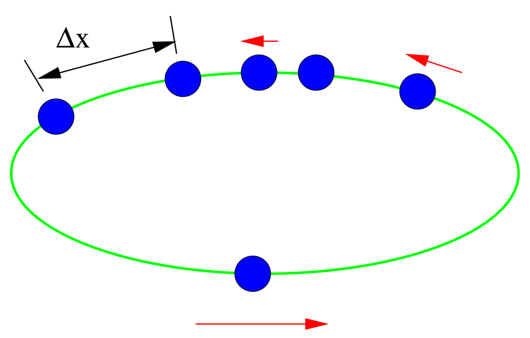
Abstract

Models for highway traffic are studied by numerical simulations. Of special interest is the spontaneous formation of traffic jams. In a thermodynamic system the traffic jam would correspond to the dense phase (liquid) and the free flowing traffic would correspond to the gas phase. Both phases depending on the density of cars can be present at the same time. A model for a single lane circular road has been studied. The model is called the optimal velocity model (OVM) and was developed by Bando, Sugiyama, et al. We propose here is a reformulation of the OVM into a description in terms of potential energy functions forming a kind of Hamiltonian for the system. This will however not be globally defined Hamiltonian but a locally defined one as it is a dynamical model. The model defined by this Hamiltonian will be suitable for Monte-Carlo simulations.

1. Bando Model

Bando Model is a deterministic model for traffic flow. We suggest a reformulation of the particle like description in the Bando model to a description on a macroscopic level.
We have a 1 dimensional single lane circular road.

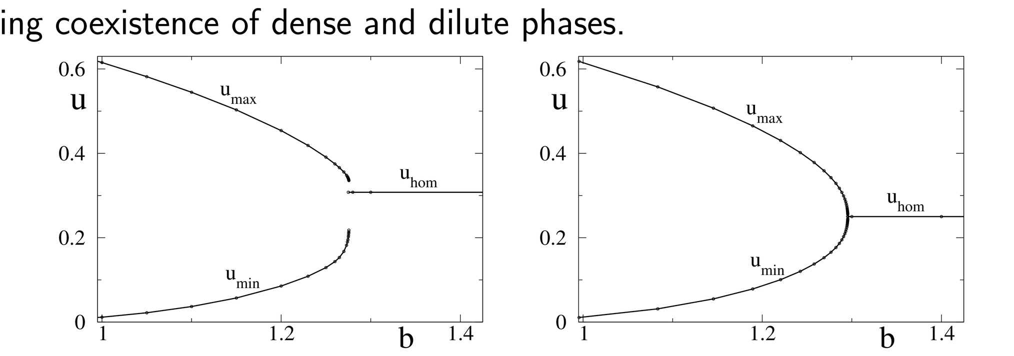


Velocity of cars v and position x, (in dimensionless formulation u, y).
Bando model:

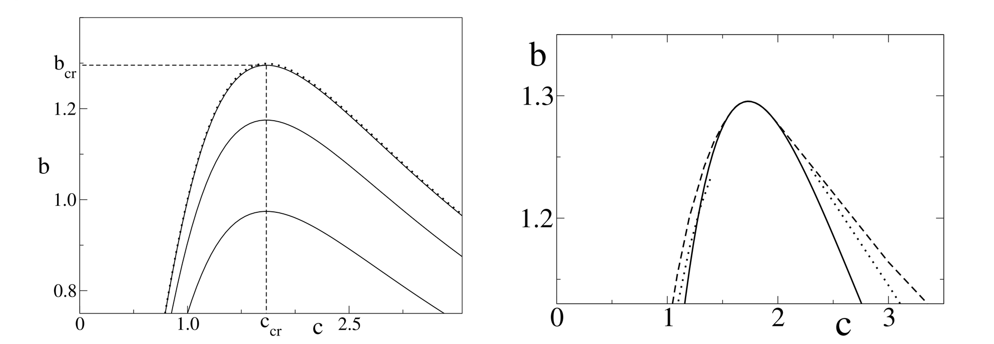
$$\begin{cases} \frac{d}{dt}v_i = \frac{1}{\tau}\left(v_{opt}(\Delta x_i) - v_i\right) & \frac{d}{dt}u_i = \left(u_{opt}(\Delta y_i) - y_i\right) \quad (1) \\ \frac{d}{dt}x_i = v_i & \frac{d}{dt}y_i = \frac{1}{b}u_i \quad (2) \\ v_{opt}(\Delta x_i) = v_{max}\frac{(\Delta x)^2}{D^2+(\Delta x)^2} & u_{opt}(\Delta y_i) = \frac{(\Delta y)^2}{1+(\Delta y)^2} \quad (3) \\ & b = \frac{D}{v_{max}\tau} \quad (4) \end{cases} \tag{1}$$

- Control parameters v_{max}, τ and D (interaction distance), $v_{opt}(\Delta x)$ is the optimal velocity function and $\Delta x_i = x_{i+1} - x_i$ is the headway (bumper-to-bumper distance). The average density of cars is $c = \frac{N}{L}$.
- The acceleration of cars is given by $a_i = \frac{dv}{dt}$. Force $F = ma = \frac{dv}{dt}$. Finally $F \cdot$ movement $= E$.
- Instead of solving the Bando equations with 4 order Runge–Kutta integration on a microscopic level. We will reformulate the model with a kind of Hamiltonian and solve it with the Metropolis method.

Numerical results (4th order Runge–Kutta) for the Bando model, showing coexistence of dense and dilute phases.



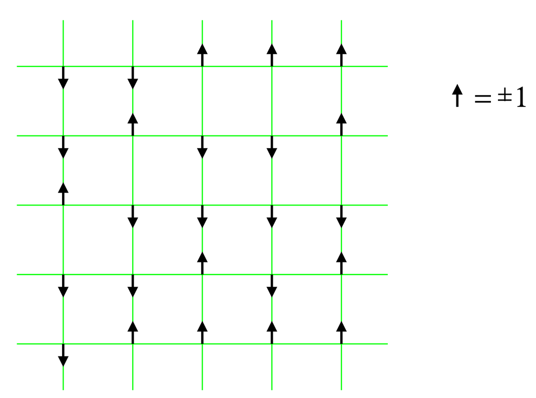
Sub critical bifurcation diagram ($c = 1.5$) and critical ($c = \sqrt{3}$) ($N = 60$).



Phase diagram of the Bando Model ($b_{cr} = 3\sqrt{3}/4$).

2. Monte Carlo Simulations

With Monte Carlo simulations one can analyze many complex problems such as: magnetic systems, gases, superconductors, atoms, nuclear decay, telephone switchboard,... An example the 2D Ising model:



Every state of the system has an energy according to the Hamiltonian of the system

$$H = -\sum_{<i,j>} S_i \cdot S_j \ .$$

- The thermodynamic properties are given by the partition function $Z = \sum_l e^{-H_l/k_BT}$. From Z we can calculate "any" thermodynamic property of the system.
- Most (nearly all) systems are to complicated to be solved analytically. One has to revert to Monte – Carlo simulations.

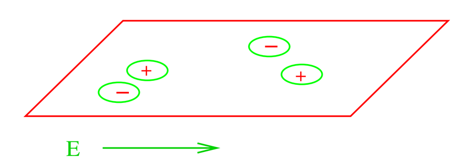
For a configuration of spins S_i. The Metropolis procedure is:

1. Generate a new state by a change to one of the spins $s_j \to s_j + \Delta s_j$.
2. Calculate the energy difference ΔE.
3. Accept the new state if $\Delta E < 0$. If $\Delta E > 0$, accept the new state if $r < e^{-\Delta E/k_BT}$ where r is a random number $r \in [0,1]$, otherwise keep the old value.
4. Goto step 1.

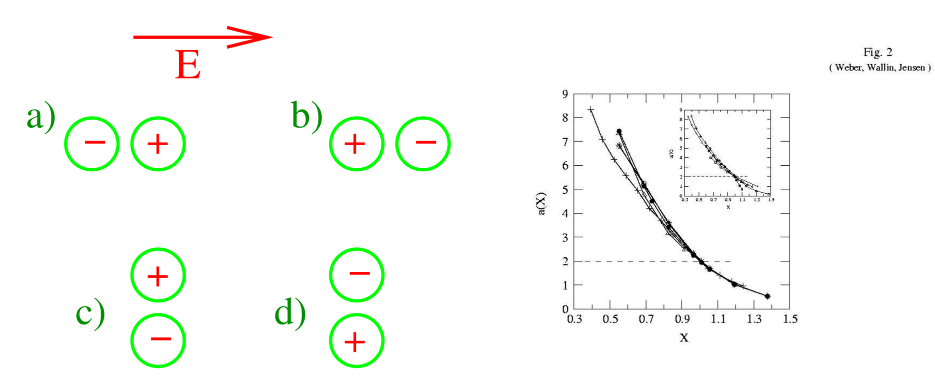
Monte–Carlo usually used for equilibrium properties, but can be used for dynamics as well. There are other Monte – Carlo procedures as well as the heat bath method.

3. Driven System, an Example

- Monte–Carlo simulation of the (current–voltage) IV characteristics of a Superconducting film.
- In a SC film vortex pairs a thermally excited.
- Vortices interact logarithmically $V(r) = \ln(r)$ and hence system is a 2D Coulomb gas.
- Monte–Carlo move consists of adding $\pm$–pairs (charge neutral).



Dynamics: IV current – voltage characteristics. The electric field → Lorentz force gives created pairs different energy according to their orientation.
a) and b) have different energy and c) and d) are not effected by the presence of the field E.



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The energy contribution due to the Lorentz force introduces a local part into the Hamiltonian → No global Hamiltonian!

$$V \propto I^a \quad a = 3 \text{ at } T_c$$

Non linear IV characteristics from experiments and Monte–Carlo simulations compare very well.

4. Traffic Flow

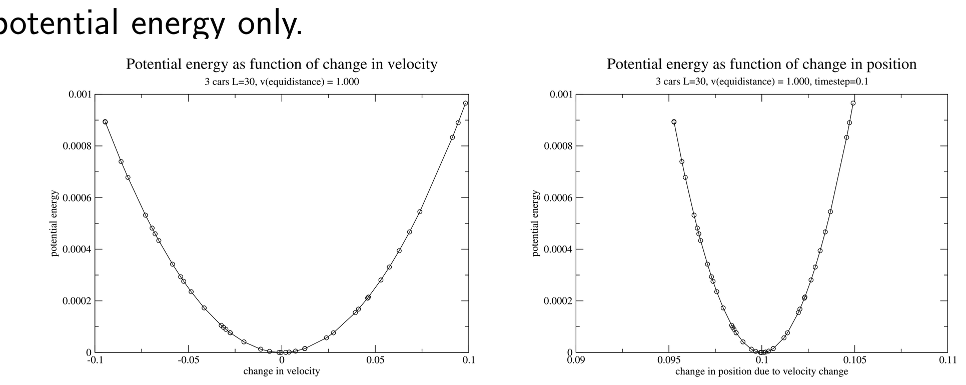
Velocity of cars v and position x. The average density of cars is $c = \frac{N}{L}$. The acceleration of cars is given by $a_i = \frac{dv}{dt}$. Force $F = ma = \frac{dv}{dt}$. And finally $F \cdot$ movement $= E$.

$$\text{Bando model:} \quad \begin{cases} \frac{d}{dt}v_i = \frac{1}{\tau}\left(v_{opt}(\Delta x_i) - v_i\right) \\ \frac{d}{dt}x_i = v_i \\ v_{opt}(\Delta x_i) = v_{max}\frac{(\Delta x)^2}{D^2+(\Delta x)^2} \end{cases} \tag{2}$$

The Metropolis procedure for the cars:

- For a car i make a random change in velocity $\Delta v \in [-\Delta v_{max}, \Delta v_{max}]$.
- The force F is known from the Bando model to the car ahead and behind.
- Calculate the change in energy ΔE (pot + kinetic) due to the proposed change in velocity Δv of the car.
- Use Metropolis to determine if the change Δv is accepted.
- Move the car with either its new or old velocity in time step Δt.
- There is an extra parameter in the problem the ratio m/T.
- The potential energy is a function of **velocity** v_i and **position** x_i.
- Note, T is not real temperature it is a measure of the strength of fluctuations.
- Under certain conditions the traffic separates into two phases. A dense (=jam) and a dilute (=free flow) one.
- Very much like a liquid-gas transition, use the difference in densities as order parameter.

3 cars in a circular road $L = 30.0$ and parameters given so that v(equidistant) $= 1.000$ and $c = \Delta x = 10.0$. Graphs below are for potential energy only.



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