The Partial Element Equivalent Circuit Method - Modeling and Experimental Verification of PCB Structures

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Abstract

To help products comply with international Electromagnetic Compatibility (EMC) regulations or as a help in a design process numerical simulation of electromagnetic (EM) characteristics are a valuable tool. With the development of high-speed computers the complexity of EM simulation programs and the systems they can simulate has increased considerable. But still, problems must be partitioned due to computer resource and/or EM simulation technique limitations.

In this thesis, four different EM simulation techniques are described and the nature of these are discussed. The focus is on the partial element equivalent circuit (PEEC) method for which the following improvements and investigations have been proposed in the enclosed papers.

First, a recent proposed formulation for the direct simulation of the radiated electric field from a device is compared against traditional post-processing equations and measurements. The results show that the proposed direct method, the electric field sensor, is unreliable for arbitrarily implementations since the length of the sensor strongly affects the results.

Second, a technique to obtain simplified PEEC models are presented. The first step is to use a discretisation procedure where partial elements with small effect on the complete PEEC model are excluded. Then, instead of using numerical integration, closed-form equations are used to calculate the partial elements. The obtained simplified PEEC models are shown to comply well against measurements.

Third, an introductory paper to the PEEC method is presented. The international interest for the method has been gaining rapidly for the past years resulting in considerable progress for the technique. But, in the Nordic countries the research effort has been low. The paper presents the technique using simple antenna examples, both printed and free space, and illustrations. For verification, simulations have been compared against analytical solutions and measurements.

Contents

Acknowledgements

I would like to start to thank the great staff at EISLAB at the department of computer science and electrical engineering for all kind of help and encouragement. A special thanks to Prof. Jerker Delsing who gave me the oppurtunity to work with this exciting area within computational electromagnetics and for his guidance. Finally, a thanks to my wonderful mother and father.

Chapter 1

Thesis Introduction

1 Introduction

Mandatory international Electromagnetic Compatibility (EMC) regulations [18] has introduced economical incentives to product developers to produce electromagnetic compatible products. If a product does not comply with regulations it has to be redesigned or subjected to post-production methods like screening and filtering. This can result in extended product development time (higher costs), degraded physical performance and/or impaired aesthetical appearance. This has created a need for the so called EMC engineer. His/hers job is, ideally, to work parallel with other engineering disciplines trying to bring different opinions into line with each other into a product that comply with EMC regulations. Traditionally, this work has been done by skilled engineers using experience and rules of thumb. With the development of high-speed computers and electromagnetic (EM) simulation programs his toolbox has expanded considerably. But, not even with today state of the art computers is it possible to model and simulate most complete designs. Instead, simplified models and problem partitioning is used to make the task possible.

EM simulation programs are based on some EM modeling technique suitable for some class of problem. Meaning that there is no universal technique applicable to all problems and that the EMC engineer should, ideally, master them all. Below, the most common techniques are briefly presented together with their pros and cons and some examples of software based on that specific technique.

The research activity in the area of EM simulations is intense. Conferences and symposiums around the world are well visited forums for researchers and companies who want to exchange ideas and report progress in their research field. I am certain that it was on one of these conventions the idea behind my Ph.D. service was born, somewhere around 1999.

1.1 Solving electromagnetic problems using numerical techniques

EM simulations can be used by researchers and product developers to make products comply with international EMC regulations or as a help in a design process. It is possible to identify two main EM simulation problem classes. First, there are EM problems that has to be solved for a restricted domain. Typical examples are printed circuit boards in a metallic enclosure or field penetration through apertures. Second, there are open air problems which are specified for very large computational domains. The radiation pattern from antennas, radiated emission tests and cable radiation are examples in this class.

Both problem classes require the solution of Maxwell's equations, equation (1) - (4) for a linear medium [13]. Maxwell's equations are a set of coupled partial differential equations with two unknown quantities, the electric field intensity, **E**, and the magnetic field intensity, **H**. The complicated nature of these equations makes an analytical solution possible only for a few simple cases. That is why a numerical technique has to be used. There are a variety of numerical techniques at hand but the choice is strongly dependant on the problem that has to be solved.

Maxwell's equations

Differential form *Integral form*

$$\nabla \times \mathbf{H} = \varepsilon \frac{\delta \mathbf{E}}{\delta t} + \mathbf{J} \qquad \oint_C \mathbf{H} \cdot dL = I + \int_S \varepsilon \frac{\delta \mathbf{E}}{\delta t} \cdot dS \tag{1}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\delta \mathbf{H}}{\delta t} \qquad \oint_C \mathbf{E} \cdot dL = \int_S \mu \frac{\delta \mathbf{H}}{\delta t} \cdot dS \tag{2}$$

$$\nabla \cdot \varepsilon \mathbf{E} = \rho \qquad \int_S \varepsilon \mathbf{E} \cdot dS = \int_V \rho dV \tag{3}$$

$$\nabla \cdot \mu \mathbf{H} = 0 \qquad \int_S \mu \mathbf{H} \cdot dS = 0 \tag{4}$$

, where

E - *Electric field intensity,* $[\frac{V}{m}]$	**H** - *Magnetic field intensity,* $[\frac{A}{m}]$
D - *Electric flux density,* $[\frac{C}{m^2}]$	**B** - *Magnetic flux density,* $[\frac{Wb}{m^2}]$
ρ - *Electric charge density,* $[\frac{C}{m^3}]$	**J** - *Electric current density,* $[\frac{A}{m^2}]$
ε - *Capacitivity of the medium,* $[\frac{F}{m}]$	μ - *Inductivity of the medium,* $[\frac{H}{m}]$

To solve for the EM field in a restricted domain, techniques based on a differential formulation of Maxwell's equations, left column of equation (1) - (4), is suitable. For these techniques the whole computational domain, including the air around it up to a specific distance, has to be divided into cells for which Maxwell's equations has to be solved. These techniques are also well suited for complex structures, like a heat sink, or structures consisting of different materials. But, for long and thin (wire-like) structures the computational domain increases considerably which results in a computationally inefficient method. Examples on differential based techniques, also entitled volume based techniques, are the finite difference time domain (FDTD) mathod and the finite element method (FEM) described below.

There are techniques based on an integral formulation of Maxwell's equations, right column of equation (1) - (4). For these methods, the air around the equipment under test (EUT) is excluded from the computation. These thechniques are ideal for large or unbounded domains consisting of perfectly electric conductors (PEC). The method of moments (MoM) and the partial element equivalent circuit (PEEC) technique are examples on integral based or surface current techniques.

These and other differences between the available computational techniques has triggered the use of hybrid techniques. Where, advantages of, often, two techniques are combined in a hybrid method to solve a specific class of problems.

In the following sections, four different types of EM computational techniques are briefly presented. The first three (FDTD, FEM and MoM) are the most common techniques used today for simulating EM problems. The last, the PEEC method, is the technique used in the three papers, paper A-C, presented in this licentiate thesis. This technique is rapidly growing and combines the solution of mixed circuit and EM field problems in the same model. The technique is for example used for the simulation of the PCB's in IBM laptops.

1.1.1 Finite Difference Time Domain (FDTD) Method

The FDTD method is a subclass of finite difference methods (FDM) [12] where finite difference equations are used to solve Maxwell's equations for a restricted computational domain. Typical problems solved using the FDTD method are field penetration through apertures and EMC/EMI problems inside metallic enclosures. There is also a finite difference frequency domain (FDFD) method, but since the FDTD method is the most popular technique for solving EM problems it is presented here.

The FDTD method is employed in the time domain and one correct simulation delivers results for a wide spectrum of frequencies. This is almost always desirable for EMI/EMC simulations where results for one single frequency is of small interest. But, for the opposite case, when considering only a small set of frequencies a time domain solution can be very inefficient.

Since FDTD is a volume based EM simulation technique, the whole computational domain must be divided into volume elements, often called cells, for which Maxwell's equations has to be solved for. The volume element size is determined by considering two factors [2]:

1. *Frequency*. The cell size should not exceed $\frac{\lambda}{10}$, where λ is the wavelength of the largest frequency of interest.

2. *Structure*. For finer structures, like very thin conductors, the frequency condition can be to rough. Then the FDTD method require a finer discretization i.e. smaller cells.

The volume elements are not restricted to cubical cells, parallelepiped cells can also be used with a side to side ratio not exceeding 1:3 [2], mainly to avoid numerical problems.

In the three dimensional FDTD method based on the Yee-cell [42], the electromagnetic field components, E_X, E_Y, E_Z, H_X, H_Y, and H_Z, are defined according to Figure 1. The

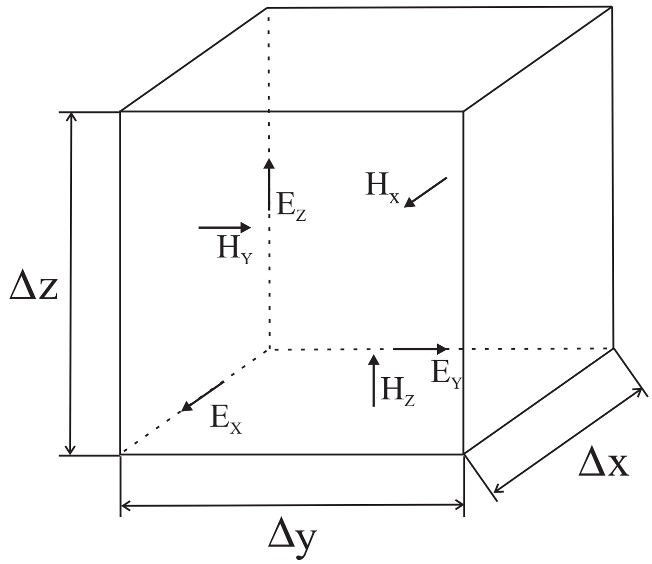

Figure 1: FDTD cell for Yee formulation

main feature of the Yee-cell is that the electric field components are shifted half a cell length to the magnetic field components. There are other FDTD methods that are not based on the Yee cell and thus have another definition of the field components. To be able to apply Maxwell's equations to the Yee cell we start by writing out equation (1) and (2) for a rectangular coordinate system according to:

$$\begin{bmatrix} \frac{\partial E_Z}{\partial y} - \frac{\partial E_Y}{\partial z} \\ \frac{\partial E_X}{\partial z} - \frac{\partial E_Z}{\partial x} \\ \frac{\partial E_Y}{\partial x} - \frac{\partial E_X}{\partial y} \end{bmatrix} = \begin{bmatrix} -\mu \frac{\partial H_X}{\partial t} \\ -\mu \frac{\partial H_Y}{\partial t} \\ -\mu \frac{\partial H_Z}{\partial t} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} \frac{\partial H_Z}{\partial y} - \frac{\partial H_Y}{\partial z} \\ \frac{\partial H_X}{\partial z} - \frac{\partial H_Z}{\partial x} \\ \frac{\partial H_Y}{\partial x} - \frac{\partial H_X}{\partial y} \end{bmatrix} = \begin{bmatrix} \varepsilon \frac{\partial E_X}{\partial t} \\ \varepsilon \frac{\partial E_Y}{\partial t} \\ \varepsilon \frac{\partial E_Z}{\partial t} \end{bmatrix} + \begin{bmatrix} J_x \\ J_Y \\ J_Z \end{bmatrix} \tag{6}$$

And by substituting time and spatial derivatives using finite difference expressions based on the Yee cell the following discrete finite difference equations for Maxwell's equations are obtained if the source term **J** are excluded :

$$H_X^{n+\frac{1}{2}}(i,j,k) = H_X^{n-\frac{1}{2}}(i,j,k) + \frac{\Delta t}{\mu_{ijk}}\left[\frac{E_Y^n(i,j,k+1) - E_Y^n(i,j,k)}{\Delta z} - \frac{E_Z^n(i,j+1,k) - E_Z^n(i,j,k)}{\Delta y}\right] \tag{7}$$

$$H_Y^{n+\frac{1}{2}}(i,j,k) = H_Y^{n-\frac{1}{2}}(i,j,k) + \frac{\Delta t}{\mu_{ijk}}\left[\frac{E_Z^n(i+1,j,k) - E_Z^n(i,j,k)}{\Delta x} - \frac{E_X^n(i,j,k+1) - E_X^n(i,j,k)}{\Delta z}\right] \tag{8}$$

$$H_Z^{n+\frac{1}{2}}(i,j,k) = H_Z^{n-\frac{1}{2}}(i,j,k) + \frac{\Delta t}{\mu_{ijk}}\left[\frac{E_X^n(i,j+1,k) - E_X^n(i,j,k)}{\Delta y} - \frac{E_Y^n(i+1,j,k) - E_Y^n(i,j,k)}{\Delta x}\right] \tag{9}$$

$$E_X^{n+1}(i,j,k) = E_X^n(i,j,k) + \frac{\Delta t}{\varepsilon_{ijk}}\left[\frac{H_Z^{n+\frac{1}{2}}(i,j+1,k) - H_Z^{n+\frac{1}{2}}(i,j,k)}{\Delta y} - \frac{H_Y^{n+\frac{1}{2}}(i,j,k+1) - H_Y^{n+\frac{1}{2}}(i,j,k)}{\Delta z}\right] \tag{10}$$

$$E_Y^{n+1}(i,j,k) = E_Y^n(i,j,k) + \frac{\Delta t}{\varepsilon_{ijk}}\left[\frac{H_X^{n+\frac{1}{2}}(i,j,k+1) - H_Z^{n+\frac{1}{2}}(i,j,k)}{\Delta z} - \frac{H_Z^{n+\frac{1}{2}}(i+1,j,k) - H_Z^{n+\frac{1}{2}}(i,j,k)}{\Delta x}\right] \tag{11}$$

$$E_Z^{n+1}(i,j,k) = E_Z^n(i,j,k) + \frac{\Delta t}{\varepsilon_{ijk}}\left[\frac{H_Y^{n+\frac{1}{2}}(i+1,j,k) - H_Y^{n+\frac{1}{2}}(i,j,k)}{\Delta x} - \frac{H_X^{n+\frac{1}{2}}(i,j+1,k) - H_X^{n+\frac{1}{2}}(i,j,k)}{\Delta y}\right] \tag{12}$$

Where the field at spatial location $(i\Delta x, j\Delta y, k\Delta z)$ and time step $n\Delta t$ is denoted:

$$\mathbf{E}(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = \mathbf{E}^n(i,j,k) \tag{13}$$

To be able to solve equation (7) - (12) the following must be specified [2]:

1. *Initial conditions (Excitation).* The initial electromagnetic field components for each discrete point in the discretised structure must be specified. The excitation of the structure is also specified at this point. The most common source modelled is the impressed current source which is located at the same spatial point as the electric field and at the same time point as the magnetic field.

2. *Boundary conditions.* Many problems in EMI/EMC simulations involves open region problems that are impossible to discreticise in the FDTD method. Instead this problem can be solved using for ex. mathematical formulations, absorbing boundary conditions (ABC), or absorbing material at the computational boundary. The specification of boundary conditions are named *mesh truncation* techniques in the FDTD method.

3. *Time step, Δt.* To assure that the transfer between the nodes do not exceed the speed of light a time step condition, Courant's condition in equation (14), has to be fulfilled.

$$\Delta t \leq \frac{1}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2} \cdot \sqrt{\frac{1}{\mu\varepsilon}}} \tag{14}$$

, where Δx, Δy and Δz are the spatial step sizes and μ and ε are material constants.

The equations are then solved by

1. Calculating the electric field components, equation (10) - (12), for the complete structure.

2. Advance time by $\frac{t}{2}$.

3. Calculate the magnetic field components, equation (7) - (9), for the complete structure based on the electric field components calculated in 1.

4. Advance time by $\frac{t}{2}$ and continue to 1.

Computer codes utilizing the FDTD method are for example EZ-FDTD from EMS-Plus or XFDTD from Remcom Inc.

1.1.2 Finite Element Method (FEM)

The finite element method [32] is a volume based numerical technique applicable both in the time and frequency domain. In the method, partial differential equations are solved by a transformation to matrix eqautions [12]. This is done by minimizing the energy for a PDE using the mathematical concept functional, F [34].

The procedure [11, 2] can be explained by considering a PDE described by the function u with corresponding driving, excitation, function f as:

$$L(u) = f \tag{15}$$

, where L is a PDE operator. For the following functional (the variational form):

$$F(v) =< L(v), v > -2 < f, v >$$

, where $< a, b >$ is the inner product of two functions a and b defined as:

$$< a, b >= \int_{\Omega} abd\Omega$$

to have a minimum at u, we have:

$$F(u) \leq F(u + \varepsilon v) = F(u) + 2\varepsilon\left[< L(u), v > - < f, v >\right] + \varepsilon^2 < Lv, v >$$

for small ε. Since $\delta F = 0$, the first variation is equal to zero. This gives $< L(u), v > =< f, v >$, which implies $L(u) = f$, where u is the solution to equation (15).

In FEM this procedure can be desribed as:

1. The three dimensional computational domain is discreticised, in the most simple case, into tetrahederal elements, Figure 2.

2. The functional for each FEM element, F_e, is then constructed. The computational domain functional is thus: $F = \sum\limits_{\Omega} F_e$.

3. The unknown function, u_e, for each element is expresssed as a series of known functions, u_i, with unknown coefficients, α_i, as: $u_e = \sum\limits_{i=1}^{n} \alpha_i u_i$. Where n are the number of nodes in the elements.

4. The last step is to sum up and minimise the functional for the entire region according to:

$$\delta F = [0] = \frac{\partial F}{\partial [\alpha]^T}$$

and solve for the unknown coefficients, α_i.

Since the FEM is a volume based technique, like the FDTD method, the computational domain has to be terminated using different techniques to avoid reflection at the computational domain. This can be done using [11, 2] :

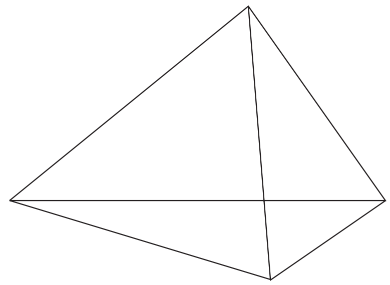

Figure 2: Tetrahederal elements for 3D FEM discretisation

1. *Infinite elements.* In this technique, one surface of the outer most finite elements are extended to infinity. Unfortunately, the integrals associated with these elements can diverge depending on the finite elements used.

2. *Absorbing boundary conditions (ABC).* A technique where specific properties are enforced on the computational domain to absorb the outgoing field components.

The method offer great flexibility to model complicated geometries with the use of nonuniform elements as illustrated in Figure 3.

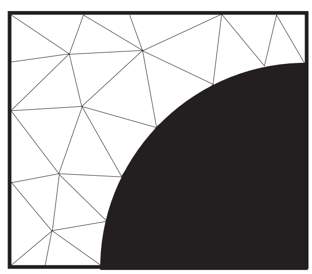

Figure 3: Example of FEM discretisation using nonuniformal elements

Computer codes utilizing the FEM method are for example PAN-CEM from ESI Group or Flux3D from Magsoft Corporation.

1.1.3 Method of Moments (MoM)

Method of moments (MoM) [25] is a surface based technique meaning that the air around the EUT can be excluded in the discretized model. This facilitates the analysis of large problems in terms of computation speed and memory usage. One disadvanage with the technique is the limited possibility to model different materials. The method is usually employed in the frequency domain but can also be applied to time domain problems.

In the MoM, integral based equations, describing as an example the current distribution on a wire, are transformed into matrix equations easily solved using matrix inversion.

When using this technique on surfaces a wire-grid approximation of the surface is often made [2], see Figure 4. This wire formulation of the problem considreably simplifies

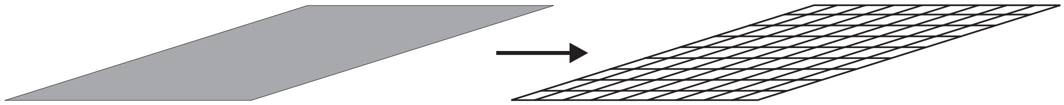

Figure 4: Conversion from surface to wire-grid approximation for MoM modeling

the calculations and are often used for far-field calculations. The starting point for the theoretical derivation [11, 2], for a wire structure, is a linear operator, L, applied to a unknown function, u, where f is the excitation function for the system as:

$$L(u) = f$$

If the wanted function, u, is expanded into as a series of known functions, u_i, with unknown coefficients, α_i, as:

$$u = \sum_{i=1}^{n} \alpha_i u_i \tag{16}$$

, where u_i are called basis functions and α_i can be for example current amplitudes on a wire segment, see Figure 5. If the summation is infinite, $n \to \infty$, eqaution (16) excactly

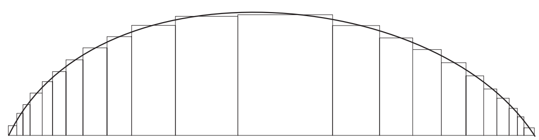

Figure 5: Staircase-approximation of current distribution on a conductor

represents the function, u. Since a infinite summation is practically impossible a residual error (RE) is defined as:

$$RE = L(u) - L(\sum_{i=1}^{n} \alpha_i u_i) \tag{17}$$

The RE is then minimized using a weighting process where weighting functions, w_j, are multiplied to both sides of equation (17) and the result is integrated over the entire wire.

$$\int REw_j dl = \int (L(u) - L(\sum_{i=1}^{n} \alpha_i u_i))w_j dl; j = 1, 2, ..., n$$

The residual error is set to zero and the resulting eqaution becomes

$$\sum_{i=1}^{n} \alpha_i \int w_j L(u_i) dl = \int w_j f dl; j = 1, 2, ..., n \tag{18}$$

This can be written as a matrix equation if the elements are defined as:

$$Z_{ij} = \int L(u_i) w_j dl$$
$$E_j = \int w_j f dl$$

Thus, equation (18) can be written as:

$$\begin{aligned}
&Z_{11}\alpha_1 + Z_{12}\alpha_2 + ... + Z_{1n}\alpha_n = E_1 \\
&Z_{21}\alpha_1 + Z_{22}\alpha_2 + ... + Z_{2n}\alpha_n = E_2 \\
&. \\
&. \\
&. \\
&Z_{n1}\alpha_1 + Z_{n2}\alpha_2 + ... + Z_{nn}\alpha_n = E_n
\end{aligned}$$

or using matrix notation to solve for the unknown coefficients as:

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ . \\ \alpha_n \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & . & . & . & Z_{1n} \\ Z_{21} & Z_{22} & . & . & . & Z_{2n} \\ & & . & & & \\ & & . & & & \\ & & . & & & \\ Z_{n1} & Z_{n2} & . & . & . & Z_{nn} \end{bmatrix}^{-1} \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

Computer codes utilizing the MoM are for example em Suite from Sonnet Software Inc. or NEC 2 (Numerical Electromagnetic Modeling), freeware.

1.1.4 Partial Element Equivalent Circuit (PEEC) Method

The EM simulation technique utilized in the enclosed papers is named the PEEC method [39, 5, 38, 41]. The main research activity in this area has been and are performed, by Dr. Albert E. Ruehli at IBM Thomas J. Watson Research Center in New York, starting with a publication in 1972, the same year I was born. At that time the foundation of the PEEC method was presented, i.e. the calculation of the partial inductances. The PEEC method is not one of the most common techniques used in EM simulation software or as a research area. But it has just been starting to gain recognition and for the first time there is a session at the 2001 IEEE EMC Symposium named after the technique.

In the mid 90's, two researchers from the university of L'Aquila in Italy, Professor Antonio Orlandi and Professor Giulio Antonini, published their first PEEC paper and are now together with Dr. Ruehli considered the top researchers in the area. There is a well established corporation between Prof. Orlandi, Prof. Antonini and Dr. Ruehli which have resulted in several joined publications.

The main research activity concerning the PEEC method can be summarized as follows:

1970	
.	Inductance calculations
.	Capacitance calculations
.	Introduction of PEEC
1975	
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.	Improved inductance calculations
1980	
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1985	
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1990	Simulate radiated emission using PEEC
.	Retarded PEEC models
.	Dielectric inclusion
.	Excitation of PEEC models using incident fields
1995	Stable PEEC models
.	
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.	
2000	PEEC speed-up*
.	
.	
.	
2005	

* Where the speed-up of the technique has been performed by using wavelets and Hartens scheme [1].

There are some advantages for the PEEC method compared to FDTD, FEM and MoM that makes the technique interesting, for example:

- A specialized solver is not needed since a circuit simulation program like SPICE solves for the currents in the structure.
- The inclusion of additional circuit elements like signal sources, transmission lines and discrete components are easy in a SPICE like environment.
- The same model can be used to obtain results both in the time- and frequency domain.
- The technique is both intuitive and instuctive.

These advantages makes the technique unique and suitable for a specific class of problems like combined circuit and electromagnetic problems.

There are some downsides with the method like:

- Large problems generate large matrices in the SPICE solver which results in large computation times.
- For radiated far field simulations a post-processor is needed to calculate the radiated fields. This step introduces an extra amount of work and also the potential for man made errors.

My work with the PEEC method started off early in 1999 with a masters thesis [19]. In which I presented basic PEEC theory and one example with a one layer printed circuit board (PCB). The research after that has been concentrated towards the application of the PEEC method to PCB problems. Mainly because this is an ideal problemtype for the PEEC method.

After that, we have obtained a specialized PEEC circuit solver developed at the university of Magdeburg [24]. This solver makes it possible to include a finite travel time between the partial mutual elements in the PEEC model [15]. With this solver, we can used the PEEC method as a complete 'full-wave technique' comparable with all other EM simulation methods. The PEEC method have been studied extensively by reading articles and by practical experiments for the last two years. Unfortunately, the lack of literature and examples have made it a struggle to perform this work from time to time. This will probably change to the better in the future since I have scheduled a longer visit to the university of L'Aquila under the supervision of Prof. Orlandi.

The technique is very different from the other methods since is has been developed particulary to solve mixed circuit- and field problems. Like other integral based EM simulation techniques, the PEEC method is suitable for long/thin structures consisting of simple materials like PCB structures.

The starting point for the theoretical derivation [20] is the total electric field, at observation point $\overline{r}$, in a multiconductor system expressed in terms of the vector magnetic potential $\overline{A}$ and the scalar electric potential Φ as:

$$\overline{E}(\overline{r},t) = -j\omega\overline{A}(\overline{r},\omega) - \nabla\Phi(\overline{r},\omega) \tag{19}$$

The vector potential term is given by [13]

$$\overline{A}(\overline{r},\omega) = \frac{\mu}{4\pi}\int_{v'}\frac{\overline{J}(\overline{r}',\omega)}{|\overline{r}-\overline{r}'|}dv' \tag{20}$$

where $\overline{J}$ is the volume current density at a source point $\overline{r}'$. The scalar potential term [13] is given by

$$\Phi(\overline{r},\omega) = \frac{1}{4\pi\varepsilon}\int_{v'}\frac{q(\overline{r}',\omega)}{|\overline{r}-\overline{r}'|}dv' \tag{21}$$

, where v' is the volume of the condutor and q is the charge density of the conductor. If equation (20) and (21) is substituted into equation (19) an electric field integral equation (EFIE) in the unknown variables J and q are obtined as:

$$\overline{E}(\overline{r},t) = -j\omega\frac{\mu}{4\pi}\int_{v'}\frac{\overline{J}(\overline{r}',\omega)}{|\overline{r}-\overline{r}'|}dv' - \nabla\frac{1}{4\pi\varepsilon}\int_{v'}\frac{q(\overline{r}',\omega)}{|\overline{r}-\overline{r}'|}dv' \qquad (22)$$

Equation (22) is then solved by using a 'method of moment'-type of solution by expanding each unknown, J and q, into a series of pulse basis functions with unknown amplitude [20]. As in a MoM-Galerkin solution pulse functions are also selected for the weighting functions. Then, each part of equation (22) can be interpretated as circuit elements [20]:

- The term on the left hand side can be shown to equal the dc current resistance between the nodes.
- The first term on the right hand side is the partial inductance between the nodes (self partial inductance) and the mutual partial inductance between the self partial inductances.
- The second term on the right hand side is the partial self capacitance to each node (self partial capacitance) and the mutual partial capacitance between the self partial capacitances.

This solution requires that the structure is discretized into fixed nodes from which one inductive and one capacitive partition is made. The partitions are then used to calculate the partial elements from the size and position of each element in the partition. This is illustrated in figure 6. The PEEC model is then 'solved' by using a circuit solver program as SPICE [35]. This makes the inclusion of excitation, by using current- and voltage sources, and additional discrete components easy in the model.

The PEEC method has been extended to:

- Handle finite travel times between the partial mutual elements. This is done using delayed coupled current- or voltage sources [29].
- Include finite lossy dielectric areas with the use of dielectric cells [31].
- Handle structures excited by incident field/fields [21].

A computer code that is partly based on the PEEC method is named PCBMod and is supplied by Simlab GmbH.

2 Paper Summaries

2.1 Paper A: Experimental Verification of PEEC Based Electric Field Simulations

When the radiated electric field is computed using integral based EM simulation techniques, like the PEEC method, this is usually done in two steps. First, the currents in the structure are determined using a suitable technique, in this case the PEEC method.

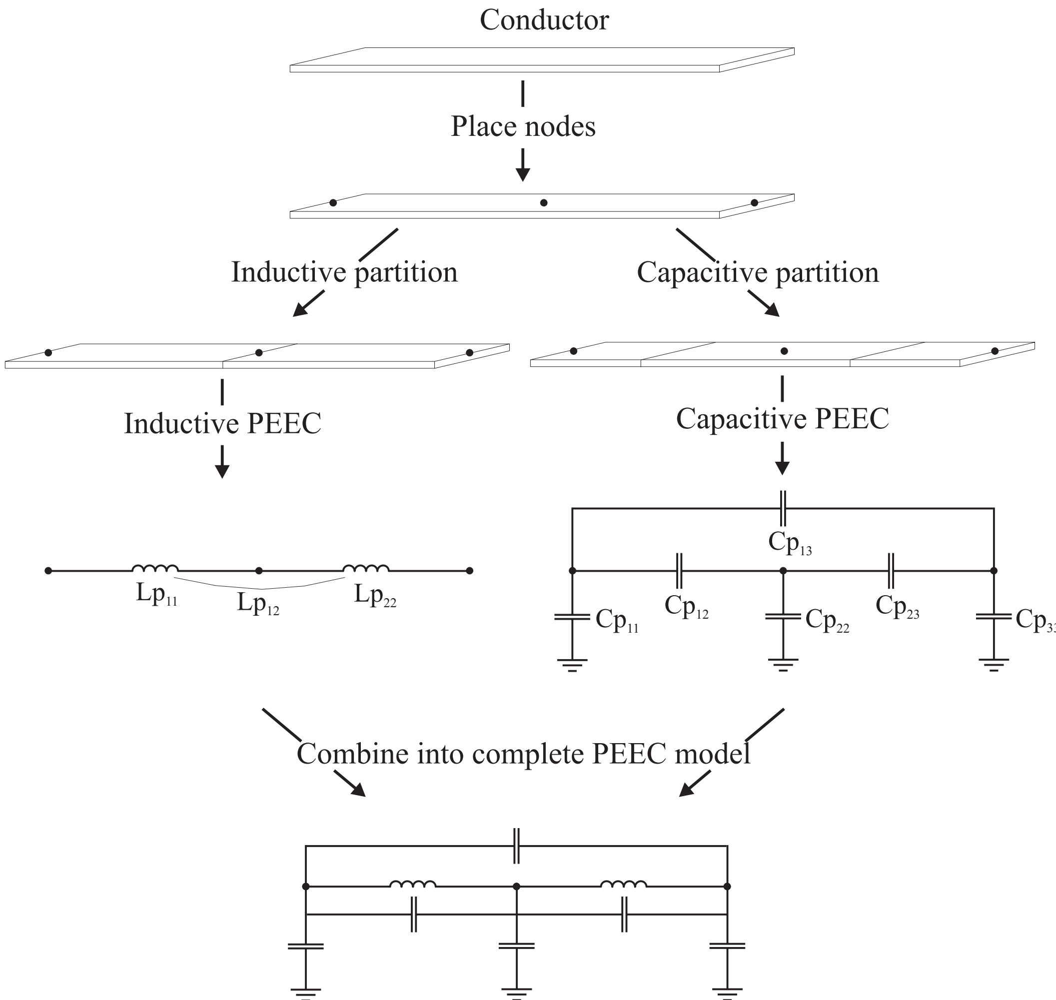

Figure 6: Procedure for creating PEEC models

Then, post-processing equations are employed using the simulation result and the physical dimensions of the structure to compute the field components. In a international journal an alternative formulation was presented, excluding post-processing totally. The proposed electric field sensor is a a virtual PEEC one-cell that measures the electric field at one or several observation points during the computation. The sensor had been verified for a simple dipole radiating in free space and compared to a MoM code. In this paper the sensor has been tested for a printed circuit board and compared against measurements and post-processing equations. It was shown that the electric field sensor formulation is unreliable for arbitrarily implementations since the length of the sensor strongly affects the predicted electric field. The use of simplified versions of post-processing equations are shown to agree well with measurements and are also easy to use.

2.2 Paper B: Simplified PEEC Models for PCB Structures and Comparison to Experimental Data

The calculation of the partial elements used in the PEEC model involves the evaluation of complex second and fourth order integrals. These integrals are usually solved using numerical integration techniques resulting in increased computation times and susceptibility to numerical errors. The paper presents a technique to obtain simplified PEEC models using a discretization procedure where partial elements with small effect on the solution are excluded and where closed-form eqaution are used for the calculation of the partial elemants. These simplifications makes the PEEC method acessible in the sense that only three eqautions are used for the partial element calculation and the resulting PEEC model can be solved using a free verison of for example SPICE (Simulation Program with Integrated Circuit Emphasis). The proposed technique has been used to model and simulate one layer PCB's with results that comply well against measurements. For a complex two layer PCB, a 9 element log-periodic dipole array (LPDA), measurements shows that the technique is to approximative.

2.3 Paper C: Analysis of Printed Antenna Structures using the Partial Element Equivalent Circuit Method

This paper was written as an introduction to the PEEC method and is to be presented at a Nordic conference. Since the development of the method during the 70's and the 80's the main research activity has been performed by Dr. A. E. Ruehli at IBM Research, NY, USA. The complexity of the method and the lack of information has restricted the research and development to a couple of 'hot spots' outside the Nordic countries. The method is presented using full and simplifed PEEC model examples with illustrations and comparison to analytical solutions and measurements are made.

3 Conclusions and suggested future work

My work and studies around the PEEC method has shown for us that the technique is very powerful and also important for us at **EIS**LAB due to the possibility to solve mixed circuit and electromagnetic problems. To combine different expertices at **EIS**LAB using SPICE as a platform to perform unique simulations is today a reality and for **EIS**LAB a powerful tool.

The possibility to perform radiated electric field simulations using the PEEC method has been investigated. Two different formulations were tested on a one layer PCB. First, an automatic electric field sensor directly incorporated in the computer simulation. Second, post processing equations, programmed in Matlab, utilizing the PEEC model currents and the geometry of the PCB. It was shown that the radiated electric fields that were computed using the post processing equations are more reliable then the electric field sensor.

A method to obtain simplified PEEC models has been presented. The main objective to use simplified PEEC models is to speed up the computation time. This is done in two steps in this paper. First, a special discretization procedure enables the use of closed form equations for the calculation of the partial elemants. Second, macromodeling techniques were used where cells with a small impact on the solution were excluded from the resulting PEEC model. This results in approximate PEEC models of the original structure with easy calculated partial elements and shortend computation times. The models were shown to comply well against measurements.

The work with PEEC's for PCB structures will continue. But, attention will be given to the solving of the circuit matrix resulting from the modified nodal analysis (MNA) in the SPICE solver. The main area of interest is to be able to speed up the solving procedure with the use of problem partitioning and identifying sub-circuits in the circuit matrix.

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Paper A

Experimental Verifcation of PEEC Based Electric Field Simulations

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Experimental Verification of PEEC Based Electric Field Simulations

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Abstract— **Different ways to simulate the measured radiated electric field for printed circuit boards using Partial Element Equivalent Circuit (PEEC) models has been investigated. Full and simplified PEEC models have been used with post-processing algorithms and a electric field sensor direct incorporated in the PEEC simulations. Calculated and simulated field strengths for a simple PCB are compared against measurements taken in a anechoic EMC chamber. It is shown that the post-processing equations compares better to measurements then the electric field sensor.**

Introduction

Due to international EMC regulations and to ensure functionality, product developers sometimes need to be able to simulate the electromagnetic behavior of printed circuit board layouts and other complicated structures prior to construction. To detect non compliance at this point in a product cycle is valuable since appropriate actions now can improve the EM characteristic in a very cost effective way. There are a variety of EM simulation techniques, one more suitable then the others for a specific problem. Since the Partial Element Equivalent Circuit (PEEC) method is based on an integral equation formulation this technique is ideal for 'open air' problems like radiations from PCB´s. The PEEC method was developed by Ruehli[1],[2],[3] and is based on the conversion of the Mixed Potential Integral Equation (MPIE) to the circuit domain. By using a specialized discretization, the original structure is converted into a network of discrete inductances, capacitances and resistances, called the partial elements. This results in a electromagnetic accurate model where additionally discrete components like transmission lines and voltage/current sources are easily included. The partial elements are calculated either by using numerical integration techniques or simplified closed form solutions. The resultant equivalent circuit can be solved with a conventional circuit solver like SPICE[10], where the same equivalent circuit can be used to obtain results in the time and frequency domain.

The calculation of the radiated fields for indirect EM simulation techniques like the PEEC method is done in two steps. First, the PEEC model is used to solve for the currents in the volume cells. Second, the calculated currents are used in a post-processing tool that calculates the field components. This introduce an extra amount of work for the user and additional error sources for the result, hence it is desirable to exclude this extra step. The e-field sensor[12] is a proposed solution. The sensor is a special PEEC one-cell that is coupled to the test object using mutual partial elements, and the radiated field strength can be plotted direct after a successful circuit solver analysis. In this paper the proposed sensor, full and simplified PEEC models using post-processing algorithms have been compared to measurements taken in an anechoic chamber for the radiated electric field strength of a simple PCB.

Derivation of the PEEC model

The starting point for the theoretical derivation is the equation for the sources of electric field at any point in a conductor

$$\overline{E} = \overline{E}^i + \overline{E}^s \tag{1}$$

where $\overline{E}^i$ and $\overline{E}^s$ is the incident and scattered electric field respectively. This equation can be rewritten using the current density $\overline{J}$, conductivity σ, vector magnetic potential $\overline{A}$ and the scalar electric potential Φ, resulting in

$$\frac{\overline{J}(\overline{r},t)}{\sigma} = \overline{E}^{i} + \left[-\frac{\partial}{\partial t}\overline{A}(\overline{r},t) - \nabla\Phi(\overline{r},t)\right] \quad (2)$$

For a system containing K conductors the free-space Green's function potentials are given by

$$\overline{A}(\overline{r},t) = \sum_{k=1}^{K} \frac{\mu}{4\pi} \int_{v_k} \frac{\overline{J}(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k \quad (3)$$

and

$$\Phi(\overline{r},t) = \sum_{k=1}^{K} \frac{1}{4\pi\varepsilon} \int_{v_k} \frac{q(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k \quad (4)$$

, where

$$t' = t - \frac{|\overline{r}-\overline{r}'|}{v} \quad (5)$$

, denotes the retardation time in the medium with propagation speed v. The charge density q considers both the bound charges and the charges bounded in the dielectric regions. The expression for the current density $\overline{J}$ must be modified[7] to include the conduction current density $\overline{J}_C$ and the polarization current density in the dielectric medium according to

$$\overline{J} = \overline{J}_C + \varepsilon_o(\varepsilon_r - 1)\frac{\partial \overline{E}}{\partial t} \quad (6)$$

Combining equations (2), (3), (4) and setting the incident field, $\overline{E}^{i}$, to zero gives equation (7) if the field point is inside a conductor

$$\frac{\overline{J}(\overline{r},t)}{\sigma} + \sum_{k=1}^{K} \frac{\mu}{4\pi} \int_{v_k} \frac{\partial}{\partial t} \frac{\overline{J}(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k +$$
$$\sum_{k=1}^{K} \varepsilon_o(\varepsilon_r - 1) \frac{\mu}{4\pi} \int_{v_k} \frac{\partial^2}{\partial t^2} \frac{\overline{E}(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k + \quad (7)$$
$$\sum_{k=1}^{K} \frac{1}{4\pi\varepsilon_o} \nabla \left[\int_{v_k} \frac{q(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k\right] = 0$$

To solve the system of equations in (7), the current and charge densities are discretized into volume and surface cells respectively, Figure 1. The current volume cells lead the current between the nodes and the charge surface cells represent the node charge. Inside the cells the variables are constant. Applying the Galerkin method K equations are obtained for the K volume cells of the structure. The interpretation of these equations as a loop[11] leads to the structure of the equivalent circuit for a PEEC cell, Figure 2. In Figure 2, Rm is the volume cell dc resistance calculated as

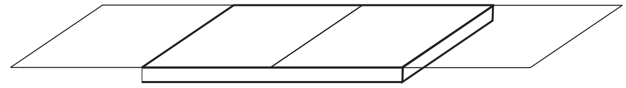

Fig. 1. Volume cell with two surface cells

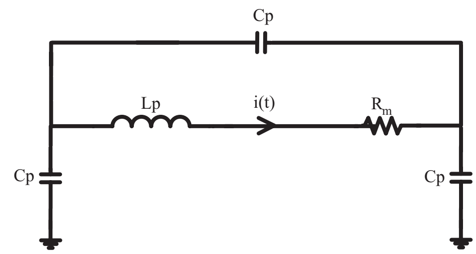

Fig. 2. Equivalent circuit model for a PEEC cell

$$R_m = \frac{l_m}{a_m \cdot \sigma} \quad (8)$$

where l_m is the cell length in the direction of the current flow, a_m is the cell cross-section and σ is the conductivity of the specific material. Lp is the partial inductance calculated as

$$Lp_{mn} = \frac{\mu}{a_m a_n} \frac{1}{4\pi\,|\overline{r}-\overline{r}'|} \int_{a_m}\int_{a_n}\int_{l_m}\int_{l_n} da_m da_n d\bar{l}_m \cdot d\bar{l}_n \quad (9)$$

If $n = m$, equation (9) represents the self partial inductance of the m:th cell. And if $n \neq m$, equation (9) represents the mutual partial inductance between the m:th and the n:th cell. Cp is the partial capacitance that can be calculated from the coefficients of potential P_{ij}[4]

$$P_{ij} = \frac{1}{\varepsilon_o a_{si} a_{sj}} \frac{1}{|\overline{r}-\overline{r}'|} \int_{a_{si}}\int_{a_{sj}} da_{si} da_{sj} \quad (10)$$

If $i = j$, the coefficients of potential in equation (10) can be recalculated to the self partial coefficient of potential of the j:th surface cell. And if $i \neq j$ the coefficients can be recalculated to the mutual partial coefficient of potential between the i:th and the j:th surface cell. For PEEC models where dielectric regions must be considered, the PEEC method has been extended through the use of dielectric cells[7].

Time retardation

Time retarded PEEC models should be used if the discretized structure is electrically large or the active frequency spectrum is high to ensure the accuracy of the PEEC solution[8]. The concept of time retardation can be seen in equation (3) where the vector magnetic potential $\overline{A}$ at point $\overline{r}$ at time t is calculated from using the current densities $\overline{J}$ in all the discretized volume cells at a retarded time t'. To extend the PEEC model to be a full-wave solution of Maxwell's equations the representation of the mutual partial elements must be revised. In the proposed retarded PEEC model[9] all mutual partial capacitances are replaced by current-controlled current sources or voltage-controlled voltage sources depending on the problem at hand. The mutual partial inductances are best represented by a voltage-controlled voltage source formulation. The controlled sources are derived from basic circuit theory and the resultant structure of the equivalent circuit for a TD-PEEC cell is given in Figure 3. The importance of time

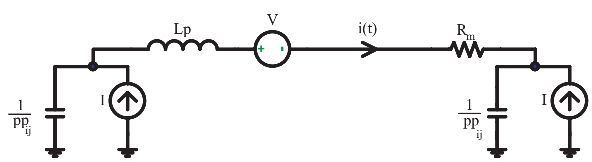

Fig. 3. Equivalent circuit model for a PEEC cell

retardation for EM field simulations have been verified in the past[6] and this formulation will be used in this paper with a modified SPICE solver[12], capable of handling the resulting delay differential equations(DDE).

Post-processing equations

For integral equation based EM models, post-processing equations are normally used to convert the calculated current and charge distributions in the structure to the radiated electric and/or magnetic field strengths. This is easy when the PEEC model is applied to PCB geometries[5]. The traces on PCB's are usually very narrow and thin in reference to the length and thus each volume cell can be seen as a wire antenna of finite length with a well defined current flow given by the circuit solver from the PEEC model. One way to derive the electric field from a wire antenna is to start by using the rotation of the vector magnetic potential to describe the magnetic flux density, $\overline{B}(\overline{r}, t)$. The resultant expression can be converted to describe the electric field intensity as

$$\overline{E}(\overline{r}, t) = \frac{\mu_o}{4\pi} \sum_N \int \left[\frac{\overline{j}(\overline{r}', t') \times \overline{r}}{r^3} + \frac{\frac{\delta \overline{j}}{\delta t}(\overline{r}', t') \times \overline{r}}{cr^2} \right] dv \tag{11}$$

, where the summation is over each of the N conducting volume cells of the discretized structure. Equation (11) can be simplified for far field calculations by excluding the first term, prop $\frac{1}{r^3}$, in the brackets. This approximation have been used for the 3 m electric field calculations in this paper since the frequencies are high, $>$ 200 MHz, and the distance to the observation point, $\overline{r}$, is large, 3 m.

Electric field sensor

To be able to exclude the post-processing step in a electric field simulation would decrease the work load for the user and also minimize the potential for man made errors.

In the proposed solution[12] the sensor is realized as a PEEC one-cell with some modifications. First, the partial self inductance of the sensor is excluded. Second, the sensor is not coupled to the test object. But, the test object is coupled to the sensor in a normal fashion using partial mutual inductances and partial mutual coefficients of potentials. The electric field is calculated from the sensor end potentials(node potentials) and the induced voltage in the one-cell. This is a powerful formulation since it allows for sensors at indefinitely many locations in one single run. For instance, sensors could be placed around a test object in a sphere configuration to detect e-field maximums. The increase in unknowns are also acceptable, for a n inductive and m capacitive partitioned PEEC the additional coupling terms are $\leq n + 2m$ for one single sensor. The main drawback is the numerical difficulties arising when calculating the partial mutual couplings for the EUT and the sensor. Even for distances no more than 3 m the couplings typically are $< 4 \cdot 10^{-4}$. The length of the sensor was also shown to be a critical parameter. The 'calculated' field strengths differs significantly, in the order of $\pm$10 dB, with the sensor length. In the simulations the sensor length, inductive partition, was set to no more then $\frac{\lambda_{\min}}{10}$, according to established PEEC design rules. The sensor width and thickness were set to $\frac{1}{10}$ of the length.

Experimental setup

The measurements were performed in a anechoic EMC chamber. The PCB was placed on a plastic table 1 m above the floor and fed using a Rohde & Schwartz SME03E signal generator. The radiated emissions were measured with a Chase BiLog antenna connected to a Rohde & Schwartz ESPC EMI test receiver. The absolute accuracy for the emission measurements performed at the test location has been estimated to ±7 dB, indicated with the light grey dashed lines in Figure 5 and 6.

The PCB used in this paper, Figure 4, is a 61×40 mm RC circuit with surface mounted components originally presented in [3]. The PEEC model, used in the simulations, consists of 15 inductive and 13 capacitive partitions. All the mutual coupling terms and conductor resistances are used but the dielectric medium was not considered. The field strengths at a 1 and 3 meter distance from the PCB to the reference point at the antenna were measured. The observation point, $\overline{r}$, was shifted to four different locations centered and perpendicular to the PCB traces. The lower frequency limit, 200 MHz, is set to ensure far field conditions for the post-processing equation. The upper frequency limit, 1 GHz, is set by the PEEC model partitioning and the receiver antenna.

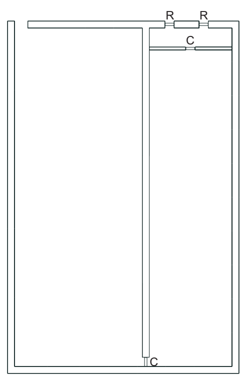

Fig. 4. Test circuit

Results

Typical results for the calculated and simulated electric field strengths compared against measurements at a 1 and 3 meter distance can be seen in Figure 5 and 6 respectively. As expected, the measurements and the two simulation techniques do not show perfect agreement. The sensor result is slightly better for near field simulations while the post-processing technique is better for far field calculation. The typical results shows also an underestimating of the electric field strength under 500 MHz and a overestimating at frequencies over 500 MHz. This has been observed before, for example in [6]. The tests has also shown that the length of the sensor is a critical parameter. By changing the sensor length, the field strength curve is either raised or lowered. Since the mutual coupling terms between the PCB and the sensor are $< 4 \cdot 10^{-4}$ for a 3 m distance, the use of the sensor at a 10 m distance could introduce possible numerical problems.

Conclusions

The study performed has shown that the use of post-processing tools are more reliable then the use of an electric field sensor. The possible reasons are the sensitivity to the sensor length and the potential numerical problem with the weak coupling coefficients for far field prediction. This indicates that the sensor formulation has to be improved.

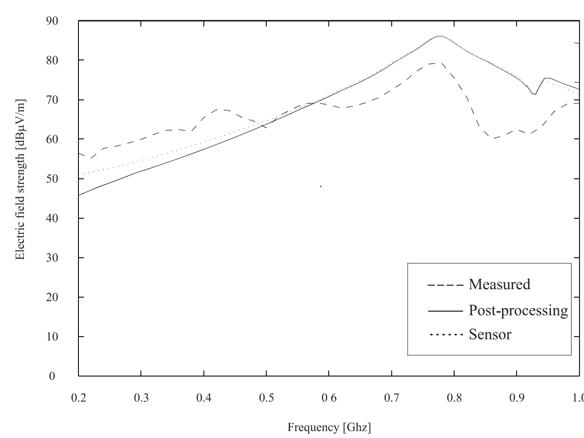

Fig. 5. Magnitude of electric field strength at 1m

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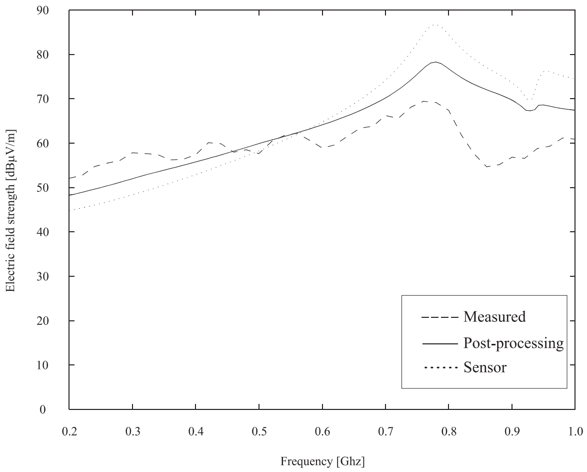

Fig. 6. Magnitude of electric field strength at 3m

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Paper B

Simplified PEEC Models for PCB Structures and Comparison to Experimental Data

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Simplified PEEC Models for PCB Structures and Comparison to Experimental Data

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Abstract— **In this paper a technique to obtain simple Partial Element Equivalent Circuit (PEEC) models for single layer PCB structures is presented. The partial elements are easily calculated using modified versions of closed form equations. The input impedance of a single layer PCB and the reflection coefficient for a 9 element LPDA measured using a network analyzer are compared against simplified PEEC model simulations. It is shown that the simplified PEEC models show good agreement for PCB problems.**

I. Introduction

The PEEC method was developed by Ruehli [1], [5], [8] and is based on the conversion of the Mixed Potential Integral Equation (MPIE) to the circuit domain. By using a specialized discretization, the original structure is converted into a network of discrete inductances, capacitances and resistances, called the partial elements. The capacitive and inductive couplings are modelled using partial mutual elements which results in a electromagnetic correct model where additionally discrete components like transmission lines and voltage/current sources are easily included. The partial elements are calculated either by using numerical integration techniques or simplified closed form equations. The resultant equivalent circuits are solved with conventional circuit solvers like SPICE [12] and the same equivalent circuit can be used to obtain results in the time and frequency domain.

The calculation of the partial elements involves the evaluation of second and fourth order integrals. These integrals are often solved using numerical integration resulting in increased computation time and susceptibility to numerical errors. By using closed form equations [1]-[5] for the partial elements the PEEC method grows to an accessible EM simulation technique requiring only a general circuit simulation program as the 'solver'. But, for a full-wave solution, time retardation must be included to account for finite travel times between the inductive and capacitive partitions.

A technique to produce simplified PEEC models for i.e. printed antennas and circuit board layouts is presented. The use of closed form equations to calculate the partial elements are combined with a discretization procedure where partial elements with small effect on the final PEEC model are excluded. This results in simplified equivalent circuits with good agreement useful in a design process. This technique facilitates the development of the equivalent circuits and has been used to analyze several PCB structures, and the results have been compared to measurements.

II. Derivation of the PEEC model

The starting point for the theoretical derivation is the summation of the electric field, E, in a multiconductor system expressed in terms of the vector magnetic potential $\overline{A}$ and the scalar electric potential Φ

$$\overline{E}(\overline{r},t) = -\frac{\partial}{\partial t}\overline{A}(\overline{r},t) - \nabla\Phi(\overline{r},t) \tag{1}$$

For a system containing K conductors the free-space Green's function potentials are given by

$$\overline{A}(\overline{r},t) = \sum_{k=1}^{K} \frac{\mu}{4\pi} \int_{v_k} \frac{\overline{J}(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k \tag{2}$$

and

$$\Phi(\overline{r},t) = \sum_{k=1}^{K} \frac{1}{4\pi\varepsilon} \int_{v_k} \frac{q(\overline{r}',t')}{|\overline{r}-\overline{r}'|} dv_k \tag{3}$$

, where

$$t' = t - \frac{|\overline{r}-\overline{r}'|}{v} \tag{4}$$

, denotes the retardation time in the medium with propagation speed v. The charge density q considers both the bound charges and the charges bounded in the dielectric regions. The expression for the current density $\overline{J}$ must be modified [11] to include the conduction current density $\overline{J}_C$ and the polarization current density in the dielectric medium according to

$$\overline{J} = \overline{J}_C + \varepsilon_o(\varepsilon_r - 1)\frac{\partial \overline{E}}{\partial t} \tag{5}$$

The total electric field at the surface of a conductor, $\overline{E}(\overline{r}, t)$, can be expressed using the current density and conductivity, σ, if no incident field is considered, and by combining equations (1), (2), (3) this gives equation (6)

$$\begin{aligned}\frac{\overline{J}(\overline{r}, t)}{\sigma} + \sum_{k=1}^{K} \frac{\mu}{4\pi}\int_{v_k} \frac{\partial}{\partial t}\frac{\overline{J}(\overline{r}', t')}{|\overline{r} - \overline{r}'|}dv_k + \\ \sum_{k=1}^{K} \varepsilon_o(\varepsilon_r - 1)\frac{\mu}{4\pi}\int_{v_k} \frac{\partial^2}{\partial t^2}\frac{\overline{E}(\overline{r}', t')}{|\overline{r} - \overline{r}'|}dv_k + \\ \sum_{k=1}^{K} \frac{1}{4\pi\varepsilon_o}\nabla\left[\int_{v_k} \frac{q(\overline{r}', t')}{|\overline{r} - \overline{r}'|}dv_k\right] = 0\end{aligned} \tag{6}$$

To solve the system of equations in (6), the current and charge densities are discretized into volume and surface cells respectively, Figure 1. The current volume cells lead the current between the nodes and the charge surface cells represent the node charge. Inside the cells the variables are constant. Applying the Galerkin method K equations are obtained for the K volume cells of the structure. The interpretation of these equations as a loop [13] leads to the structure of the equivalent circuit for a PEEC cell, Figure 2. In Figure 2, Rm is the volume cell dc resistance calculated as

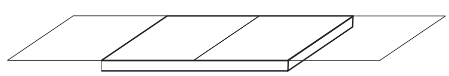

Fig. 1. Volume cell with two surface cells

$$R_m = \frac{l_m}{a_m \cdot \sigma} \tag{7}$$

where l_m is the cell length in the direction of the current flow, a_m is the cell cross-section and σ is the conductivity of the specific material. Lp is the partial inductance calculated as

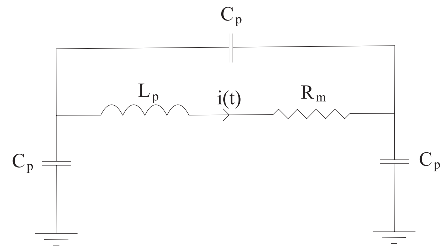

Fig. 2. Equivalent circuit model for a PEEC cell

$$Lp_{mn} = \frac{\mu}{a_m a_n}\frac{1}{4\pi\,|\overline{r} - \overline{r}'|}\int_{a_m}\int_{a_n}\int_{l_m}\int_{l_n} da_m da_n d\overline{l}_m \cdot d\overline{l}_n \tag{8}$$

If $n = m$, equation (8) represents the partial self inductance of the $m:th$ cell. And if $n \neq m$, equation (8) represents the partial mutual inductance between the $m:th$ and the $n:th$ cell. Cp is the partial capacitance that can be calculated from the coefficients of potential P_{ij} [6], where

$$P_{ij} = \frac{1}{\varepsilon_o a_{si} a_{sj}}\frac{1}{|\overline{r} - \overline{r}'|}\int_{a_{si}}\int_{a_{sj}} da_{si} da_{sj} \tag{9}$$

If $i = j$ in equation (9), the partial self coefficient of potential of the $j:th$ surface cell is calculated. And if $i \neq j$, equation (9) represents the partial mutual coefficient of potential between the $i:th$ and the $j:th$ surface cell.

For PEEC models where dielectric regions must be considered, the PEEC method has been extended through the use of dielectric cells [11].

III. Simplified PEEC models

If the partial elements are calculated using equation (8) and (9) some numerical integration technique has to be used. For general PEEC models this is the correct approach to ensure the accuracy of the partial elements. But for simple structures, like single layer PCB structures, it is possible to use only three closed form equations for the calculations. This makes the PEEC method an accessible EM simulation technique requiring only a general circuit simulation program like SPICE as the solver. The PEEC method can now be used for example in undergraduate courses to facilitate the understanding of EM theory or as an aid in a design process.

The approach is to try to discreticise the PCB structures into parallel rectangular surface and rectangular parallelepiped volume cells, which is required for the closed form equations. This can be accomplished by using a minimum of three nodes at every orthogonal interconnection. This is illustrated in Figure 3 where three nodes have been placed in the corner separating one x-directed and one y-directed inductive partition. Without these nodes the resulting capacitive partition is L-shaped and the coefficients of potentials are more difficult to calculate. The drawback of this method is the increase of unknowns for the final PEEC model, but the additional number of unknowns can be reduced by:

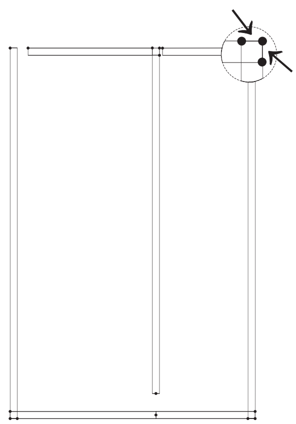

Fig. 3. Inductive and capacitive partition for PCB example circuit

1. Excluding the self-capacitance and capacitive couplings to the small corner cells (one indicated with an arrow in Figure 3(right)).
2. Excluding the inductive coupling terms for the two extra inductive cells of each corner (one pair indicated with arrows in Figure 3(left)).
3. Excluding the cell resistances for the extra inductive cells of each corner.

Excluding partial elements like this can be done if the magnitude of the partial elements are small compared to the other, this has been observed in [9]. This result in an simplified EM model of the original circuit with easy calculated partial elements from the closed form equations. All partial self and mutual coefficients of potentials (capacitive elements) are calculated using one single equation, and for the partial self and mutual inductive elements two equations are used.

IV. Equations for partial element calculations

In this section a complete set of equations for the calculation of the partial elements are presented. The equations are originally published in [1]-[5] for three-dimensional structures but have been modified to fit single layer PCB structures.

A. Partial self inductance

When calculating the partial self inductance of a volume cell, equation (10) from [1] has been used. Using the notations in Figure 4 and the following normalizations $u = l/w$ and $\omega = t/w$ the partial self inductance of a $10 \times 1 \times 0.05$ mm ($l \times w \times t$) cell becomes 6.96 nH.

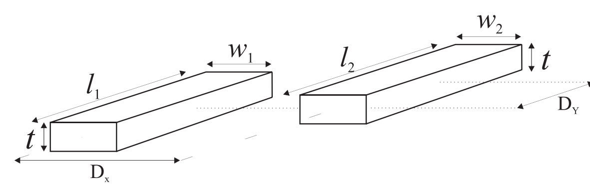

Fig. 4. Notations for partial inductance calculations

B. Partial mutual inductance

The partial mutual inductance between two inductive partitions correspond to the magnetic coupling in the PEEC model. For the calculations equation (11) is used with the notations from Figure 4. Equation (11) was originally published in [2], but is presented in a modified form suitable for single layer PCB's.

$$\begin{aligned} Lp_{km} &= \frac{\mu}{4\pi}\frac{1}{w_1 w_2}\sum_{i=1}^{4}\sum_{j=1}^{4}(-1)^{i+j}\left[\frac{b_j^2 a_i}{2}\ln(a_i+\rho_{ij})\right. \\ &\quad \left. +\frac{a_i^2 b_j}{2}\ln(b_j+\rho_{ij}) - \frac{\rho_{ij}}{6}(b_j^2+a_i^2)\right] \quad (11) \end{aligned}$$

where

$$\rho_{ij} = \sqrt{a_i^2 + b_j^2}$$

$$a_1 = D_x - \tfrac{l_1}{2} - \tfrac{l_2}{2} \quad a_2 = D_x + \tfrac{l_1}{2} - \tfrac{l_2}{2}$$
$$a_3 = D_x + \tfrac{l_1}{2} + \tfrac{l_2}{2} \quad a_4 = D_x - \tfrac{l_1}{2} + \tfrac{l_2}{2}$$

$$b_1 = D_y - \tfrac{w_1}{2} - \tfrac{w_2}{2} \quad b_2 = D_y + \tfrac{w_1}{2} - \tfrac{w_2}{2}$$
$$b_3 = D_y + \tfrac{w_1}{2} + \tfrac{w_2}{2} \quad b_4 = D_y - \tfrac{w_1}{2} + \tfrac{w_2}{2}$$

Using the cell from the previous section, the mutual inductance between two volume cells with a

$$
\begin{aligned}
\frac{Lp_{ii}}{l} = & \frac{2\mu}{\pi}\{\frac{\omega^2}{24u}\left[\ln(\frac{1+A_2}{\omega})-A_5\right]+\frac{1}{24u\omega}\left[\ln(\omega+A_2)-A_6\right] \\
& +\frac{\omega^2}{60u}(A_4-A_3)+\frac{\omega^2}{24}[\ln(\frac{u+A_3}{\omega})-A_7]+\frac{\omega^2}{60u}(\omega-A_2)+\frac{1}{20u}(A_2-A_4) \\
& +\frac{u}{4}A_5-\frac{u^2}{6\omega}\tan^{-1}(\frac{\omega}{uA_4})+\frac{u}{4\omega}A_6-\frac{\omega}{6}\tan^{-1}(\frac{u}{\omega A_4})+\frac{A_7}{4} \\
& -\frac{1}{6\omega}\tan^{-1}(\frac{u\omega}{A_4})+\frac{1}{24\omega^2}\left[\ln(u+A_1)-A_7\right]+\frac{u}{20\omega^2}(A_1-A_4) \\
& +\frac{1}{60\omega^2 u}(1-A_2)+\frac{1}{60u\omega^2}(A_4-A_1)+\frac{u}{20}(A_3-A_4) \\
& +\frac{u^3}{24\omega^2}\left[\ln(\frac{1+A_1}{u})-A_5\right]+\frac{u^3}{24\omega}\left[\ln(\frac{\omega+A_3}{u})-A_6\right] \\
& +\frac{u^3}{60\omega^2}\left[(A_4-A_1)+(u-A_3)\right]\}
\end{aligned}
\tag{10}
$$

, where

$$
\begin{array}{llll}
A_1=\sqrt{1+u^2} & A_2=\sqrt{1+\omega^2} & A_5=\ln(\frac{1+A_4}{A_3}) \quad A_6=\ln(\frac{\omega+A_4}{A_1}) \quad A_7=\ln(\frac{u+A_4}{A_2}) \\
A_3=\sqrt{\omega^2+u^2} & A_4=\sqrt{1+\omega^2+u^2} &
\end{array}
$$

---

center to center distance of 0 and 10 mm (D_x, D_y) is 0.94 nH.

C. Partial coefficients of potential

Due to a central difference approximation used in the theoretical derivation of the PEEC method, the surface cells are shifted one half cell length to the volume cells, as indicated in Figure 1. The magnitude of the capacitive coupling between the surface cells are calculated from the coefficients of potential. If no delay times between the surface cells are considered, discrete capacitances can be used to model the coupling. But, for finite delay times coupled sources are used [10]. For alternative capacitive representations see [6].

Equation (12), for 3D version see [5], is used to calculate both the partial self and mutual coefficients of potentials. For the self term, with the notations from figure 5, use $a_x = b_x, a_y = b_y$ and $d_x = d_y = 0$.

$$
\begin{aligned}
ps_{ij} = & \frac{1}{4\pi\varepsilon}\frac{1}{a_x a_y b_x b_y} \\
& \sum_{k=1}^{4}\sum_{m=1}^{4}(-1)^{m+k}[\frac{b_m^2 a_k}{2}\ln(a_k+\rho_{km}) \\
& +\frac{a_k^2 b_m}{2}\ln(b_m+\rho_{km})-\frac{\rho_{km}}{6}(b_m^2+a_k^2)]
\end{aligned}
\tag{12}
$$

where

$$\rho_{km} = \sqrt{a_k^2+b_m^2}$$

$$
\begin{array}{ll}
a_1 = d_x - \frac{a_x}{2} - \frac{b_x}{2} & a_2 = d_x + \frac{a_x}{2} - \frac{b_x}{2} \\
a_3 = d_x + \frac{a_x}{2} + \frac{b_x}{2} & a_4 = d_x - \frac{a_x}{2} + \frac{b_x}{2}
\end{array}
$$

$$
\begin{array}{ll}
b_1 = d_y - \frac{a_y}{2} - \frac{b_y}{2} & b_2 = d_y + \frac{a_y}{2} - \frac{b_y}{2} \\
b_3 = d_y + \frac{a_y}{2} + \frac{b_y}{2} & b_4 = d_y - \frac{a_y}{2} + \frac{b_y}{2}
\end{array}
$$

For a surface cell of 10×1 mm the self coefficient of potential is 6.34 pF^{-1}. The mutual coefficient of potential between two 'touching' 10×1 mm surface cells is 1.22 pF^{-1} and between a 10×1 mm and a 5×1 mm surface cell, 1.66 pF^{-1}.

V. Experiments

The technique described has been used to analyze PCB structures such as dipoles, LPDA's and patch antennas where the discreticised partition ratios has been; $0.03 < \frac{T}{W} < 0.1$ and $5 < \frac{L}{W} < 100$.

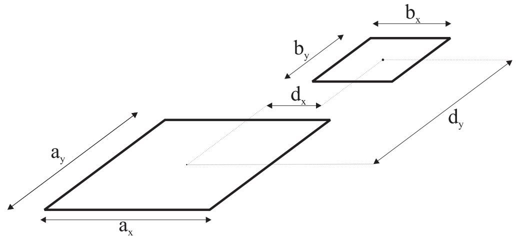

Fig. 5. Notations for coefficients of potential calculation

Simple Java programs have been used to generate the resulting circuit-files and a specialized SPICE based retarded solver [7] has been used for the simulations. In the experiments, the dielectric material has not been take into account, except for the LPDA where an effective ε_r have been used [14].

The upper frequency limits for the experiments are set by the largest PEEC model cell length to assure that all cells are less then $\frac{\lambda}{20}$, where λ is the smallest wavelength.

A. $\frac{\lambda}{2}$ Dipole

The calculated partial elements from the previous sections can be used to simulate the resonance frequency for a simple half-wavelength dipole. If each arm of the dipole is modelled using five volume cells, the resulting 100 mm dipole would have a theoretical resonance frequency of 1.5 GHz. If the inductive and capacitive couplings are realized using mutual elements only between the 'touching' volume/surface cells the resonance frequency is predicted to 1444 MHz. Further simulations made on the $\frac{\lambda}{2}$ dipole indicates that a finer discretization and the inclusion of all mutual elements improve the accuracy of the PEEC model.

B. One layer PCB circuit

In Figure 7 the simulated input impedance for the PCB structure in Figure 6 connected to a 0.1m long 50 Ω coax cable is compared with network analyzer measurements.

The model consists of 15 self and 18 mutual partial inductances and 13 self and 156 mutual partial capacitances calculated using equations (10)-(12). The agreement compared to measurements are good up to about 1 GHz. Tests performed on the dipole indicates that for higher frequencies a finer discretization could improve the accuracy of the model.

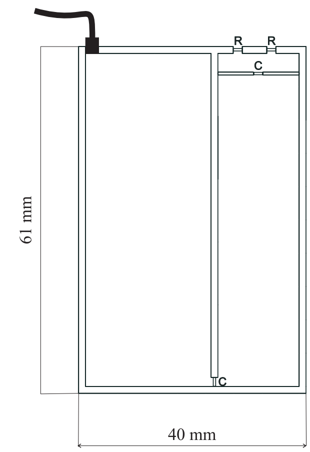

Fig. 6. PCB test circuit

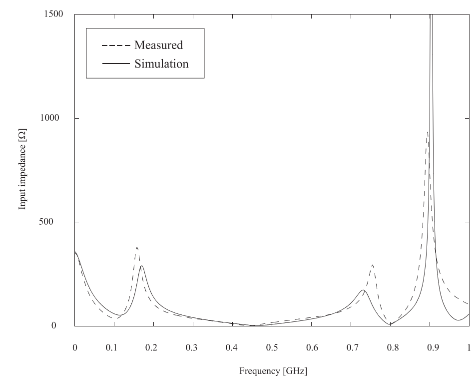

Fig. 7. Input impedance for PCB test circuit

C. LPDA

In Figure 9 the reflection coefficient for the nine element Log-periodic dipole array, Figure 8, printed on a 1.6mm thick dielectric plate is compared to network analyzer measurements.

This is a more complicated geometry and the model consists of 246 self and a maximum of 34308 mutual partial inductances and 196 self and a maximum of 196^2 mutual partial capacitances. For this PEEC model, the unmodified three-dimensional equations from [1]-[5] was used with the proposed discretization strategy. The 3D equations must be used because the LPDA has antenna elements on both sides of the dielectric substrate. The model accurately predicts two LPDA resonance peeks at 800 and 1125 MHz but fails to predict the magnitude of the reflection coefficient at lower frequencies. To model the LPDA at higher frequencies a finer discretization could improve the accuracy of the model. And, since the dielectric regions are of more importance for this structure, the use of dielectric cells could also improve the accuracy instead the use of an effective ε_r.

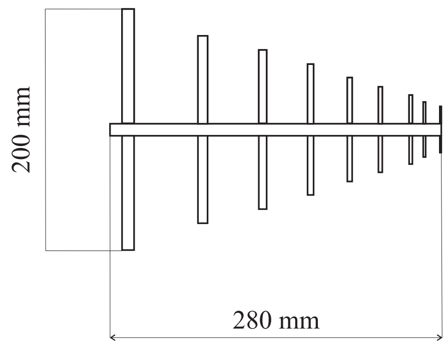

Fig. 8. Log periodic dipole

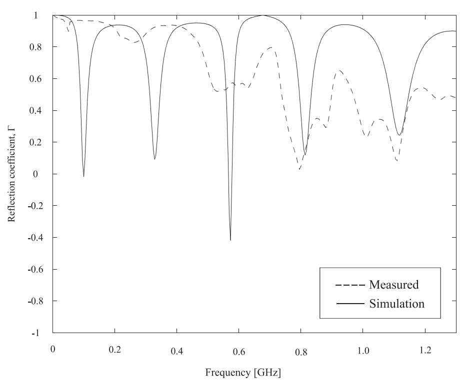

Fig. 9. Reflection coefficient for 9 element LPDA

VI. Conclusions

The proposed method to generate simplified PEEC models for PCB structures have been verified by comparing simulations against measurements for several prototype PCB's. The agreement is very good for the investigated single layer PCB's in experiment A and B where the PEEC models are obtained using equations (10)-(12). For more complicated geometries, like the presented 9 element LPDA, the accuracy is not that good. It is possible that the simplifications are to extensive for two layer structures.

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Paper C

Analysis of Printed Antenna Structures using the Partial Element Equivalent Circuit Method

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Analysis of Printed Antenna Structures using the Partial Element Equivalent Circuit (PEEC) Method

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Abstract— **The partial element equivalent circuit (PEEC) method is a electromagnetic simulation technique suitable for mixed circuit and field problems. The technique is numerically equivalent to a method of moments solution using Galerkin solution. In this paper, the PEEC method is illustrated and applied to printed antenna structures where measurements are compared to simulations and analytical solutions. The possibility to use simplified PEEC models to decrease computation time is discussed with illustrative examples.**

I. Introduction

The Partial Element Equivalent Circuit (PEEC) method is a full wave technique for the solution of mixed circuit and field problems in both the time and frequency domain. The international interest for the method has been gaining rapidly for the past years but in the Nordic countries the research effort has been low. This paper can be considered as an fundamental introduction to this electromagnetic computation technique.

The method is based on the conversion of the Mixed Potential Integral Equation (MPIE) to the circuit domain. By using a specialized discretization, the original structure is converted into a network of discrete inductances, capacitances and resistances, called the partial elements. The capacitive(electric field) and inductive(magnetic field) couplings are modelled using partial mutual elements which results in a electromagnetic correct model. The partial elements are calculated either by using numerical integration techniques or simplified closed form equations. The resulting equivalent circuits are solved by using a commercial circuit simulation program like SPICE. The use of SPICE-like circuit solvers facilitates the inclusion of discrete components, transmission lines, current/voltage source etc in the resulting PEEC model.

Since the method is based on an integral equation formulation the analysis of 'open air' problems like radiation from antennas are computationally efficient. To have efficient electromagnetic computation techniques for printed antenna structures can be important to speed up the development of for example mobile embedded internet systems.

II. Derivation of the PEEC model

The starting point for the theoretical derivation is the summation of the electric field, E, at a field point, $\overline{r}$, in a multiconductor system expressed in terms of the vector magnetic potential $\overline{A}$ and the scalar electric potential Φ at a source point $\overline{r}'$.

$$\overline{E}(\overline{r},t) = -\frac{\partial}{\partial t}\overline{A}(\overline{r},t) - \nabla\Phi(\overline{r},t) \tag{1}$$

For a system containing K conductors the free-space Green's function retarded potentials are given by

$$\overline{A}(\overline{r},t) = \sum_{k=1}^{K}\frac{\mu}{4\pi}\int_{v_k}\frac{\overline{J}(\overline{r}',t')}{|\overline{r}-\overline{r}'|}dv_k \tag{2}$$

and

$$\Phi(\overline{r},t) = \sum_{k=1}^{K}\frac{1}{4\pi\varepsilon}\int_{v_k}\frac{q(\overline{r}',t')}{|\overline{r}-\overline{r}'|}dv_k \tag{3}$$

, where

$$t' = t - \frac{|\overline{r}-\overline{r}'|}{v} \tag{4}$$

, denotes the retardation time in the medium with propagation speed v. The charge density q considers both the bound charges and the charges bounded in the dielectric regions. The expression for the current density $\overline{J}$ must be modified [12] to include the conduction current density $\overline{J}_C$ and the polarization current density in the dielectric medium according to

$$\overline{J} = \overline{J}_C + \varepsilon_o(\varepsilon_r - 1)\frac{\partial \overline{E}}{\partial t} \tag{5}$$

Since the total electric field at the surface of a conductor, $\overline{E}(\overline{r}, t)$, can be expressed using the current density and conductivity, σ, and if no incident field is considered equation (6) is obtained by combining equations (1), (2), (3).

$$\frac{\overline{J}(\overline{r}, t)}{\sigma} + \sum_{k=1}^{K} \frac{\mu}{4\pi} \int_{v_k} \frac{\partial}{\partial t} \frac{\overline{J}(\overline{r}', t')}{|\overline{r} - \overline{r}'|} dv_k +$$

$$\sum_{k=1}^{K} \varepsilon_o(\varepsilon_r - 1) \frac{\mu}{4\pi} \int_{v_k} \frac{\partial^2}{\partial t^2} \frac{\overline{E}(\overline{r}', t')}{|\overline{r} - \overline{r}'|} dv_k + \tag{6}$$

$$\sum_{k=1}^{K} \frac{1}{4\pi\varepsilon_o} \nabla \left[\int_{v_k} \frac{q(\overline{r}', t')}{|\overline{r} - \overline{r}'|} dv_k \right] = 0$$

To solve the system of equations in (6), the current and charge densities are discretized into volume and surface cells respectively, Figure 1(top). The solution requires also that the surface cells are shifted half a cell length to the volume cells as indicated in Figure 1(top). The current volume cells lead the current between the nodes and the charge surface cells represent the node charge. Inside the cells the variables are constant. Applying the Galerkin

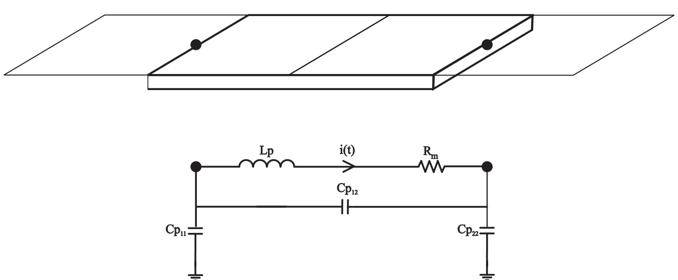

Fig. 1. Conductor discretisation(top) and corresponding PEEC model(bottom)

method K equations are obtained for the K volume cells of the structure. The interpretation of these equations as a loop [15] leads to the structure of the equivalent circuit for a PEEC cell, Figure 1(bottom).

The practical implications of this solution requires that fixed nodes are placed on the structure under test. From these nodes two partitions are made. The *inductive partition* is based on the volume cells between two subsequent nodes. And, the *capacitive partition* is based on the surface cells associated to each node. This is illustrated in Figure 2. The two partitions are used for the calculation

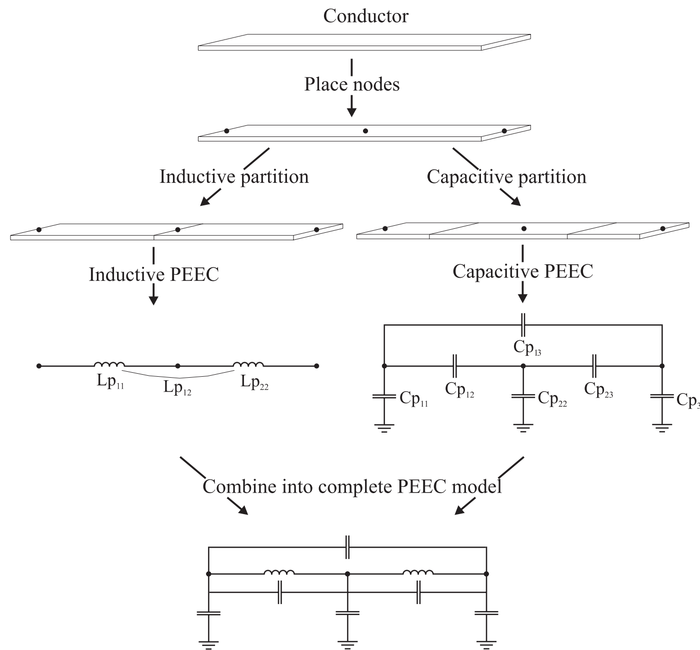

Fig. 2. PEEC method procedure

of the partial elements according to the follwowing section.

A. *Partial elements*

The discrete components in Figure 1 and 2 are denoted partial elements and the calculations are performed based on the geometrical shape of the inductive and capacitive partitions using numerical integration or closed-form equations.

A.1 Partial resistance

In Figure 1(bottom), R_m is the volume cell dc resistance of the volume cells in the inductive partitions. The resistance in the PEEC model correspond to conductive losses and the inclusion results in a *(R)PEEC model*. The resistance is simply the dc resistance of the conductor calculated as:

$$R_m = \frac{l_m}{a_m \cdot \sigma} \tag{7}$$

where l_m is the volume cell length in the direction of the current flow, a_m is the cell cross-section and σ is the conductivity of the specific material.

A.2 Partial inductance

In Figure 1 and 2 Lp is the partial inductance calculated using the inductive partition and equation (8).

$$Lp_{mn} = \frac{\mu}{a_m a_n} \frac{1}{4\pi \left|\bar{r} - \bar{r}'\right|} \int_{a_m} \int_{a_n} \int_{l_m} \int_{l_n} d\bar{l}_m \cdot d\bar{l}_n da_m da_n \tag{8}$$

If $n = m$, equation (8) represents the partial self inductance of the $m : th$ cell. This is the internal inductance of the volume cell and is connected in series with the partial resistance between the nodes.

If $n \neq m$, equation (8) represents the partial mutual inductance between the $m : th$ and the $n : th$ cell. This corresponds to the magnetic field coupling between the volume cells. In SPICE this effect is modeled using the K command. This representation of the partial mutual inductance is instantaneous, meaning that the current in one conductor affects all other conductors at once. For volume cells that are far apart, or for high frequencies, this representation becomes unvalid since the field couplings occur at finite travel times. This problem is solved by the use of delayed, retarded, current- or voltage sources [11]. This is described in Figure 3(left) where current i_{2_1} is instantly induced in conductor 2 due to the current in conductor one through the partial mutual inductance Lp_{12}. In Figure 3(right) the partial mutual inductance is coupled to a current source with a specified delay time.

But since delayed sources are not supported by most commercial circuit simulation tools like SPICE a specialized solver must be used, as described in [16]. The inclusion of partial inductances and retardation in a PEEC model is denoted a *(Lp,τ)PEEC* model.

Since the calculation of the partial inductances using equation (8) is both complex and time consuming a set of closed form equations has been presented, see [1] and [7].

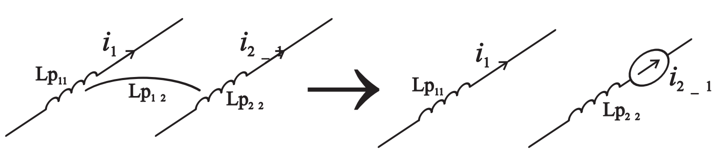

Fig. 3. Magnetic field coupling using delayed current sources

A.3 Partial capacitance

Cp in Figure 1 and 2 is the partial capacitance that can be calculated from the coefficients of potential P_{ij} [5], where

$$P_{ij} = \frac{1}{\varepsilon_o a_{si} a_{sj}} \frac{1}{\left|\bar{r} - \bar{r}'\right|} \int_{a_{si}} \int_{a_{sj}} da_{si} da_{sj} \tag{9}$$

The coefficients of potential is an alternative capacitive representation relating the surface potential, V, to the surface charge, Q, according to

$$V = PQ \tag{10}$$

The regular capacitance notation, C, is the inverse to the coefficients of potential since

$$CV = Q \tag{11}$$

If $i = j$ in equation (9), the partial self coefficient of potential of the $j : th$ surface cell is calculated. This capacitance is connected between the corresponding nodes, surface cell, and the node at infinity.

If $i \neq j$, equation (9) represents the partial mutual coefficient of potential between the $i : th$ and the $j : th$ surface cell. This correspond to the electric field coupling between the surface cells. Since equation (10) is a instantaneous relationship the inversion of the coefficients of potential matrix, P, to the capacitance matrix, C, can only be done when retardation times are neglected. Then the diagonal elements in the C matrix represents the partial self capacitance to each surface cell, Cp_{ii} in Figure 1 and 2. The off diagonal elements are the partial mutual capacitances between the surface cells, Cp_{ij} in Figure 1 and 2.

When retardation times must be considered, delayed sources are used in a similar manner as for the partial inductances [11]. The inclusion of capacitances or coefficients of potential in a PEEC model is denoted a *(C)PEEC* or *(P)PEEC* model respectively.

Closed form equations for the calculation of the partial coefficients of potential, thus also partial capacitances, has been presented in [2].

A.4 Extended PEEC models

There are numerous features with the PEEC method that is not covered in this paper, as an example:

• For PEEC models where dielectric regions must be considered, the PEEC method has been extended through the use of dielectric cells [12].
• The excitation of the structure is usually performed using current- or voltage sources. In [13] the formulation was extended to include excitation using incident fields.
• An efficient Skin effect model were presented in [17].

III. Examples

In this section two examples are presented and the corresponding PEEC models are discussed. In the PEEC model examples the partial elements have been calculated using the closed form equations suggested in the previous section. For the retarded current/voltage source simulation a specialized solver has been used [16]. The accuracy of the PEEC models are compared against analytical solutions and measurements.

A. Half wavelength dipole

The $\frac{\lambda}{2}$dipole and monopole antennas are common radiating structures in electronic systems and is therefor of great importance. The simplicity of these structures makes it an ideal example for simple PEEC models.

To model the resonance frequency of an 40 cm free space $\frac{\lambda}{2}$ dipole, the upper frequency limit, corresponding $\lambda_{\min}$, must be specified. This must be known to assure that the dimension of the volume and surface cells in the PEEC model does not exceed $\frac{\lambda_{\min}}{10}$. The use of 20 mm volume elements makes the PEEC model valid up to 1.5 GHz and since the theoretical resonance frequency for the dipole is 375 MHz, this is choosen.

The first step in the modeling is to make the inductive and capacitive partition based on the cell lengths. This is shown in Figure 4 where all inductive volume elements are 20 mm long (the dipole cross-section was choosen to 1x1 mm). Since the capacitive cells are shifted half a cell length to the inductive cells the two antenna elements consists of 11 capacitive cells each (two 10 mm and nine 20 mm). Second, all partial elements are calculated using the proposed closed form equations. Third, the inductive and capacitive elments are combined into the PEEC model, as in Figure 4, and the desired analsis is performed.

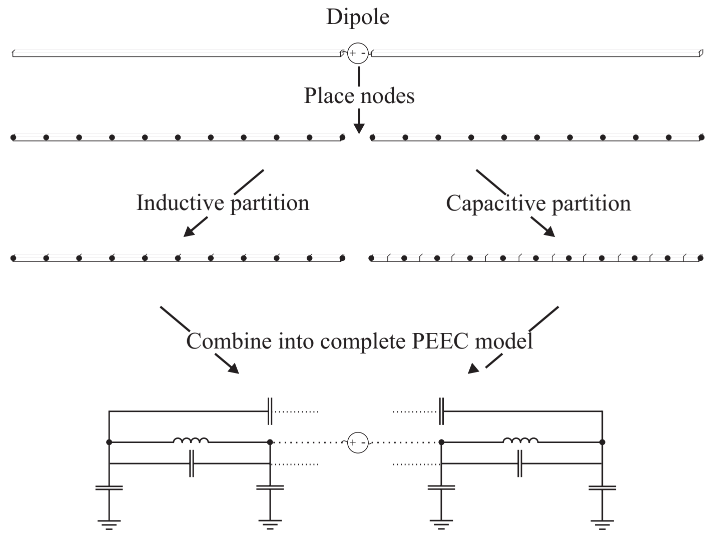

Fig. 4. The inductive and capacitive partitions for the $\frac{\lambda}{2}$dipole example

At first a simplified (R,L,C)PEEC model for the $\frac{\lambda}{2}$dipole was implemented without the partial mutual inductive and capacitive elements. The results is shown in Figure 5 where the resonance frequency is low by 20 % if no magnetic field coupling(dotted line) is included or high by 32 % if no electric field coupling(dash-dot line) is included compared to the correct solution, straight line in Figure 5. The performance of the model is improved by the use of all the partial mutual elements and the resonance frequency is predicted to 358 MHz (4.5 % off), Figure 5 (dash). If this PEEC model is upgraded to a retarded formulation using finite trave times between the partial mutual elements the resonance frequency is predicted to 376 MHz (< 0.3 % off), Figure 5 (solid).

The inclusion of time retardation in the PEEC model also introduces damping in the equivalent circuit. This is clearly visible in Figure 5 where the drive current for the non retarded formulation has a very high Q-value compared to the retarded formulation. Since the focus is on the PEEC method, the actual Q-value and drive current amplitude has not been investigated.

B. Patch antenna

The patch antenna is a very common structure in antenna applications where they appear as single patches or in array formulations. This structure is more complicated compared to the dipole antenna since the PEEC model must include (1) a

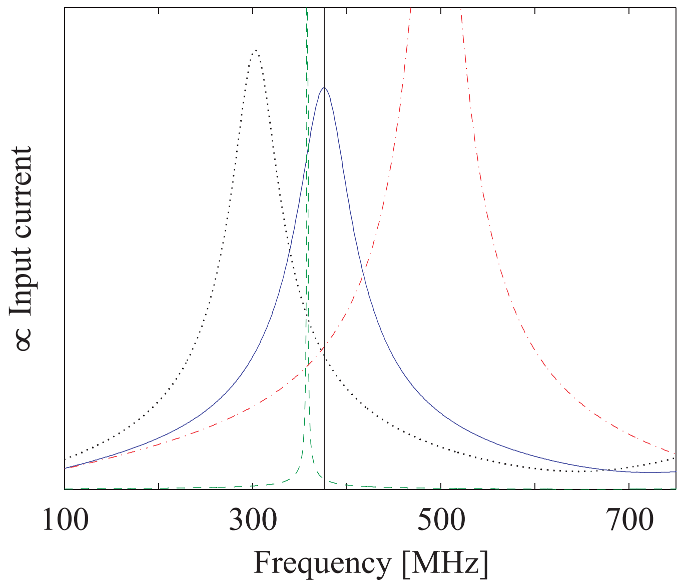

Fig. 5. Simulated resonance frequency for $\frac{\lambda}{2}$dipole using different PEEC models compared to analytical solution (line)

finite ground plane, (2) the dielectric medium between the planes and (3) a two dimensional current distriubution to correctly model the patch antenna.

To model the ground plane and the patch antenna using two dimensional current distribution [6], one x-directed and one y-directed inductive partition is made using the fixed nodes in a similar manner as described in Figure 2. This model could also be extended to include currents in the thickness of the patch and ground plane. But since three dimensional PEEC models are complicated the two dimensional representation was choosen.

In this example a 35 μm thick 62 $\times$ 99 mm patch antenna located on a 1.55 mm dielectric substrate, ε_r = 4.5, over a ground plane is modeled. The patch is shown i Figue 6 where the feeding point is marked with a ⊗-symbol. The basics of the two dimensional equivalent circuit for the patch and the ground plane is shown, in the figure top right corner, where the partial self inductances and the patial resistances are displayed. Note that the partial self capacitances and all partial mutual elements are excluded in the figure, for simplicity reasons, but not in the PEEC model.

The dielectric medium is not modelled using cells, instead an effective ε_r has been used [10]. As for the one dimensional case, the dipole, the partial elements were calculated using closed form equations.

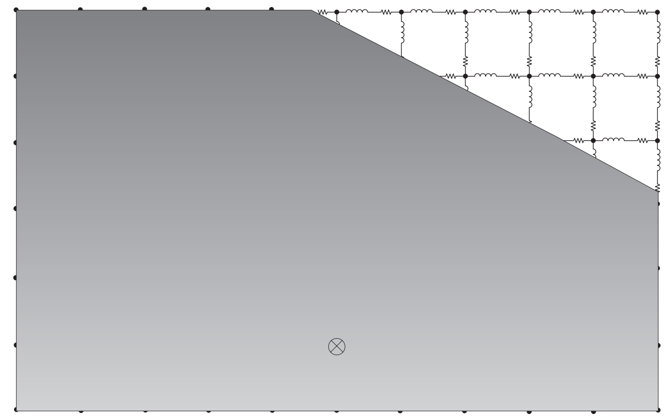

Fig. 6. Example patch antenna with cut to show the discrete components in the PEEC model

For this PEEC model the reflection coefficient, S11, was measured using a Rhode & Schwarz ZVR network analyzer. The measurements are compared to a *(Lp,P,R,τ)PEEC* model as shown in Figure 7.

The discretisation of the patch antenna into a max cell size of 20 $\times$ 20 mm results in a upper frequency limit of 1.5 GHz. This cell size is fine enough to model the first resonance around 1.2 GHz, indicated by measurements. As can be seen in the figure the maesurements and simulations are in close agreement, except for the rippel and the higher Q-value for the measured reflection coefficient.

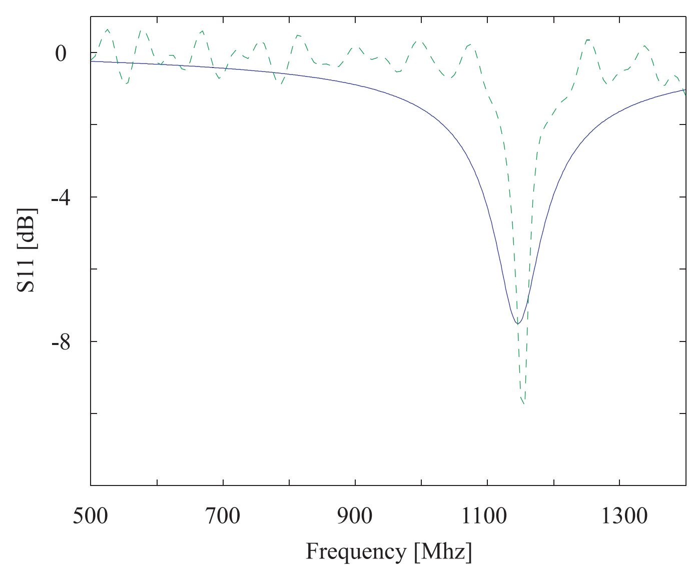

Fig. 7. Reflection coefficient for patch antenna where the dashed line is measurements and the solid line is simulation

The PEEC model used in the simulations consists of :

- 154 partial self coefficients of potential
- 154^2 partial mutual coefficients of potential
- 132 x-directed and 140 y-directed partial self inductances
- $132^2 + 140^2$ partial mutual inductances
- 272 partial resistances

And takes approximately 45 minutes to run on a PIII/750 MHz laptop computer. To speed up the computation time it is possible to exclude partial mutual elements with weak coupling coefficients, $<$ 0.15, without effecting the predicted resonance frequency more than +/- 5 %. The speed up cut computation times by approximately 25 minutes. To improve the accuracy of the PEEC model, dielectric cells and a finer partition could be used.

IV. Conclusions

The PEEC method has been shown to be a very powerful simulation technique for combined circuit and electromagnetic field problems. The first example displays the possibility to make PEEC models by using closed form equations to calculate the partial elements and a free version of SPICE as the solver. This feature makes the method possible to use in education and for simple design tasks. However, the method require a retarded circuit solver to be considered a full wave method comparable with a method of moments solution. The application of PEEC's to antennas is a valuable tool in many areas where antenna resonance frequencies are of importance. The paper shows that antennas are modelled with good agreement compared to analytical solutions and measurements.

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