Multisensor fusion and control strategies for low cost hybrid stepper motor solutions

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Master of Science Thesis in Engineering Physics and Electrical Engineering

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Abstract

This thesis has explored if it is feasible to produce a good estimation of the rotational position of a stepper motor by using sensor fusion schemes to merge a sensorless back-EMF position estimation with the measurement from a Magnetic Rotational Position Sensor. The purpose is to find a cheaper alternative for position feedback in closed loop control from conventionally used rotational encoders and resolvers. Beyond the sensor fusion a suitable position control logic was also developed to verify the concept of a low cost closed loop hybrid stepper motor solution for high precision position applications.

The sensor fusion and position control were simulated offline to test the feasibility of the implementation, after which laboratory tests were performed to assess online performance. The extended Kalman filter implemented improved the performance of the MRPS sensor that was run exclusively at lower speeds (between 0-75 rpm) by decreasing the RMSE by almost half from 0.0733 unfiltered to 0.0370 filtered (in mechanical degrees). When fusing both of the position signals in higher speeds the EKF does improve the result by the fusion. However is it not meaningful to discuss the numeric improvement as this was not a general result. This is because the two signals used for the fusion is diverging towards positive and negative error respectively positioning the fused estimate in between. This essentially causes the fusion to simply outperform two very bad signals, and is not meaningful to use as a measure of how well the fusion is actually performing.
Acknowledgements

To begin with, I would like to express my deepest gratitude to my academic supervisor Dr. Damiano Varagnolo for the effort, input and most importantly good spirits he have contributed during the work with this thesis. I also want to thank ABB Corporate Research and especially my supervisor Dr. Bin Liu for both the opportunity to conduct my thesis there and all the great collaboration. Furthermore I am very happy for the good overall experience of my stay in Västerås, a result of all the nice colleagues both regularly employed and other thesis students. Lots of laughs and interesting experiences have been shared in the fika room, something that made the whole thesis to a very positive and fun time.

Västerås, June 2017
Mattias Wallin
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<tbody>
<tr>
<td>ABB</td>
<td>ASEA Brown Boveri</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>EKF</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>EMF</td>
<td>Electromotive Force</td>
</tr>
<tr>
<td>FOC</td>
<td>Field-Oriented Control</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
</tr>
<tr>
<td>HSM</td>
<td>Hybrid Stepper Motor</td>
</tr>
<tr>
<td>MRPS</td>
<td>Magnetic Rotary Position Sensor</td>
</tr>
<tr>
<td>PWM</td>
<td>Pulse-Width Modulation</td>
</tr>
<tr>
<td>RPM</td>
<td>Revolutions per Minute</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root-Mean-Square Error</td>
</tr>
<tr>
<td>SPI</td>
<td>Serial Peripheral Interface (bus)</td>
</tr>
<tr>
<td>UKF</td>
<td>Unscented Kalman Filter</td>
</tr>
<tr>
<td>Notation</td>
<td>Meaning</td>
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<tr>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>$e_a$</td>
<td>Back-EMF induced in winding A [V]</td>
</tr>
<tr>
<td>$e_b$</td>
<td>Back-EMF induced in winding B [V]</td>
</tr>
<tr>
<td>$e_s$</td>
<td>Back-EMF resultant vector [V]</td>
</tr>
<tr>
<td>$i_a$</td>
<td>Current in winding A [A]</td>
</tr>
<tr>
<td>$i_b$</td>
<td>Current in winding B [A]</td>
</tr>
<tr>
<td>$i_s$</td>
<td>Current resultant vector [A]</td>
</tr>
<tr>
<td>$i_0$</td>
<td>Current magnitude [A]</td>
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<td>$i_d$</td>
<td>Current in $d$-reference [A]</td>
</tr>
<tr>
<td>$i_q$</td>
<td>Current in $q$-reference [A]</td>
</tr>
<tr>
<td>$J$</td>
<td>Rotor inertia [kg · m²]</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Derivative part of PID-controller</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Integrator part of PID-controller</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Proportional part of PID-controller</td>
</tr>
<tr>
<td>$K_m$</td>
<td>Motor torque constant [N · m²]</td>
</tr>
<tr>
<td>$K_v$</td>
<td>Coefficient of viscous friction [N · m · s/rad]</td>
</tr>
<tr>
<td>$L$</td>
<td>Mutual coil inductance in windings $a$ and $b$ [H]</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Resultant coil inductance [H]</td>
</tr>
<tr>
<td>$L_d$</td>
<td>Coil inductance in $d$-reference [H]</td>
</tr>
<tr>
<td>$L_q$</td>
<td>Coil inductance in $q$-reference [H]</td>
</tr>
<tr>
<td>$N_r$</td>
<td>Number of pole pairs of the motor</td>
</tr>
<tr>
<td>$R$</td>
<td>Mutual resistance in windings $a$ and $b$ [$\Omega$]</td>
</tr>
<tr>
<td>$T_D$</td>
<td>Detent torque [N · m]</td>
</tr>
<tr>
<td>$T_E$</td>
<td>Produced electromagnetic torque [N · m]</td>
</tr>
<tr>
<td>$T_L$</td>
<td>Load torque [N · m]</td>
</tr>
<tr>
<td>$u_a$</td>
<td>Terminal voltage over winding A [V]</td>
</tr>
<tr>
<td>$u_b$</td>
<td>Terminal voltage over winding B [V]</td>
</tr>
<tr>
<td>$u_s$</td>
<td>Terminal voltage resultant vector [V]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Load angle [rad]</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Rotor angular velocity [rad/s]</td>
</tr>
<tr>
<td>$\theta_e$</td>
<td>Rotor position (electrical)[rad]</td>
</tr>
<tr>
<td>$\theta_r$</td>
<td>Rotor position (mechanical)[rad]</td>
</tr>
<tr>
<td>$\Psi_r$</td>
<td>Magnetic rotor flux [Wb]</td>
</tr>
<tr>
<td>$\Psi_s$</td>
<td>Stator flux linkage vector [Wb · stator winding turns]</td>
</tr>
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Introduction

This report will present the work and findings of a M.Sc. thesis project conducted during 20 weeks in the spring of 2017. It has been executed by a student of the Engineering Physics and Electrical Engineering masters programme at Luleå University of Technology. The research project was performed at ABB Corporate Research in Västerås, Sweden.

1.1 Background

Using hybrid stepper motors for rotational actuation in industry is a interesting and acknowledged solution due to the motor’s traits regarding high torque at low speeds, good positioning in open loop, low cost and high mechanical robustness. When the motor is subjected to an external torque however it will force the rotor out of alignment with the induced magnetic fields introducing a position error named load angle. Also, if the applied load torque exceeds the motors rated holding torque it will loose synchronism introducing a large unknown position error. To deal with these problems closed loop control is needed for enabling usage of the motor in applications with high requirements on position accuracy.

The conventional way of implementing closed loop control on stepper motors is using large and expensive rotational encoders or resolvers for positional feedback, something that counteracts the benefits of reduced cost and compactness arguments in using the hybrid stepper motor.

A research project was conducted at ABB Corporate Research in 2016 [15] examining possible alternatives to the costly encoders and resolvers for position feedback as well as implementing a high precision position controller for hybrid stepper motors. The project examined two alternative versions of position estimation, a low cost magnetic sensor solution and a sensorless approach estimating the position based on the voltage produced by the back electromotive force. Due
to delays and a lot of effort spent on producing a professional lab bench setup the project had to end before a position controller could be developed.

The research project concluded the possibility of fusing the lower quality results from the low cost magnetic sensor measurements together with the back EMF estimation, something that now will be explored in this thesis, as well as the development of a position controller to verify the complete concept of a low cost solution of the hybrid stepper motor.

1.2 Purpose

The underlying purpose of this thesis research project is to verify the concept of a low cost solution of high precision hybrid stepper motors. This would open up new possibilities for wider usage of the motor in many different industrial applications. This cheaper rotational actuation would then allow for more affordable and accessible machinery in industry.

1.3 Problem definition

The goal of this thesis is to prove the concept of a low-cost solution for closed loop position control of a hybrid stepper motor. The aim is to by means of sensor fusion combine a back-EMF voltage derived estimation of the position with that of a low cost magnetic rotational position sensor (MRPS) and hence achieve satisfying position estimation. This position estimation will then be used for closed loop position control at a lower cost than conventionally used rotational encoders and resolvers.

In conclusion the thesis project will consist of two major subtasks, namely:

- Sensor fusion - By merging the position estimation from the back-EMF measurements with the measurements from the low cost MRPS sensor produce a better estimate of the position than would have been possible by using any of the estimation solutions exclusively.

- Position error compensating control - Implementing position control logic to overcome the positional drawbacks of the hybrid stepper motor such as the load angle, loss of synchronism, resonance and vibrations as well as increasing energy efficiency.

1.4 Scope

As the work presented in this thesis is a continuation of an older project, it also comes with some already determined factors. Furthermore there exists a lot of diversity in both motor brands, sizes as well as different drivers and sensors that theoretically could be used. In the scope of this project the choice of the motor, the motor driver and the magnetic sensor is accepted as is, and the work will be performed to prove the concept on this specific hardware. Finally, the back
1.5 Scientific contribution

EMF-measurements/estimation is something that has been developed outside the scope of this thesis and will also be accepted as is.

To summarize, these items are predetermined variables of the project.

- The currently used stepper motor and driver.
- The current magnetic sensor.
- The given back-EMF estimation.

Seeing these items and theories were inherited with the ongoing research project and not decided in the scope of this thesis, no major effort will be put on discussing and defending the choice of this hardware or between different theories. Instead they will rather be introduced and explained to give a proper background to their function within this thesis work.

1.5 Scientific contribution

The scientific contribution from this thesis work will consist mainly of research on the non linear problem of sensor fusion on the position estimation signals. The project will also evaluate the proposed sensor fusion and position control in a real test environment to prove the feasibility of the low cost solution practically.

1.6 Outline

The background of the thesis along with the project definition is what has already been presented in chapter 1. The underlying theory used in the thesis work will then be introduced in chapter 2, after which the methodology of how this theory will be used to answer the research question will be presented in chapter 3. Finally the results will be brought forward in chapter 4 and then discussed and concluded in chapter 5.
2.1 Hybrid Stepper Motors

A hybrid stepper motor (HSM) is a DC motor used in numerous industrial areas due to its properties of high torque at low speeds, simple position control in open loop, straightforward construction and overall mechanical robustness. The motor is mainly used in applications where the precision requirements on the drive is not critical and the torque requirements modest. Examples of applications include printers, disk drives as well as basic automation setups and machinery with low torque requirements such as 3D-printers.

The stepper motor is made up of two major components, a stator and a rotor. The stator (the static part) consists of a number of electromagnets, connected in two or more separate phases. As an example figure 2.1 shows a disassembled motor similar to the one used in this thesis having eight electromagnets in two phases. The rotor of a HSM is made of a permanent magnetic material constructed in a teethed fashion. The teeth introduces an effect called the minimal reluctance effect, which states that the magnetic fields want to cross the least permeability as possible. Hence the easiest path for the magnetic fields to travel will be when the teeth on the rotor aligns perfectly with the teeth on the stator, introducing an added torque. The solid rotor which has no physical contact to the stator, and the fact that the moving parts contain no windings make the construction of the HSM very durable and not subjected to much wear during operation.

From this basic construction, movement will be created by inducing magnetic fields in the stator by driving opportune currents through the electromagnets. This will cause the magnetic rotor to start moving and aligning its teeth to the stator teeth. This movement is what is referred to as a step. Using this behaviour desirable rotational movement can be found by exciting the phases in turns to create a continuous movement to a desired position. A good illustration of this
is shown in figure 2.2 where to phase $b$ just got excited, and the rotor will take a step clockwise to align to the induced magnetic fields. The resolution of the motor is depending on the number of teeth on the magnetic rotor. The more teeth the higher resolution though this will also make the motor slower and able to produce less torque.

The most basic way of controlling a stepper motor is by fully exciting each individual phase in turn. However by implementing a more elaborate control of the driving currents partial movement between poles can be implemented. This method is called microstepping with the easiest example being "half-stepping", e.g. allowing both phases to be excited at the same time, positioning the rotor exactly in between the two poles in effect doubling the resolution of the motor. Taking the concept further partial excitation of the two phases will introduce a resulting current vector between two poles/teeth that can be seen as the electrical position vector. This introduces a higher resolution of the motor steps which essentially is dependent on the resolution of the current controller supplying current to the motor phases. Further presentation of the various excitation strategies will be shown in section 2.2.1. Microstepping will always be used in the scope of this thesis.

### 2.1.1 Hybrid stepper motor mathematical model

The equations describing the dynamic model of a hybrid stepper motor can be divided into two parts: i) the equations governing electrical performance, and ii) the equations concerning the mechanical relationships. The final model of a stepper motor derived here is commonly used in related literature and the assumptions made will be introduced briefly.

Initially the electrical equations of the system (the voltage in the two phases) can be modeled as [19]:

**Figure 2.1:** Disassembled stepper motor (ABB).

**Figure 2.2:** Schematic view of a simplified HSM with four electromagnets in two phases and 15 pole pairs in the rotor [15].
\[ u_a = R_i a + L \frac{d i_a}{d t} + e_a \quad (2.1) \]

\[ u_b = R_i b + L \frac{d i_b}{d t} + e_b \quad (2.2) \]

where \( u_a \) and \( u_b \) is the voltage across phases \( a \) and \( b \) [V], \( R \) is the phase resistance [\( \Omega \)], \( i_a \) and \( i_b \) is the current going through each phase [A], \( L \) is the inductance in both phases [H] and finally \( e_a \) and \( e_b \) are the voltages [V] from back electromotive force. In [4] Butcher et. al. argue that the inductance in the coils are position dependent due to a varying gap distance between rotor and stator thus introducing a change in reluctance. This is something that is generally not considered in literature within the field and will also be not be considered in this thesis. Further are these equations based on the assumption that mutual inductances between the phases are zero, as well as the simplification that resistance and inductance are identical for both windings, an assumption that also is common in literature [2][3].

The mechanical equations describing the HSM can be written as [1]:

\[ T_E - T_L - T_D = J \frac{d \omega_r}{d t} + K_v \omega_r \quad (2.3) \]

\[ \omega_r = \frac{d \theta_r}{d t} \quad (2.4) \]

where

\[ T_E = K_m (-i_a \sin(N_r \theta_r) + i_b \cos(N_r \theta_r)), \quad (2.5) \]

in which \( T_E \) is the produced electromagnetic torque \([N \cdot m]\), \( T_L \) is the load torque \([N \cdot m]\), \( T_D \) is the detent torque \([N \cdot m]\), \( J \) is the inertia of the system \([kg \cdot m^2]\), \( \omega_r \) is the rotational speed \([rad/s]\), \( K_v \) is the coefficient of viscous friction \([N \cdot m \cdot s/rad]\), \( K_m \) is the torque constant \([V \cdot s/rad]\), \( N_r \) is the number of rotor teeth (pole pairs) and finally \( \theta_r \) is the rotor position \([rad]\). The detent torque term \( T_D \) originates from the magnetic rotors interaction with the magnetic material of the stator poles. However in their paper [1] Balakrishnan et. al. claim that the detent torque is of a much lesser magnitude than the other torque components and can be neglected.

The back-EMF voltages in the electric equations can be rewritten according to Schweid et. al. [16] as:

\[ e_a = -K_m \omega_r \sin(N_r \theta_r) \]
\[ e_b = K_m \omega_r \cos(N_r \theta_r) \quad (2.6) \]

To describe the system in state space form, the state has to be declared. Seeing the terms of current derivatives in the electrical equations (2.1) and (2.2) the state can be chosen advantageously as:

\[ x^T = [i_a \ i_b \ \omega_r \ \theta_r] \quad (2.7) \]
Using equations (2.5) and (2.6) (as well as neglecting the detent torque) the system equations (2.1)-(2.4) can be rewritten in non-linear state space form as:

\[
\begin{align*}
\frac{di_a}{dt} &= \frac{1}{L} \left( u_a - Ri_a + K_m \omega_r \sin(N_r \theta_r) \right) \\
\frac{di_b}{dt} &= \frac{1}{L} \left( u_b - Ri_b - K_m \omega_r \cos(N_r \theta_r) \right) \\
\frac{d\omega_r}{dt} &= \frac{1}{J} \left( -K_m i_a \sin(N_r \theta_r) + K_m i_b \cos(N_r \theta_r) - K_v \omega_r - T_L \right) \\
\frac{d\theta_r}{dt} &= \omega_r
\end{align*}
\]  

(2.8)

where the voltages \( u_a \) and \( u_b \) can be considered input for the system and the load torque \( T_L \) can be treated as a disturbance to the system.

### 2.1.2 Electrical degrees

When discussing the hybrid stepper motor two different kinds of angles have to be considered: the rotor position denoted \( \theta_r \), and the phase of the electrical field denoted \( \theta_e \). The electrical degrees is the position of the current resultant vector between two closest poles and is vital when considering the hybrid stepper motor in microstepping control. The relationship between rotor and electrical position can be described as:

\[
\theta_e(t) = N_r \theta_r(t)
\]  

(2.9)

A more illustrative explanation of the electrical degree will be given in the section about microstepping, section 2.2.1.

### 2.1.3 \( ab - dq \) transformation

To simplify the equations governing stepper motors the \( dq \) transformation is sometimes used, for example when applying field oriented control (FOC) strategies on the hybrid stepper motor (which will be introduced briefly in section 2.2.3).

The transformation translates the fixed \( ab \)-coordinate system aligned with the phases \( a \) and \( b \) of the stator to instead be rotating along with the rotor of the motor. The two \( dq \)-coordinates are the direct-axis component which is aligned with the magnetic flux vector of the permanent magnet rotor and the quadrature-axis component being perpendicular to the direct-axis coordinate (leading the direct-axis by 90 electrical degrees). The \( dq \)-transformation is given by the following relation [14] [13]:

\[
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_e(t)) & \sin(\theta_e(t)) \\
-\sin(\theta_e(t)) & \cos(\theta_e(t))
\end{bmatrix} \begin{bmatrix}
i_a(t) \\
i_b(t)
\end{bmatrix},
\]

(2.10)

additionally the inverse transform back into \( ab \)-reference is then given as:
\[
\begin{bmatrix}
i_a(t) \\
i_b(t)
\end{bmatrix} = 
\begin{bmatrix}
\cos(\theta_e(t)) & -\sin(\theta_e(t)) \\
\sin(\theta_e(t)) & \cos(\theta_e(t))
\end{bmatrix}
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix}.
\] (2.11)

### 2.1.4 Load angle and its relation to electromagnetic torque

As the main objective of this research is to control the rotation and position of the motor, it is meaningful to consider the electromagnetic torque produced by the motor. The torque is determined by the cross product between the stator flux linkage vector \( \Psi_s \) and the stator current resultant vector \( i_s \) [17]:

\[ T_e = \Psi_s \times i_s. \] (2.12)

If saturation effects are neglected, the total stator flux linkage vector can be expressed as the sum of the magnetic rotor flux \( \Psi_r \) and the two flux linkages originating from the two phases of the stator. Thus equation (2.12) can be rewritten (in dq-reference frame) as:

\[ T_e = (\Psi_r + i_d L_d + i_q L_q) \times i_s \] (2.13)

Performing this cross-product an expression for the motor torque is found, given as a function of current vector \( i_s \) and the load angle \( \delta \) (defined as the angle between the current vector \( i_s \) and the rotor flux \( \Psi_r \), (see figure 2.3):

\[ T_e = \Psi_r i_s \sin(\delta) + \frac{L_d - L_q}{2} i_s^2 \sin(2\delta). \] (2.14)

The first term of (2.14) describes the torque produced by the interaction between rotor flux \( \Psi_r \) and the direct current \( i_s \) while the second term describes the minimum reluctance effect. Due to the sinusoidal dependence of the load angle, the produced torque is zero if there exists no load angle. This pinpoints the fact why the load angle is needed, or in a sense why it will always be present during movement of a stepper motor. Due to the sinusoidal dependence the produced torque will increase with an increasing load angle until the torque reaches its maximum point, after which the torque will start rapidly decreasing and a loss of step and synchronism is plausible. Seeing the load angle is depending on the load torque applied to the motor, the relationship can be seen as the produced torque will increase (as the load angle increases) to meet the applied load torque.

### 2.2 Position control of hybrid stepper motors

When discussing position control for the hybrid stepper motor there are a few aspects to take into consideration. Firstly the means of how one actually excites the different phases of the motor to create movement can be done in a few different ways. An overview of the most common phase excitation strategies will be briefly introduced. After this an introduction to how the position control essentially is implemented as well as finally some examples of different closed loop control strategies will be presented.
2.2.1 Excitation strategies

Full-stepping

The most basic version of exciting the two phases of the stepper motor is called full-stepping. In essence the two phases will be excited individually in sequence, causing the rotor to align to the currently induced magnetic field. This method is the simplest way of controlling a stepper motor, as the supplied current in the coils does not have to be regulated or controlled but rather throttled on and off. A brief illustration of the full-stepping excitation principle is shown in figure 2.4 on a simplified motor model with two pole pairs.

Half-stepping

An upgrade from the full-stepping strategy is to allow both phases to be excited simultaneously. As a result, when both phases are inducing a magnetic field, the rotor will align itself between these two fields, as both poles on the rotor will try to align to its closest corresponding magnetic field from the stator. This implementation then doubles the resolution of steps for the motor, as a discrete step is added in between each pole compared to the full stepping implementation. An illustration of the excitation strategy when half-stepping is shown in figure 2.5. While increasing the resolution of the motor, it does however require both phases to be energized simultaneously, reducing the efficiency of the motor.

Also worth noting, is that full-step excitation could be utilized with both phases excited simultaneously, (going between state two and four in figure 2.5) increasing the electromagnetic torque produced, however at lower efficiency. That is because while the torque produced by the magnetic flux linkages double, the benefits of minimum reluctance effect gets lost as the teeth of the rotor are not aligning perfectly with the stator teeth.
2.2 Position control of hybrid stepper motors

Figure 2.4: Illustration of full step excitation scheme for a hybrid stepper motor.

Figure 2.5: Illustration of half step excitation scheme for a hybrid stepper motor.
Micro-stepping

The strategy micro-stepping is found by extending this concept of half-stepping. Instead of fully exciting both coils simultaneously or individually, partial excitation of the coils is introduced to allow the rotor to align at even smaller increments between the poles. If for example phase \( a \) is excited with 75% of the maximum current magnitude, while phase \( b \) is excited with 25% of the current magnitude the rotor will align itself proportionally between the two poles. With this method the resolution of the stepper motor is increased to the resolution of the current generation through the coils.

A common method of micro-stepping is called sine/cosine micro-stepping, in which the currents supplied to the coils will be sinusoidal with 90 degree phase shift between them. This relationship is illustrated in figure 2.6. As the magnetic fields will align according to the current resultant \( i_s \) the relationship between the electric angle \( \theta_e \) and the currents in the two coils are then:

\[
\begin{align*}
    i_a &= i_0 \cos(\theta_e) \\
    i_b &= i_0 \sin(\theta_e)
\end{align*}
\]  

where \( i_0 \) is the magnitude of the current resultant vector \( i_s \). This expression also further illustrates the relationship between electrical degrees to the mechanical degrees introduced in equation (2.9). As compared to the true motor, instead of finalizing a full mechanical rotation after one electrical rotation as in this simplified model it will rather reach next pole pair and start rotating towards the next closest pole. So for this simple model with only one pole pair the relationship then turns out as \( \theta_e = \theta_r \). But for a motor with 50 pole pairs it must instead rotate 50 electrical rotations (a full electrical revolution between each pole pair) to finalize a mechanical revolution.

![Figure 2.6: Illustration of half step excitation scheme for a hybrid stepper motor.](image)

2.2.2 Open loop control

As previously discussed, what is essentially needed when controlling the position of a hybrid stepper motor, is to control the produced electromagnetic torque. By using the micro-stepping currents found in the previous section in a reference
notation with the expression for produced torque (2.5) in the means of current electrical position the following relationship is found:

\[ T_E = K_m i_0 \left( -\cos(\theta_{e,ref}) \sin(\theta_e) + \sin(\theta_{e,ref}) \cos(\theta_e) \right). \]  (2.16)

Using the trigonometric identity that \( \sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b) \), equation (2.16) can be rewritten as:

\[ T_E = K_m i_0 \sin(\theta_{e,ref} - \theta_e) \]  (2.17)

This then leads to no produced torque and a stable position when the actual electrical position has reached the reference electrical position.

### 2.2.3 Closed loop control

While the introduced open loop control opens up the possibility of controlling the position of the stepper motor it has three primary drawbacks. With no information about the current position of the rotor, no feedback is given about what kind of load the motor is subject to. As seen from equation (2.17) the torque produced is directly proportional to the magnitude of current supplied through the coils. Because of this the current magnitude in open loop control is set at the maximum rating to deal with as high as possible load and torque disturbances. This kind of over-compensation is essentially then resulting in low energy efficiency because the motor is run at maximum produced torque and high currents even if there is no load and hence no need of producing torque. Furthermore if the torque produced does not succeed to meet the demand of the load, the load angle will rise to exceed the point of maximum torque production with a dropping torque and resulting loss of synchronism. The rotor will then align to the next pole pair that manages to meet the load torque. This loss of synchronism is also often referred to a loss of step or skipping steps. Seeing no feedback about the true rotor position exists this will introduce a constant error because the open loop position control thinks the rotor is still aligned to the electrical reference position. Finally, as it has already been established, the position error from the load angle is something that will always be needed to produce torque and any movement at all. Even if this error is small (generally less than 0.9 deg) this error might be detrimental in, for example, a multi joint robot where position errors are additive.

To come to means with these problems closed loop control have to be used. In the coming section a few examples of interesting closed loop control implementations will be presented.

**P/PI speed controlled position control**

A simple solution to close the loop of the hybrid stepper motor is to utilize the open loop micro-stepping position control proposed in subsection 2.2.2. By integrating the position reference, the position control translates into a speed controller (with the integrator adding up, it is in a sense a continuously moving position reference).
By implementing the PI control on the current magnitude and a P control on the position reference but in speed control sense the reference currents can be written as functions of the position error $\theta_{\text{ref}} - \theta$ as:

$$
\begin{align*}
    i_{a,\text{ref}} &= K_m \left(k_p (\theta_{\text{ref}} - \theta) + k_i \int_0^t (\theta_{\text{ref}} - \theta) \, dt \right) \cdot \cos \left( \int_0^t (\theta_{\text{ref}} - \theta) \, dt \right) \\
    i_{b,\text{ref}} &= K_m \left(k_p (\theta_{\text{ref}} - \theta) + k_i \int_0^t (\theta_{\text{ref}} - \theta) \, dt \right) \cdot \sin \left( \int_0^t (\theta_{\text{ref}} - \theta) \, dt \right)
\end{align*}
$$

With this then the torque equation (2.17) can be rewritten as:

$$
T_E = K_m \left(k_p (\theta_{\text{ref}} - \theta) + k_i \int_0^t (\theta_{\text{ref}} - \theta) \, dt \right) \cdot \sin \left( \int_0^t (\theta_{\text{ref}} - \theta) \, dt - \theta_e \right)
$$

**Field Oriented Control (FOC)**

Field oriented control (FOC) is a common strategy in hybrid stepper motor control. It uses the park transformation (into $dq$-reference frame) to control the position of the rotor by keeping the direct current at zero and then controlling the quadrature current. Depending on the implementation it might also control the magnitude of the current to increase the torque produced. Kim et.al. [12] proposes a new commutation scheme in which the $dq$ transformation does not have to be used, and also proves that the control still takes the form as a conventional FOC. In their paper the authors propose a PID control law with velocity feed-forward in which the torque is viewed as the system input as:

$$
\tau = k_p (\theta_{\text{ref}} - \theta) + k_i \int_0^t (\theta_{\text{ref}} - \theta) \, dt + k_d (\dot{\theta}_{\text{ref}} - \dot{\theta}) + B \omega_{\text{ref}} + J \dot{\omega}_{\text{ref}}
$$

However seeing that the torque is not a valid input to the system, but it rather has to be controlled by the currents, the authors propose a commutation scheme for translating the torque control into deciding the reference currents as:

$$
\begin{align*}
    i_{a,\text{ref}} &= -\frac{\tau}{K_m} \sin(N_r \theta) \\
    i_{b,\text{ref}} &= \frac{\tau}{K_m} \cos(N_r \theta)
\end{align*}
$$

With these reference currents it is then showed that the proposed commutation scheme is equivalent to conventional micro-stepping. By applying the Park transform, (equation (2.10)) on the proposed currents in (2.21) the reference direct and quadrature current then becomes:
\[ i_{d,\text{ref}} = 0 \]
\[ i_{q,\text{ref}} = \frac{\tau}{K_m} \]  

(2.22)

where it can be seen that the proposed reference currents actually take the form of a conventional FOC control without the need of first transforming the measurements into \( dq \) reference frame and then doing an inverse transformation to later translate the desired \( dq \) currents into \( ab \)-reference to actuate the currents to the coils.

### 2.3 Back-EMF based sensorless position estimation

To implement the closed loop position control some feedback of the true rotor position is required. One suggested low cost solution for this is a sensorless estimation based on back electromotive force (EMF).

By definition the load angle \( \delta \) is the difference between the desired rotor position (in the form of the current vector \( i_s \)) and the true position (in the form of the rotor magnetic flux \( \Psi_r \)). This relationship is clearly illustrated in figure 2.3. As such, seeing that the supplied current vector is already known (since being measured in the current control loop on the motor driver hardware), the problem of finding the true position of the rotor can be reduced to the problem of finding a way of estimating the load angle.

Derammellaere et. al. [6] suggest using a solution that utilizes the back electromotive force (back EMF). The back EMF originates from Lenz’s law, in essence that as the magnetic fields from the rotor is rotating past the windings of the electromagnets in the stator, it will produce a current in these coils. Essentially this is then counteracting the current sent in the coil to produce movement in the first place. The back-EMF voltage vector \( e_s \) can be written as (voltage being the voltage drop caused by the induced back-EMF):

\[ e_s = C \frac{d\Psi_r}{dt} \]  

(2.23)

where \( C \) is a constant.

This implies a phase lead of \( \pi/2 \) between the back-EMF vector \( e_s \) and the magnetic flux vector \( \Psi_r \) [6]. Therefore it follows that the angle between the back-EMF vector \( e_s \) and the current vector \( i_s \) is \( \pi/2 - \delta \), which in turn leads to an expression of the load angle as:

\[ \delta = \frac{\pi}{2} - (e_s - i_s) \]  

(2.24)

Seeing that the resultant current vector \( i_s \) can be easily found by measuring the currents supplied to the motor, what is left to find the position of the rotor is determining the back emf vector \( e_s \). The relationship between load angle and back EMF can be seen in figure 2.7.
Figure 2.7: Illustration describing the back emf vector [6].

From the phase voltage equation (2.6) the resultant back EMF could be found from rewriting the equation:

\[ u_s = R_s i_s + L_s \frac{d}{dt} i_s + e_s \]  

(2.25)

However determining the back EMF with the help of the voltage and current will lead to noisy signals especially due to the derivative term of the current resultant \( i_s \). Instead Deramellaere et. al. [6] suggest to rewrite (2.25) in the frequency domain as:

\[ E_s(j\omega) = U_s(j\omega) - j\omega L_s I_s(j\omega) - R_s I_s(j\omega) \]  

(2.26)

With this the authors later show that with knowledge about the fundamental current and voltage, the fundamental back EMF can be estimated and used to determine an estimated load angle as [6]:

\[ \hat{\delta} = \frac{\pi}{2} - (\angle(E_1) - \angle(I_1)) \]  

(2.27)

where the subscript \( 1 \) indicates the fundamental component. While this estimation scheme neglects higher harmonic current and voltage components, it will only have a minor contribution to the average torque. When this load angle is computed the true position of the rotor can then be found as the load angle plus the reference position (current reference), essentially coming from the definition of load angle being the offset from the desired position.

Finally an important notion regarding the back EMF is to consider the equation concerning EMF from the system model (equation 2.6), where it can be seen that the back EMF in the coils are directly proportional to the rotational speed \( \omega_r \). Hence as the magnitude of EMF voltage will be small at lower speeds, the fundamental EMF measured in the two coils will be much more corrupted with noise and disturbances at lower speeds making the angle between the components un-
2.4 Magnetic Rotary Position Sensor - MRPS

The other suggested position feedback source presented in the previous thesis [15] was to use a MRPS sensor. The magnetic rotary position sensor (MRPS) is in essence a set of Hall elements mounted in quadrature. The sensor is coupled with a permanent magnet mounted on the rotor parallel to the fixed MRPS. As the magnetic flux from the magnet rotates over the static sensor it will give rise to varying voltages over the different hall elements which then are translated to a discrete rotational position. Further elaboration on the MRPS sensor is found in the previous thesis [15].

2.5 Sensor fusion

Within the scientific community there has been some ambiguous use of the term sensor fusion in technical literature. The phrase has a multitude of conceptually similar synonyms as for example "information fusion", "data fusion" or "multi-sensor data fusion". This is due to the existence of different fields of science using the same words and underlying ideas however applying it within their specific applications and own terminology. From higher level perspectives, an example of a sensor providing data may be a person, who are producing opinions that will be the actual information which will have to be fused. This is conceptually similar to an electric sensor producing quantified data to fuse but still very different in application. Effort have however been made [8][18] to formalise the field of "fusion" and work have been conducted to establish a generalised vocabulary within the community to reduce ambiguity and form a common framework for "fusion" ranging over all different fields.

In his paper from 2001, Dasarthy defines a general concept for fusion of any kind of information as [5]:

"Information Fusion encompasses theory, techniques and tools conceived and employed for exploiting the synergy in the information acquired from multiple sources (sensor, databases, information gathered by human, etc.) such that the resulting decision or action is in some sense better (qualitatively or quantitatively, in terms of accuracy, robustness, etc.) than would be possible if any of these sources were used individually without such synergy exploitation."

Building on this definition Elmenreich later uses this concept in defining sensor fusion as being a subset of the information fusion domain [7]:

"Sensor Fusion is the combining of sensory data or data derived from sensory data such that the resulting information is in some sense better than would be possible when these sources were used individually"
This formulation of sensor fusion is what will be used in this thesis, in taking two signals describing the current position of the stepper motor, into producing an improved and more accurate estimation of the position than would be available with any of the two position estimations individually.

As mentioned previously, the term sensor fusion is generally more of a concept and the actual implementation of the fusion can look very different. Hence there is a need of finding sensor fusion algorithms that will be suitable for the application at hand. A state-of-the-art review made in 2011 [11] examined the sensor fusion field and discussed different algorithms in regard to their benefits and shortcomings. A few interesting algorithms are presented below.

### 2.5.1 Extended Kalman Filter - EKF

The basic Kalman filter was introduced in the 1960s [10] as an optimal filter for linear systems in the least square mean error sense. Building from this introduction it has since become a core algorithm in state estimation and has been deeply studied and applied throughout the field [11]. The Kalman filter can be proven to be optimal for a linear system having normally distributed random variables (such as measurement noise and modeling errors). However the assumption of linearity is rarely true in real world applications and hence an extension to the filter was needed.

The easiest adaptation of the Kalman filter to nonlinear problems is by using Taylor series expansion to linearize the system. Nevertheless while the linearization allows the system to be treated with Kalman filter theory, it also introduces errors in the estimation. Beyond the obvious errors introduced from linearization, the random variables which are assumed Gaussian will also be transformed through the nonlinear system resulting in a non-Gaussian variable. These effects results in making the EKF suboptimal and approximative when dealing with non-linear problems and the performance of the filter will be dependent on the systems non-linearities.

The algorithm is implemented as follows. First, a prediction of the state is found by using the linearized process function:

\[
\hat{x}_{k+1|k} = \hat{x}_{k|k} + t \cdot f(\hat{x}_{k|k}, u_k)
\] (2.28)

Then the prediction error covariance matrix is predicted as:

\[
P_{k+1|k} = F_k P_{k|k} F_k^T + Q_k
\] (2.29)

where \(F_k\) is the Jacobian matrix of the system function \(f\) and \(Q_k\) is the constant process error covariance matrix. Following this the Kalman gain matrix \(K\) is calculated as:

\[
K_{k+1} = P_{k+1|k} H_k^T (H_k P_{k+1|k} H_k^T + R_k)^{-1}
\] (2.30)

where \(H_k\) is the Jacobian matrix of the output function \(h\) and \(R_k\) is the constant measurement error covariance matrix. Next, the prediction of the state is corrected by using the calculated Kalman gain as:
Finally the prediction error covariance matrix is corrected according to:

\[ P_{k+1|k+1} = P_{k+1|k} - K_{k+1} H_k P_{k+1|k} \]  (2.32)

This is then repeated for each iteration. Important to note is as the process function is not linear, both the process function \( f \) and its Jacobian \( F \) has to be re-evaluated every iteration. This results in longer computational times than the classic Kalman filter.

### 2.5.2 Unscented Kalman Filter - UKF

To reduce the erroneous impact of the linear approximation and the computational times of the EKF discussed in the previous section a new algorithm was presented for the filtering of nonlinear problems [9]. The unscented theory introduced by Julier and Uhlmann treats the nonlinearity not by linearization but instead instead parametrizes the mean and covariance of the probability distributions of the transformation. This is done by choosing a specific set of weighted points which will be transformed through the nonlinear function, and then finding the mean and covariance of the resulting transformed data points. In a sense the difference between the EKF and the UKF is that the unscented theory approximates the Gaussian distribution resulting after the nonlinear transformation, instead of approximating the nonlinear function or transformation. In their introduction of the new filter the authors comment on the methods resemblance of a Monte Carlo approach, but point out the fundamental difference that for the UKF the data points are not chosen randomly but according to a deterministic algorithm [9].

#### Unscented transformation

The unscented transformation algorithm is implemented as follows [9]. A \( n \)-dimensional random variable \( \mathbf{x} \) with mean \( \bar{x} \) and covariance \( P_{xx} \) is approximated by first choosing \( 2n + 1 \) weighted points according to:

\[
\begin{align*}
\chi_0 &= \bar{x} \\
\chi_i &= \bar{x} + \left( \sqrt{(n + \kappa)P_{xx}} \right)_i \\
\chi_{i+n} &= \bar{x} - \left( \sqrt{(n + \kappa)P_{xx}} \right)_i \\
W_0 &= \kappa/(n + \kappa) \\
W_i &= 1/2(n + \kappa) \\
W_{i+n} &= 1/2(n + \kappa)
\end{align*}
\]  (2.33)

where \( \kappa \) is a real number (\( \kappa \in \mathbb{R} \)), \( \left( \sqrt{(n + \kappa)P_{xx}} \right)_i \) is the \( i \)th row or column of the matrix square root of \( (n + \kappa)P_{xx} \) and \( W_i \) is the weight connected with the \( i \)th point. The transformation through the nonlinear function then follows:

\[ \gamma_i = f[\chi_i], \]  (2.34)
in which a set of transformed sigma points are calculated. The mean is then
determined as the weighted average of the transformed points:

$$\bar{y} = \sum_{i=0}^{2n} W_i \gamma_i.$$  (2.35)

Finally the covariance is found as the weighted outer product of the transformed
points:

$$P_{yy} = \sum_{i=0}^{2n} W_i (\gamma_i - \bar{y})(\gamma_i - \bar{y})^T.$$  (2.36)

**Implementation of unscented filter**

The unscented theory can be implemented for filtering in the same fashion as a
Kalman filter. However instead of the linearization used in the EKF, unscented
transformation is used to approximate the nonlinearity. The procedure in appli-
cation is then the following [20].

First, the state vector is augmented with process and measurement noise terms:

$$x^a(k) = \begin{bmatrix} x(k) \\ v(k) \\ m(k) \end{bmatrix}.$$  (2.37)

Then the process model is rewritten to be a function of the augmented state $x^a(k)$:

$$x(k+1) = f[x^a(k), u(k)].$$  (2.38)

The calculations are initialized by determining the initial state andcovariance,
i.e.,

$$\hat{x}_0^a = [x_0^T 0 0]^T,$$  (2.39)

and

$$P_0^a = \begin{bmatrix} P_0 & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_m \end{bmatrix}.$$  (2.40)

For timesteps $k \in \{1, \ldots, \infty\}$ the filtering is performed by calculating a matrix of
sigma points as equation (2.33):

$$x^a_{k-1} = \begin{bmatrix} \hat{x}_{k-1}^a, \hat{x}_{k-1}^a \pm \sqrt{(n + \kappa)P_{k-1}^a} \end{bmatrix}.$$  (2.41)

The sigma points are then transformed through the nonlinear process model:

$$x^a_{k|k-1} = f[x^x_{k-1}, x^y]$$  (2.42)

Then the prediction of the state is calculated as the weighted average of the trans-
formed sigma points as in equation (2.35):
\[ \hat{x}_{k|k-1} = \sum_{i=0}^{2n} W_i \chi_{i,k|k-1}^x. \] (2.43)

The covariance of the predicted state is then calculated as:

\[ P_k^- = \sum_{i=0}^{2n} W_i \left( \chi_{i,k|k-1}^x - \bar{x} \right) \left( \chi_{i,k|k-1}^x - \bar{x} \right)^T. \] (2.44)

Transform the sigma points through the measurement function:

\[ \gamma_{k|k-1} = h\left[ \chi_{i,k|k-1}^x, \chi_{i,k|k-1}^n \right], \] (2.45)

which is used to calculate the mean of the measurement:

\[ \hat{y}_k^- = \sum_{i=0}^{2n} W_i \gamma_{i,k|k-1}. \] (2.46)

Then the covariance of the output estimation is calculated as:

\[ P_{yy}^- = \sum_{i=0}^{2n} W_i \left( \gamma_{k|k-1} - \hat{y}_k^- \right) \left( \gamma_{i,k|k-1} - \hat{y}_k^- \right)^T. \] (2.47)

Determine the cross correlation matrix:

\[ P_{xy}^- = \sum_{i=0}^{2n} W_i \left( \gamma_{k|k-1} - \hat{y}_k^- \right) \left( \chi_{i,k|k-1}^x - \hat{y}_k^- \right)^T. \] (2.48)

This is then used to calculate the Kalman gain as:

\[ K_k = P_{xy}^- P_{yy}^{-1}. \] (2.49)

The Kalman gain is then used to correct the prediction of the state as:

\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k^-) \] (2.50)

Finally the estimation error covariance matrix is updated for the next iteration of the filter according to:

\[ P_k = P_k^- - K_k P_{yy} K_k^T. \] (2.51)
To answer the research question at hand, the different theories presented in chapter 2 will be used together in a combined system. In the coming section the setup of this system will be introduced. Further will the implementation of both the sensor fusion and position control be showcased and tested offline before being evaluated experimentally in a laboratory set up.

### 3.1 Overview of the complete control solution

The low cost solution suggested for position control of the hybrid stepper motor will utilize both the low cost MRPS sensor and the presented sensorless estimation via back-EMF for positional feedback. The two signals will then be fused using a sensor fusion filter to improve the accuracy of the current rotor position estimation. Finally this position estimate will be compared to a position reference to find a control error which will be the control signal of a closed loop position control algorithm. An overview of the system is illustrated in figure 3.1.

### 3.2 Simulation environment of the hybrid stepper motor

To achieve an environment of convenient offline testing of the system a simulation environment was set up in SIMULINK. For simulating the stepper motor the "Stepper Motor" block from SimPowerSystems library was used. A model of two H-bridges for controlling the currents in the two phases was also implemented with a current control loop in a similar fashion to what will be done in the experimental solution. An example of the resulting SIMULINK implementation with the basic open loop position control presented in 2.2.2 is shown in figure 3.2.
3.3 Offline verification of sensor fusion algorithm

Before the full system is realized in the experimental setup it is also reasonable to verify the implementation of different subsystems by themselves. For the sensor fusion algorithms this will be done in the simulation environment presented in section 3.2.

These simulations will not be done in an attempt to directly assess the performance of the sensor fusion but rather in the context of verifying the basic functionality and a correct implementation of the filters. This is mainly because the focus of the research is to verify the system experimentally. Therefore no major time and effort will be spent on modeling the errors needed to achieve a proper simulation model for acceptable assessment nor will the filters be tuned to excel in simulation for that matter based on the same reasoning.

To verify the implementation and function of the sensor fusion algorithms they are first implemented in SIMULINK, then two position measurements are created from the true simulated position output variable with some added bias and noise:
\[ \theta_1 = \theta + \alpha_1 + w_1 \]
\[ \theta_2 = \theta + \alpha_2 + w_2 \]  

(3.1)

where \( \alpha \) is a certain bias offset and \( w \) is white measurement noise. These two measurements will then be run through the sensor fusion algorithm.

### 3.4 Experimental evaluation of full control solution

When the different subsystems have been tested offline it is time to verify the full system experimentally. In the laboratory setup a high-precision rotary encoder will be treated as the true position of the rotor. The two different subtasks will be run separately with the encoder signal as comparison. For example the sensor fusion algorithms will be tested at different speeds and then the produced filtered signal and its estimation error will be compared to the true position.

Further will the position control be tested first with the encoder as feedback to allow for a fair assessment of the actual control algorithm in isolation, before implementing the full system for testing the complete low cost solution.

### 3.5 The experimental laboratory setup

The lab rig that will be used in the experiment was developed in the previous research project [15] with a real time FPGA as the interface between the computer control run in SIMULINK and the hardware (current drivers/measurements etc.). The hardware setup consists of a hybrid stepper motor that has a MRPS and permanent magnet mounted on its shaft. On the axle there is also mounted a high precision encoder as well as a torque sensor and a hysteresis brake for introducing loads.

The motor is driven using a commercial drive board, in essence an inverter driver chip controlling two H-bridges/8 MOSFETs along with some current measurement logic and protection. For interfacing the system with SIMULINK we use a realtime FPGA computer of the brand SpeedGoat, that reads various SPI-communication from the sensors and supplies the PWM-signals controlling the driver chip.

A photo of the lab rig is shown in figure 3.3.
Figure 3.3: The lab rig with its different components.
4.1 Results - simulation position control

A quick verification of the simulation environment was carried out by simulating the different position control implementation presented in section 2.2. A plot of the closed loop FOC position control from section 2.2.3 is shown in figure 4.1.

Figure 4.1: Simulation of FOC closed loop position control.
4.2 Results - simulation sensor fusion

To verify the implementation of the sensor fusion algorithms they were implemented in SIMULINK and fed with distorted measurement signals as was presented in section 3.3. A result from running the EKF algorithm with measurement distortions $\alpha_1 = 0.2, w_1 = 0.02$ and $\alpha_2 = -0.3, w_2 = 0.01$ (where the noise variable is the covariance of the Gaussian noise) is shown in figure 4.2. Some quick error analysis is presented in table 4.1.

\[ \text{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}(\hat{y}_t - y_t)^2}{n}} \quad (4.1) \]

<table>
<thead>
<tr>
<th>Filtered estimation</th>
<th>Measurement 1</th>
<th>Measurement 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.0543</td>
<td>0.2447</td>
</tr>
</tbody>
</table>

*Table 4.1: Root mean square error of the different position quantities.*

4.3 Back-EMF based sensorless position estimation

In section 2.3 it was noted that the back EMF is directly proportional to the rotational speed. This was very noticeable when running experiments in the lab setup. In figure 4.3 an example of this is shown where the magnitude of the measured back EMF voltages at 100, 50, 25 and 10 rpm is showcased.
4.3 Back-EMF based sensorless position estimation

Figure 4.3: EMF measurements at 100, 50, 25 and 10 rpm.

At the lower speeds, it can be noticed that the back EMF measurements are corrupted with noise. To illustrate this more clearly figure 4.4 shows the difference between the two desired quantities of EMF and current angle at 100 rpm versus the same signals at 10 rpm. This noisy behavior makes the position derived from back EMF useless at lower speeds as the spikes shown in figure 4.4 leads to the load angle estimation jumping around of the full spectrum of the quantity (between ±180 electrical degrees).
Figure 4.4: Current and EMF angles at 100 rpm (top) and 10 rpm (bottom).

To show the effect this behavior has on the final position estimation the position error from both the MRPS and the sensorless algorithm are compared at different speeds (100, 50, 10 and 1 rpm) in figure 4.5. Here it can be seen how the sensorless algorithm produces very noisy results especially at 10 rpm and 1 rpm. Notably the estimated load angle will at these speeds introduce an error of \(\pm 3.6 \text{ deg}\) as a result of the noisy EMF measurements. This error is essentially due to the fact that the estimation of the load angle is noisy and taking any given value in its full spectrum of a complete electrical reference revolution (\(360/50 = 7.2 \text{ deg}\)).
4.4 Position errors at higher speeds

Another interesting thing to note is that the MRPS performs increasingly worse with higher rotational speeds. The reasons for this may be either the least square fitting of the non linear error (introduced in the previous thesis [15]), or even some lag in the actual sensing of the rotating magnetic fields. This intuition comes from the apparent linear behavior of increasing error with increasing speeds. An example of the position error of the MRPS at 100, 200, 300 and 400 rpm is shown in figure 4.6.

These results suggest that the MRPS will produce better results at lower speed, but that the sensorless method should be able to produce better results at higher
speeds, as the back EMF measurements will be less affected by noise and hence more accurate at higher speeds.

However when testing this hypotheses it was indeed found that the sensorless method did produce better results than the MRPS above 100 rpm; at the same time somewhere around 300 rpm that signal also got noisier and performing increasingly worse towards 400 rpm. This behavior can be seen in figure 4.7.

![Position error sensorless estimation - increasing speeds](image)

**Figure 4.7**: MRPS position error at 100, 200, 300 and 400 rpm.

When examining the cause of this it was found that while the back EMF signal worked flawlessly at these higher speeds giving nice signals with almost no noise, instead the other half of the sensorless estimation had issues. The current controller did not manage to supply the desired current properly at these higher speeds. This is shown in figure 4.8, where the current behavior in phase a is shown at a rotational speed of 400 rpm. Basically the current controller does not manage to provide the desired currents at the higher frequency because of the inductive load, and while this behavior still makes the motor run decently, it introduces problems to our estimation scheme. Essentially this current signal introduces the same kind of error that the EMF signal causes at lower speeds, with a noisy estimation as a result. Specifically what happens is that the difference between the current angle and back EMF angle is not static for a full period of the signal as they should be (consider the behavior from the 100 rpm measurements in figure 4.4 how the angle quantities should essentially look like true triangular waves while the 400 rpm one in figure 4.8 clearly contains oscillations.)
4.5 Results - experimental evaluation sensor fusion

So, as it was found that the sensorless method based on back EMF measurement did not produce any meaningful information at lower speeds it would not make any sense to use that signal in a fusion at lower speeds. Hence, the MRPS will be used exclusively at lower speeds, and in figure 4.9 a run in the low speed range (between 75 and 1 rpm) is shown, comparing the MRPS measurement with the MRPS measurement run through the EKF filter.

What can be seen in this test is that the EKF estimates the position more accurately than the raw signal, and that the error at 75 and 50 rpm is less biased. Furthermore, the estimate is less noisy than the raw signal, something that is seen clearly at 25 and 1 rpm, where the output of the filter is following the behavior as the raw MRPS signal (however with less noise). Also included in the plot is the root mean squared error of the two different measurements, where it can be seen that the EKF almost halves the RMSE.

Following this an attempt to fuse the two position signals at higher speeds is shown in figure 4.10.
Position error - lower speeds

RMSE mrps = 0.073298
RMSE ekf = 0.036953

Figure 4.9: EKF filtered MRPS measurements at 75, 50, 25 and 1 rpm.

Position error - higher speeds

RMSE mrps = 0.25649
RMSE emf = 0.21645
RMSE ekf = 0.064086

Figure 4.10: Sensor fusion between the sensorless method and the MRPS sensor for 75, 150, 250 and 350 rpm.

In this plot it is clearly seen that the RMSE is significantly lower for the sensor fusion filtered signal comparing to the two input position signals. This is also visually obvious when examining the three plotted signals. While the sensor fusion scheme does improve the estimate of the two sensors, it is also important to realize how the EKF/fused signal estimates the position in between the erroneous MRPS and sensorless signal. This makes the improvement by the filter very dependent of the two signals being treated. If for example both of the signals would be diverging towards a positive error, the improvement of the filter would not be as significant as the result showcased in figure 4.10.
4.6 Results - experimental evaluation position control

Unfortunately the thesis came to a conclusion before any extensive results could be produced regarding the position control part of the project. This was partly caused by some delays working with a problem of the current control of the lab setup, a problem that is presented in the appendix. Therefore, as this problem was solved at a very late stage of the thesis, the decision was made to focus more on the sensor fusion part of the project and the position control part was put on hold. Some position control was used experimentally and in figure 4.11 an example of the P/PI control is shown for some step responses.

![Figure 4.11: Position control (P/PI) run with the sensor fusion estimated position.](image)

The position reference used for the first position control runs (figure 4.11) is not a good example of a position requirement used in an industrial environment. Position applications in industry usually contain some path-planning, making the position reference continuous and compliant to certain parameters for example acceleration limitations of the motor etcetera. To be able to experiment with the position control using one of these more applicable position references the position reference for one axis of an industrial robot setup was taken from a simulation program and used as trial position reference. The reference is also multiplied by a factor of 10, essentially modeling an added gearbox with the ratio 10:1. Testing the position control (a PID controlled speed control showcased in section 2.2.3) using this desired position reference is shown in figure 4.12. In these experiments the high precision encoder was used as position reference, to be able to examine the position control behavior with no interference of the sensor fusion performance.
When running the position control scheme with this continuous path-planning position reference it is clearly visible how the controlled position lags behind the reference. Even while tuning the PID parameters very aggressive this kind of lag was not able to be treated with the currently used control structure. The reasons behind this lag in position might be the concept of how the speed control is implemented by using an integrator, effectively introducing delay in the system. To treat this lag, a velocity feed-forward was implemented in the control scheme, the results of doing this is shown in figure 4.13.

As can be seen in the plot the lagging behavior is eliminated by the added velocity feedforward, improving the position control performance. Unfortunately, as the work with the position control was started so late in the project due to the problems with the lab rig, no proper error assessment has been performed. This is partly due to the current position control performance is still long way off from
desired performance with peak errors of about 5 mechanical degrees, and an average error of about 1 mechanical degree. Hence it was not purposeful to spend time on assessing the performance extensively as it was easily determined that it was not "good enough". Rather the purpose of presenting the current results of the position control is more of giving an overview of the current progress on the position control.

Finally, a last plot includes the speed reference in the system, which showcases a good speed performance which runs at max 300 rpm but generally more around 200 rpm during the continuous operation, a speed range which is quite reasonable for position control applications.

\textbf{Figure 4.14:} Position control (PID/P) with added velocity feedforward also showing the velocity reference (run with the high precision encoder).

The currently implemented PID position control with speed feedforward implementation looks as follows:

\textbf{Figure 4.15:} Position control (PID/P) with added velocity feedforward implementation in Simulink.
5

Discussion

5.1 Discussion

5.1.1 Sensor fusion

The results showcasing the offline implementation of the sensor fusion quickly verified the sensor fusion concept. However, when running the algorithms on the real laboratory setup, it was realized that the performance of the back-EMF at lower speeds were so limited that no benefits would come from using the sensorless estimation in these low speed conditions. Essentially the sensor fusion algorithms analyzed in this thesis fuse a number of different sensors/measurements into an estimate that depends on a predetermined weighting about the noises and uncertainties corresponding to each sensor and the model of the system. However the performance of the position estimation based on back-EMF at lower speeds can not be treated as mere measurement noise, as it will be significantly worse at this working point than the regular noise in the signal at higher rotational speeds (see figure 4.5). To deal with this kind of behavior would then require introducing some dynamic model that represents the noise as a quantity which depends on the different working points/speeds. In my review of literature I did not come across anything similar to this kind of fusion behavior; however it was neither something I was then specifically considering in my searches.

Unfortunately there was not time to compare the Unscented version with the Extended Kalman filter experimentally. Similar simulation results as presented for the EKF was found for the UKF, but as mentioned previously, seeing that these simulations were run with no tuning and only dummy data they provide no meaningful basis to make a comparison between the methods. So as there was no experimental data to couple with the UKF simulation this simulation data was omitted as it provides no additional content to the results of this thesis.

Further, one thing that was realized in the conclusion of the project was that
the implementation of the back-EMF based sensorless position estimation was unnecessarily complicated. As the load angle is generally a useful variable to calculate when concerning stepper motor control the theory used to find the rotor position first calculated the load angle and used it to derive the rotor position. However, the position could have been calculated directly from the back-EMF without concerning the current which would then eliminate the problems occurring with the sensorless method at higher speeds (see figure 4.8).

5.1.2 Position control

The position control part of this thesis was not qualitatively assessed because of the amount of time lost in getting the lab rig to run satisfactorily. For the major part of the thesis the current control loop contained a bug causing the current supply to the two phases to shut off momentarily. This issue both made any true position control impossible, as the current would just shut off when the speed of the motor (frequency of currents) got to the 1-20 rpm area when approaching the desired position. Furthermore it also made working with position derived from a continuous current vector impossible which was needed for the sensorless algorithm. Because of this problem considerable time was spent working around this bug, finding working conditions for the system where reasonable results could be produced draining time from the thesis. This current bug is briefly presented in appendix A.1. When the current bug was solved some two weeks before the end of the thesis time was spent working on improving the position control, but this was done too late to produce enough results to reasonably assess the control part of the thesis.

The final version of the position control with added velocity feedforward shows promise mainly due to the potential speed performance, however it also introduces some new aspects that has not been considered within this thesis. For example is the estimated velocity used for feedforward found by differentiating the encoder, a method that at this point is ad-hoc and not very reliable.

5.2 Conclusions

This thesis proves that using sensor fusion can improve low cost position estimation of a hybrid stepper motor by fusing a sensorless back EMF position estimation with the readings from a MRPS sensor. In figure 4.9 the performance of the EKF on only the MRPS in the low speed range shows how the RMSE halves by running the MRPS signal through the filter. In the high speed scenario (figure 4.10) the first five seconds (between 5 and 10 seconds) showcase this same performance of reducing overall noise, before the two fused signals starts giving unwanted behavior making a quantified result hard to discuss.

The thesis has not assessed fully if this will be satisfactory to produce results that are good enough for using hybrid stepper motors in industrial applications.
5.3 Future work

Future work stemming from this thesis would be to improve the sensor fusion by first treating the signals used for the fusion. Firstly understanding and treating the seemingly linear increase in error for the MRPS at increasing rotational speeds. As well as examining the sensorless position estimation based on back-EMF by only considering the back-EMF and not calculating the position via load angle. Then the EKF filter could be tuned further for improved results. At a later stage it could also be interesting to compare the UKF and EKF versions both for comparing the actual fusion but also the performance in computational times etcetera for plausible implementation on more limited hardware.

The second part would be to spend more time on producing a good closed loop position control based on exact requirements on the position behavior of the system. Obvious improvements from the position control scheme presented in figure 4.15 would be to improve on the speed estimation so the speed control part could be implemented as closed loop, as well as improving the feedforward performance. Further should the current magnitude be treated based on the torque requirements, something that has been experimented with before, but was put to a static magnitude at this point. Putting it to a static current magnitude was done to remove one loop complicating the system, isolating the actual position performance.

When this is done it will allow to more qualitatively assess the performance of the hybrid stepper motor compared to the conventionally used AC servo motor. Important factors to take into account in such research is gear boxes in the applications and speed requirements, something that is obviously really closely tied together.
Appendix
A.1 Current problem

In the initial phases of the project the current controller contained a problem causing the current supply to abruptly shut off for brief moments during slower operation. An example of this is shown in figure A.1. Oddly, the behaviour never occurred at higher rotational speeds (higher frequency of sinusoidal currents).

It was suspected that maybe some kind of security protection on the inverter chip controlling the MOSFETs would cause this shutoff but after thorough examination did a "FAULT" pin on the IC never give any warnings indicating that it was the IC that shut off the MOSFETs and hence the current.

![Current behaviour 20rpm (old version)](image.png)

**Figure A.1:** Testing the EKF implementation with distorted measurement signals.

Eventually after reading up some extra on the hardware chip for the inverter
it was realized that it should supply enough protection by its own, and some protection that was implemented in the FPGA software was omitted leading to removing the long times of shutoff, but the shutting off behaviour still existed, see figure A.2.

![Diagram](image.png)

**Figure A.2:** Testing the EKF implementation with distorted measurement signals.

After a lot of time spent on debugging, and changing the FPGA code, eventually this problem was fixed, by changing the implementation of how the PWM signal controlling the inverter chip opened and closed the different MOSFETs to produce the current.
Bibliography


