

A Scenario-based Predictive Control Approach to Building HVAC Management Systems

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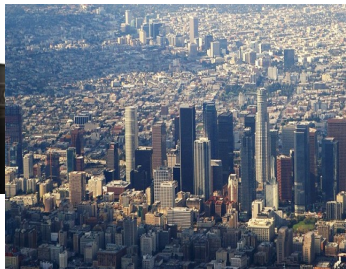
CASE 2013 – 19 August



Thanks to...

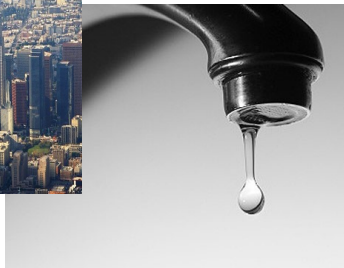
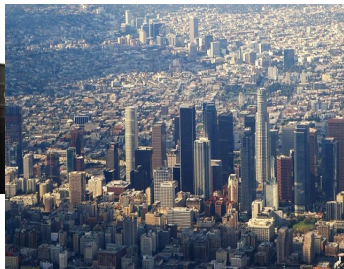


Motivations



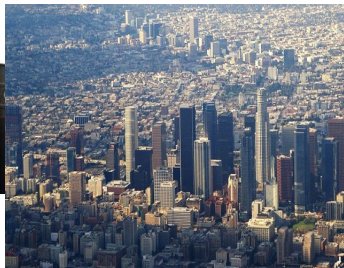
Motivations

diminish energy requirements



Motivations

diminish energy requirements



maintain comfort levels

Methodology (big vision)

How to, for HVAC systems?

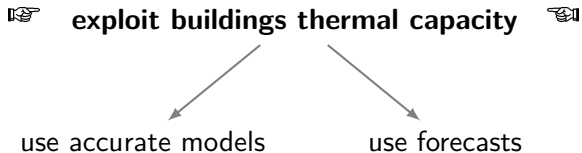
Methodology (big vision)

How to, for HVAC systems?

👉 **exploit buildings thermal capacity** 👈

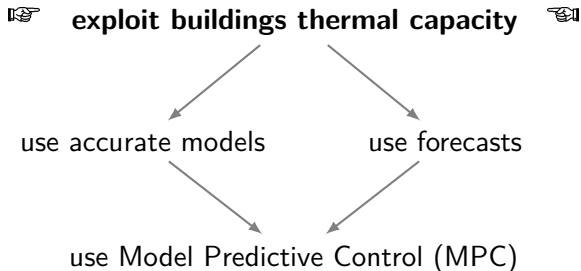
Methodology (big vision)

How to, for HVAC systems?



Methodology (big vision)

How to, for HVAC systems?



Literature review



Ma (2012)

Fast stochastic MPC with optimal risk allocation applied to building control systems

[Conference on Decision and Control](#)



Oldewurtel (2012)

Use of model predictive control and weather forecasts for energy efficient building climate control

[Energy and Buildings](#)



Salsbury (2012)

Predictive control methods to improve energy efficiency and reduce demand in buildings

[Computers and Chemical Engineering](#)



Mady (2011)

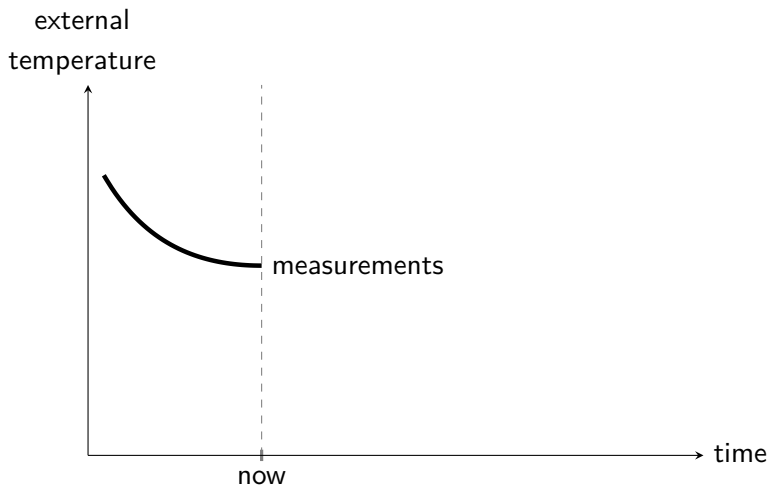
Stochastic model predictive controller for the integration of building use and temperature regulation

[Conference on Artificial Intelligence](#)

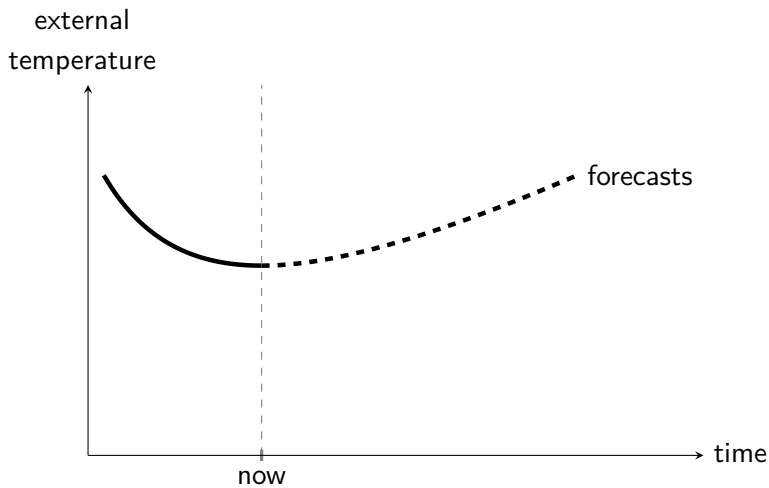
Our contributions

- address uncertainties in the forecasts (\rightarrow stochastic MPC)
- consider a peculiar description of these uncertainties

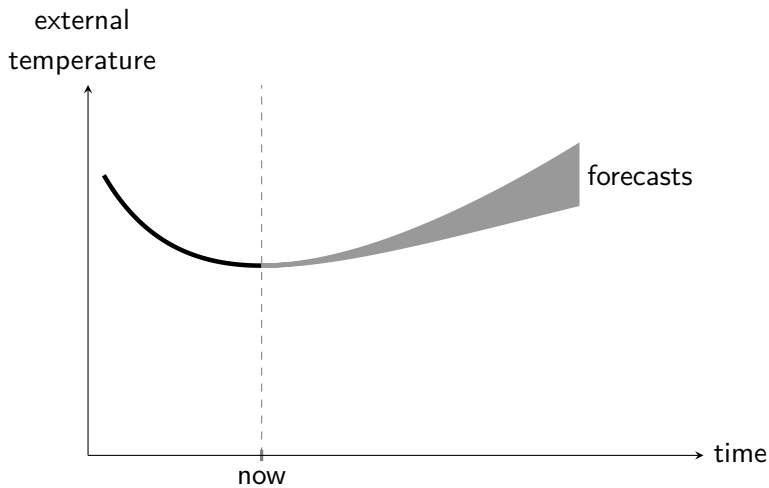
Example in words



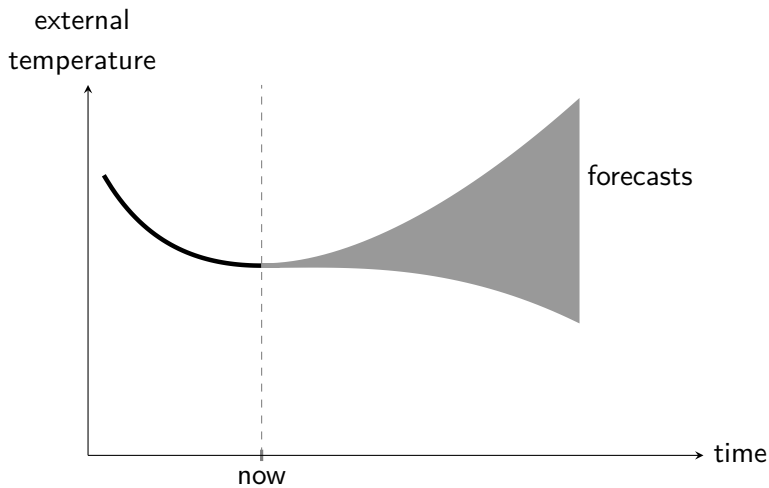
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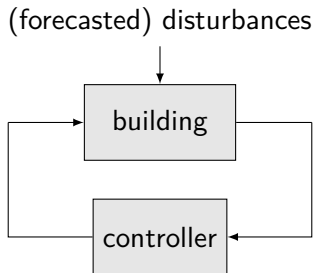
Example in words



Example in pseudo-formulas

minimize (energy usage over the forecast horizon)

subject to: \mathbb{P} [dynamics will lead to comfort violations] $\leq \varepsilon$
actuation is constrained



“subject to \mathbb{P} [dynamics will lead to comfort violations] $\leq \varepsilon$ ”
may be a formidably complicated constraint

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plausible solution: **simplify it**

caveat: **do not oversimplify it**

Aim

find controllers
accounting for forecasts uncertainties
and handling the associated computational problems

Methodology (for the current problem)

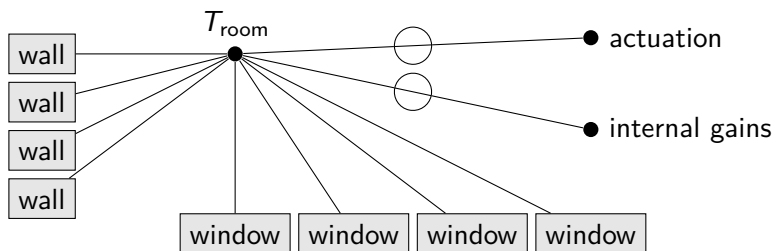
- perform realistic simulations
- describe forecasts uncertainties opportunely
- approximate $\mathbb{P}[\text{comfort violations}] \leq \varepsilon$ using scenarios

Room model

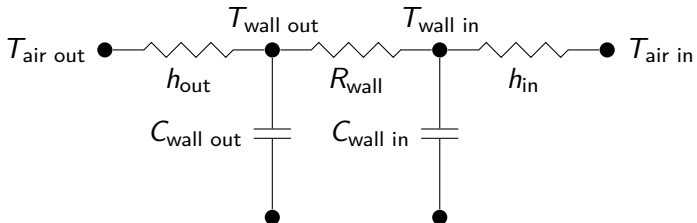
Our choice

Necessity: model should be accurate and computationally tractable

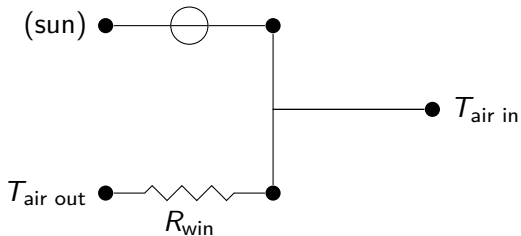
Our choice: RC-network ($R \leftrightarrow$ thermal resistance, $C \leftrightarrow$ thermal capacitance)



Wall model

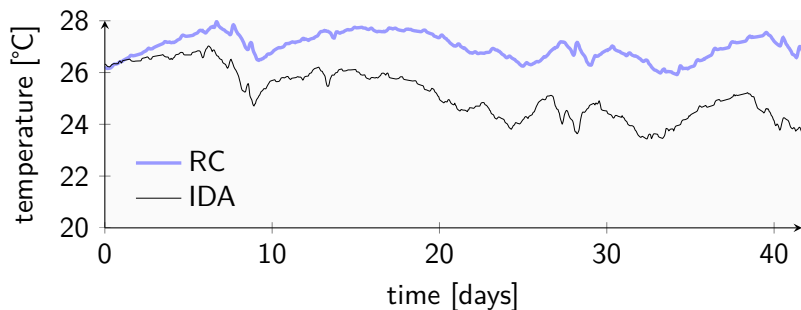


Window model



Building model

Validation against IDA-ICE

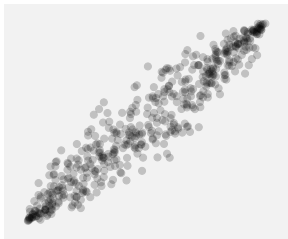


- simpler than commercial SW exploiting more complex libraries
- captures the most important buildings dynamics' characteristics

Describing uncertainties through Copulas

Motivations

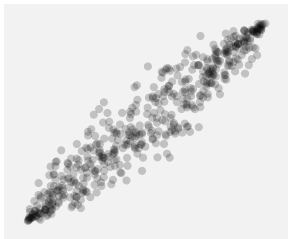
Gaussian



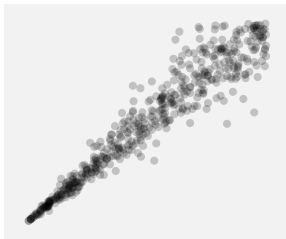
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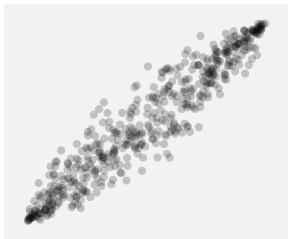
Clayton



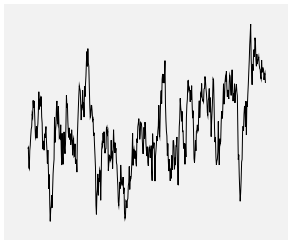
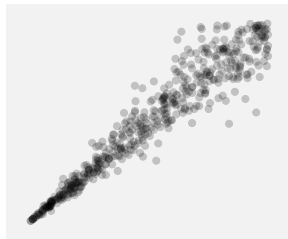
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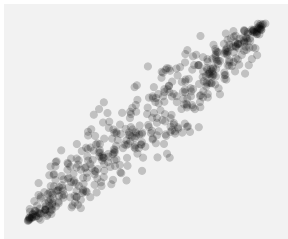
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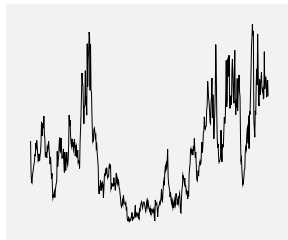
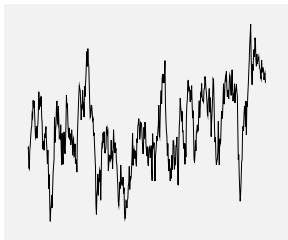
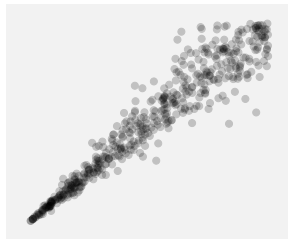
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Describing uncertainties through Copulas

Formalism

$$\mathbb{F}_{\mathbf{w}}(a_1, \dots, a_K) = \mathbb{C}(\mathbb{F}_{w_1}(a_1), \dots, \mathbb{F}_{w_K}(a_K)) \quad \mathbb{C} : [0, 1]^K \mapsto [0, 1]$$

In words, Joint distribution = Copula + Marginal distributions

Describing uncertainties through Copulas

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Pros

- completely generic
- separated modeling / learning of marginals / dependencies

Describing uncertainties through Copulas

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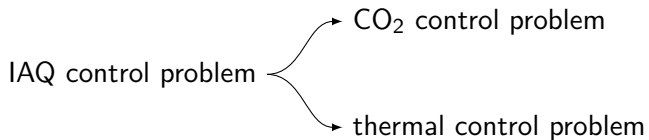
Cons

- generating *scenarios* is computationally more expensive

Scenario-based MPC

A cascade of two controllers

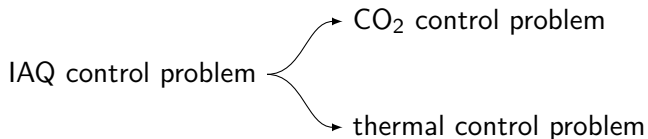
Thanks to the physics



Scenario-based MPC

A cascade of two controllers

Thanks to the physics



Thanks to the linear models

$$\begin{aligned} \min_{\mathbf{U}} \quad & \mathbf{E} \mathbf{P}_{\text{room}}^T \mathbf{U} \\ \text{s.t.} \quad & \mathbb{P} [\mathbf{G}_u \mathbf{U} + \mathbf{G}_w \mathbf{W} - \mathbf{g} \leq 0] \geq 1 - \alpha \\ & \mathbf{F} \mathbf{U} \leq \mathbf{f} \end{aligned}$$

Scenario-based MPC

Obtaining numerically tractable problems

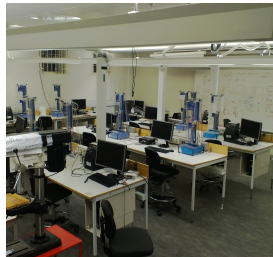
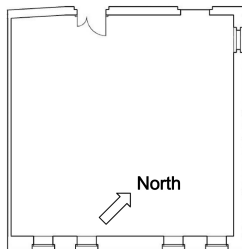
Replace \mathbb{P} with empirical \mathbb{P} :

$$\begin{aligned} \min_{\mathbf{U}, \tau} \quad & \mathbf{E} \mathbf{P}_{\text{room}}^T \mathbf{U} \\ \text{s.t.} \quad & \mathbf{F} \mathbf{U} \leq \mathbf{f} \\ & \tau + \alpha^{-1} \sum_{i=1}^{N_s} N_s^{-1} z_i \leq 0 \\ & \mathbf{G}_u^j \mathbf{U} + \mathbf{G}_w^j \mathbf{W}_i - \mathbf{g}^j - \tau - y_i^j \leq 0 \\ & z_i \geq y_i^j \quad y_i^j \geq 0 \quad z_i \geq 0 \end{aligned}$$

- *scenarios* := independent extractions of the disturbances *from their joint distributions* (i.e., copulas!!)
- N_s := number of i.i.d. scenarios extracted (the more, the better)

Numerical results

Room (hvac.ee.kth.se):

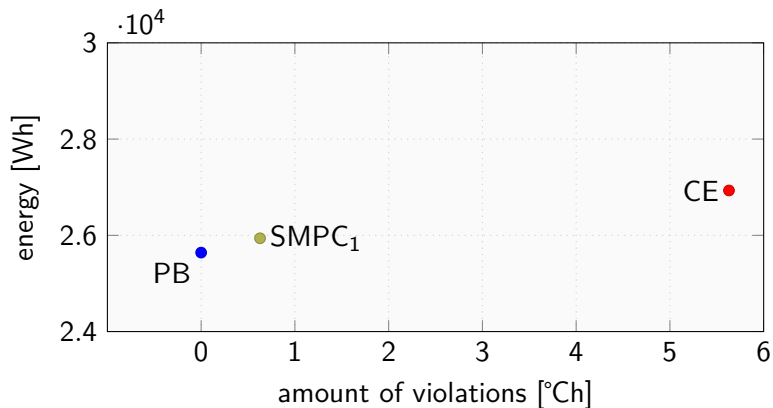


Controllers:

- Performance Bound (PB): has perfect forecasts
- Certainty Equivalence (CE): neglects forecasts uncertainties
- SMPC: our approach

Numerical results

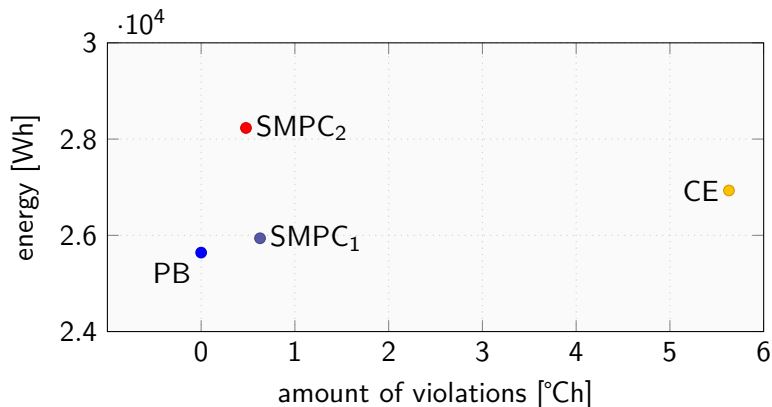
Assessment of performance



- SMPC₁: $\alpha = 0.09$, 60 scenarios

Numerical results

Assessment of performance



- SMPC₁: $\alpha = 0.09$, 60 scenarios
- SMPC₂: $\alpha = 0.06$, 120 scenarios

Summary

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aim: include forecasts' uncertainties

use scenario-based MPC

generate scenarios using copulas

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use scenario-based MPC

generate scenarios using copulas

Future extensions

- real tests (in progress right now)
- extend to buildings
- learn copulas cooperatively

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