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Finite Element Modeling of Power Electronic Circuits Containing Switches Attached to Saturable Magnetic Components

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Abstract: Finite element analysis of saturable lossy magnetic components is extended to include the effects of switches, which may be either mechanical or electronic. The switches are modeled using coupled electromagnetic and structural finite elements which are stretched to vary their electrical resistance with time. Good results are shown for a simple resistive circuit with a switched excitation and for a highly saturated transformer with a secondary load resistor that is switched on and off.

Keywords: Finite elements, switches, saturation, nonlinear, transient, eddy current, loss, transformer.

INTRODUCTION

Inductors, transformers, motors, and other magnetic components are often designed with the aid of electromagnetic finite element analysis. Such analysis can determine the saturable magnetic fields, eddy currents, and related performance parameters before a prototype magnetic component is built.

When magnetic components are used in power electronic circuits such as adjustable speed drives, switches are often present. Whether mechanical or electronic, the switches affect the electromagnetic fields and system performance.

One way of analyzing such switched components is to use state space techniques or other circuit methods [1]–[3]. The inductances of circuit components in such models can be pre–computed by finite element analysis. If eddy currents and/or saturation occur, however, their effects may be difficult to include in circuit models.

Another way to compute the interaction of circuits and magnetic components is to use a mixed–dimension finite element model. The circuits are modeled with 0D finite elements, attached to multiturn windings modeled with 1D finite elements, connected to magnetic components modeled with 2D or 3D finite elements. Such finite element models have been described in papers and books beginning in 1991 [4] – [9], and can include the effects of diodes and other nonlinear circuit devices. The nonlinear devices can act as switches, but must act at times that are functions of circuit currents and voltages.

No finite element modeling techniques can be found in the literature for switches which close and open at arbitrary times. Constraints cannot be used to model such switches, because when open their voltage is unknown, and when closed their current is unknown.

This paper presents a new method of using finite elements to model switched magnetic components. The switches can be closed and opened at any prescribed times. The paper begins by describing how coupled electromagnetic and structural finite elements can model a switch of any desired resistance as a function of time. After modeling a simple resistive circuit, the new method will be applied to a highly saturated transformer with a switched secondary resistor and with eddy current losses in its core.

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COUPLED ELECTROMAGNETIC AND STRUCTURAL FINITE ELEMENT ANALYSIS

Switches will be modeled as variable resistors using the coupled electromagnetic and structural capability of the finite element analysis software package EMAS [10] available from Ansoft Corp. This capability analyzes nonlinear transient coupled electromagnetic fields and structural stresses [11].

The electromagnetic potential vector $\{u_E\}$ consisting of magnetic vector potential **A** and time integrated electric scalar potential ψ is calculated by the matrix equation [9]:

$$[M_E]\{\ddot{u}_E\} + [B_E]\{\dot{u}_E\} + [K_E]\{u_E\} = \{F_E(t)\} \qquad (1)$$

where $[M_E]$ is the permittance matrix, $[B_E]$ is the conductance matrix, and $[K_E]$ is the reluctance matrix. The right hand side is $\{F_E(t)\}$, the electromagnetic excitation vector [4]–[8], which may involve circuit excitations of various kinds acting on multiturn windings [7],[8]. Equation (1) is solved at each time step for linear or nonlinear materials. From the computed potentials at all times, the time–varying fields and currents are calculated.

The structural displacement vector $\{u_S\}$ is calculated by the structural finite element matrix equation [9], [10]:

$$[M_S]\{\ddot{u}_S\} + [B_S]\{\dot{u}_S\} + [K_S]\{u_S\} = \{F_S(t)\} \quad (2)$$

where $[M_S]$ is the mass matrix, $[B_S]$ is the damping matrix, and $[K_S]$ is the stiffness matrix. Equation (2) is solved at every time step given the right hand side force vector $\{F_S(t)\}$. The displacements $\{u_S\}$ are added to the node locations in (1) at each time step, and thus $[M_E]$, $[B_E]$, and $[K_E]$ can vary with time. Hence the electromagnetic and structural solutions are coupled.

MODELING A 1D TIME–VARYING RESISTOR

The force vector $\{F_S(t)\}$ of (2) can be varied over time from 0 to some high value, thereby causing an electrically conductive finite element to vary in length. Because resistance is proportional to length, a variable resistor is thereby included in $[B_E]$ for the electromagnetic solution (1). A resistor varying from a very low value to a very high value simulates a switch.

To make sure that the length (displacement) of the finite element is instantaneously proportional to force $F_S(t)$, the structural properties of that element must be pure elastic. That is, it should make zero contributions to $[M_S]$ and $[B_S]$ of (2). Its $[K_S]$ contribution will be constant (linear).

The simplest way to model a variable resistive switch is with a one–dimensional (1D) finite element. It is a conductive wire element in the electromagnetic model of (1) and a rod finite element in the structural model of (2).

The electrical resistance of the 1D wire finite element is:

$$R = d/(\sigma A) \quad (3)$$

where d is the distance between the two nodes, σ is the electrical conductivity of its material, and A is the wire cross–sectional area. The 1D wire element then contributes the following partition to the electromagnetic B_E matrix of (1):

$$B_E = \frac{1}{R}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (4)$$

where the two rows and columns correspond to the electric scalar potential ψ of the two nodes of the 1D wire finite element [4], [9].

The structural stiffness of a rod (or truss) is given by [9]:

$$K = AE/d \quad (5)$$

where d is the distance between the two nodes of the rod, E is the modulus of elasticity of the rod material, and A is the cross–sectional area of the rod. Stiffness K relates displacement u_S to applied force F_S according to:

$$F_S = K u_S \quad (6)$$

A 1D rod finite element then contributes the following partition to the structural K_S matrix of (2):

$$K_S = K\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (7)$$

where the two rows and columns correspond to the displacements of the two nodes of the 1D rod finite element. If a force F is applied to one of the nodes, then according to (2) and (6), the displacement will be proportional to the force, and will be inversely proportional to K.

The values of force F_S, K, d, A, and σ should be chosen so that R changes from low to high, but is never so low or so high that the B_E matrix of (1) leads to an ill–conditioned solution matrix. In the software used here the matrix equations (1) and (2) are solved using LU decomposition, which is not nearly as sensitive to ill–conditioning as are most iterative solution methods.

A SWITCH IN A SIMPLE CIRCUIT

Figure 1 shows a simple switched ac resistive circuit. Its frequency is 1 kHz. Its switch is to be closed from t=0 to t=0.5 ms, at which time it is to be opened. Its load voltage V_L is to be computed as a function of time.

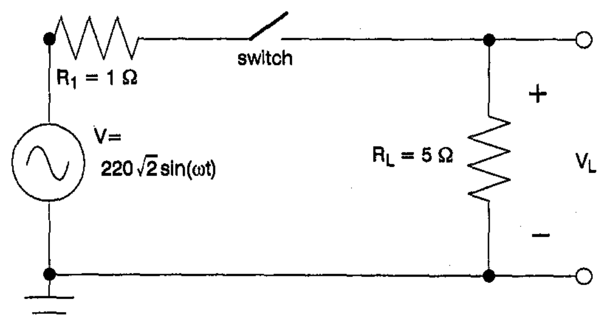

Fig. 1. Simple ac circuit with a switch.

Figure 2 shows the coupled electromagnetic/structural finite element model of Fig. 1. Note that the 1D element is of variable length and connects nodes 1 and 2. The electromagnetic excitation has been changed to a current source via Norton's theorem, and the resistors are 0D elements as in previous work [4]–[9]. Node 2 can move in the x direction. Initially, node 2 is 1 cm away from stationary node 1.

The 1D line element in Fig. 2 has σ = 1 S/m and A = 1 m^2 so that (3) gives R=0.01 ohm in its initial position, which is essentially a short circuit compared to the 1 and 5 ohm resistors of Fig. 2. The structural properties of the 1D rod element of Fig. 2 are E = 1.E–3 pascal and A = 1 m^2.

Fig. 3 shows the force waveform applied to the free (unconstrained) node on the rod. Since (5) gives K=0.1 N/m, the maximum displacement F/K = 2000 / 0.1 = 20 km. Note in Fig. 3 that no force is applied until t = 0.5 ms, which is when the switch is to be opened.

Figure 4 shows the computed displacement of node 2 as a function of time. Note that the 20 km is reached instantaneously at t = 0.5 ms, because the rod has been specified as having zero mass and zero structural damping.

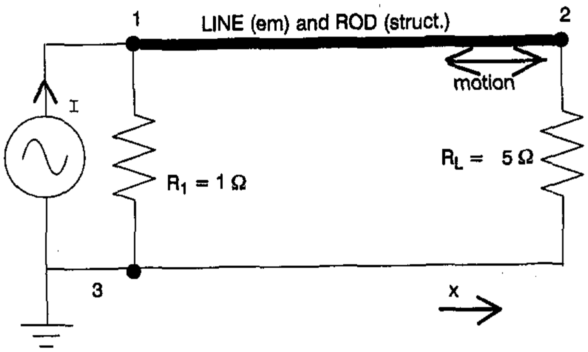

Fig. 2. Finite element model of Fig. 1, using 3 nodes and 1D electromagnetic and structural finite elements. The Norton current source has I = V/R_1=220 amps rms.

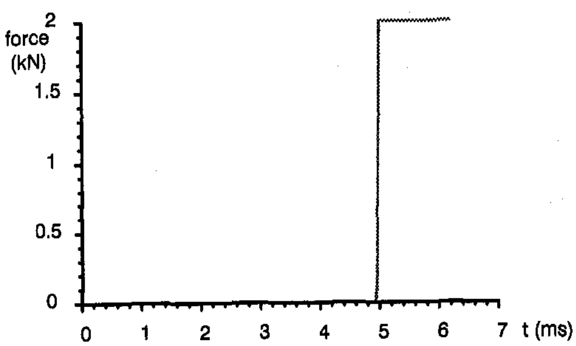

Fig. 3. Force waveform applied to node 2 of Fig. 2

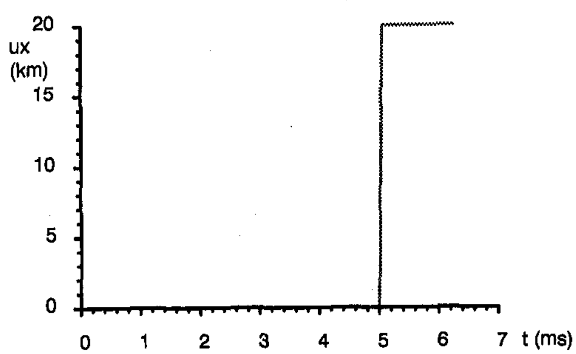

Fig. 4. Displacement (x−direction) waveform computed for node 2 of Fig. 2

The resulting time-varying voltage computed across the 5 ohm load resistor of Fig. 2 is shown in Fig. 5. It is correct because it agrees with simple hand calculations and with PSPICE results for a diode between nodes 1 and 2. Hence the new technique for modeling switches has been verified.

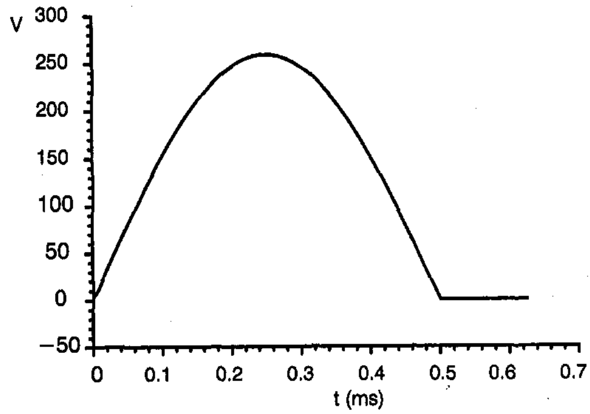

Fig. 5. Load voltage in simple circuit computed using model of Fig. 2.

MODELING A SWITCHED SECONDARY LOAD ON A SATURATING TRANSFORMER

Figure 6 shows a previously published [4] finite element model of a saturating transformer with a steel with a typical B−H curve and with zero conductivity. Its secondary current was previously computed for an 86 mΩ resistor on its secondary winding, and a current of frequency 60 Hz injected in its primary. Figure 7 shows the waveform of the input primary amp−turns, which peak at 10,000.

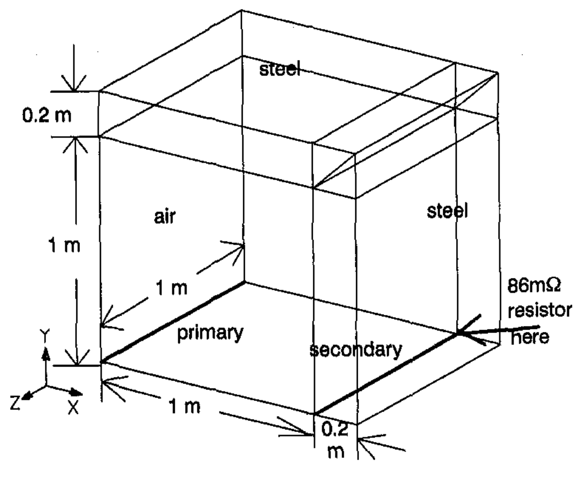

Fig. 6. Finite element model of upper right quadrant of a transformer. 3D steel finite elements are outside a hexahedral air element. The primary and secondary are modeled by highly conductive 1D elements.

Now suppose that the secondary is to have a switch (in series with its resistor) which is closed only at certain times. Hence we modify the model of Fig. 6 to have a new 1D electromagnetic/structural finite element connecting the secondary 1D element and the 86 mΩ 0D finite element. The new 1D element has electromagnetic and structural properties identical to those of Fig. 2. Its initial length is 1 mm.

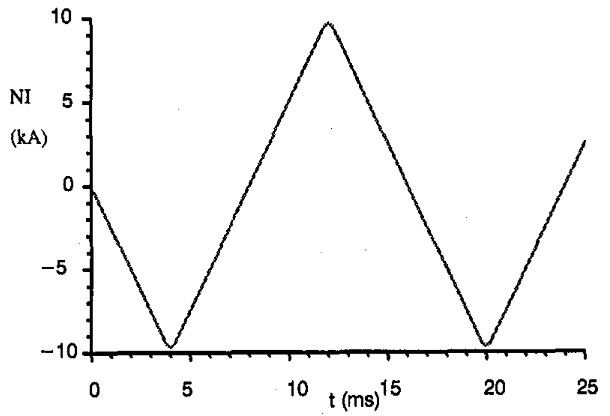

Fig. 7. Amp−turn waveform applied to primary winding of transformer of Fig. 6.

At time zero the secondary switch is closed. If it stays closed, then we apply no force to the model, and the 1D switch element maintains its length of 1 mm. Figure 8 shows the resulting computed voltage waveform across the resistor, which agrees with the previously published peaked current waveform [4] produced when the steel saturates.

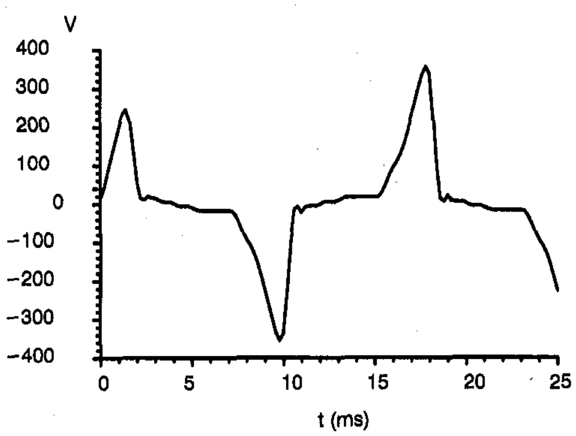

Fig. 8. Computed voltage across secondary resistor for saturating transformer with secondary switch always closed. The core conductivity is zero.

Finally, consider the case where the switch on the secondary is closed from t=0 to 15 ms, open from t=15 ms to 20 ms, and then closed from then on. Applying such a force waveform to the finite element model results in the computed voltage across the secondary resistor shown in Fig. 9.

Note in Fig. 9 that from t=0 to 15 ms, the voltage in Fig. 6 agrees with that of Fig. 8, which it should. From t=15 to 20 ms, Fig. 9 correctly shows that the voltage across the secondary resistor is zero because the switch is open. Note, however, that when the switch is again closed at 20 ms, a large inductive kick occurs which increases the voltage across the secondary resistor beyond that of Fig. 8. By t=23 ms Fig. 9 is again agreeing closely with Fig. 8.

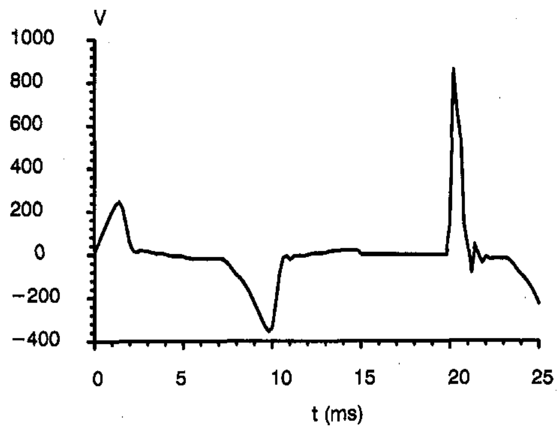

Fig. 9. Computed voltage across secondary resistor for saturating transformer with secondary switch closed except from t=15 to 20 ms. Like Fig. 8, the core conductivity is zero.

The computed core flux density waveform B(t) corresponding with the Fig. 9 is shown in Fig. 10. Note that because the secondary current is zero from 15 to 20 ms, the computed B over that range of time remains nearly constant.

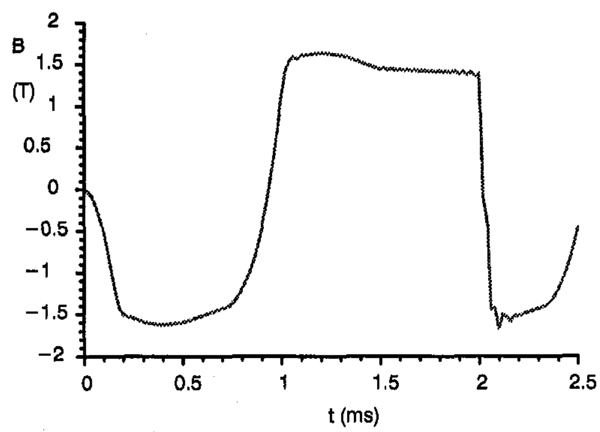

Fig. 10. Computed B in transformer with secondary switch closed except from t=15 to 20 ms. Like Figs. 8 and 9, the core conductivity is assumed zero.

CORE EDDY CURRENT EFFECTS ON THE SWITCHED SECONDARY OUTPUT VOLTAGE OF THE SATURATING TRANSFORMER

Now consider the effects of a finite nonzero electrical conductivity in the core of the switched transformer of Fig. 6. The resulting eddy currents may cause changes in the computed output voltage, and will also produce additional power losses called core loss. We will examine conductivity values in the range of from 1 to 100 S/m, which are typical for soft ferrite core materials. The core end surfaces at z=0 and z=1 in Fig. 6 are assumed to be in contact with a highly conductive clamp that provides a zero resistance return path for core eddy currents.

Figure 11 shows the switched output voltage waveform computed for the case of conductivity σ = 100 S/m. Note all three voltage peaks in Fig. 11 are now smaller than those of Fig. 9. Figure 12 shows the corresponding waveform of instantaneous core loss due to eddy currents versus time. As expected according to Faraday's Law, the eddy losses peak when the flux density B(t) of Fig. 10 has high slopes (first time derivatives).

Similar analyses were made for σ = 10 S/m and 1 S/m. Figure 13 graphs the voltage peaks at t=15 ms (the "switch" voltage) and the output voltage (the magnitude of the negative peak near 10 ms) versus a nonlinear scale of electrical conductivity of the core. Finite conductivity serves to reduce the inductive "kick" of the switch on the secondary to a greater extent than it reduces the useful output voltage.

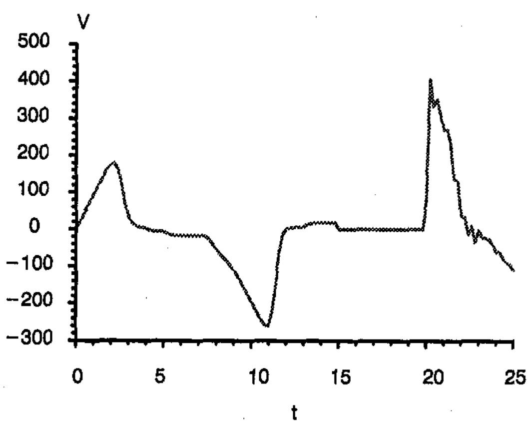

Fig. 11. Computed voltage across secondary resistor for saturating transformer with secondary switch closed except from t=15 to 20 ms. The core conductivity is 100 S/m.

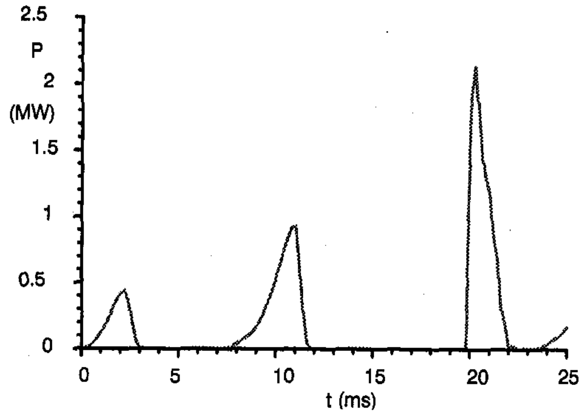

Fig. 12. Computed eddy current core losses vs. time in transformer of Fig. 6 with core conductivity = 100 S/m as in Fig. 11.

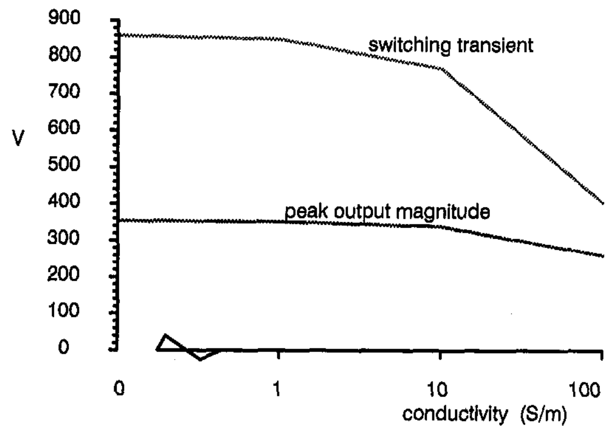

Fig. 13. Computed secondary voltage peak magnitudes versus conductivity of core material in switched transformer of Fig.6.

CONCLUSION

Switches have been modeled as variable resistors in coupled electromagnetic and structural finite element analyses. The switches have been attached to 0D and 1D finite elements in 3D finite element models. Correct results have been obtained for a simple switched resistive circuit and for a highly saturated transformer with a switched secondary resistor. The same transformer has also been analyzed with finite core conductivity, which has been shown to reduce the inductive switching transient.

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BIOGRAPHIES

John R. Brauer (S'65, M'69, SM'76, F'95) was born on April 18, 1943 in Kenosha, WI. He received three degrees in electrical engineering: a bachelor's from Marquette University, and a master's and Ph.D. from the University of Wisconsin–Madison. He is a Senior Development Engineer at Ansoft Corporation's facility in Milwaukee, WI, USA where he develops and applies electromagnetic finite element software. Prior to this position, he was a consulting engineer with the MacNeal–Schwendler Corporation and earlier with the A. O. Smith Corporation. He is the author of over 120 technical papers, many in various *IEEE Transactions* including those on *Magnetics, Power Apparatus and Systems, Industry Applications, Electromagnetic Compatibility, Electron Devices, Microwave Theory and Techniques,* and *Antennas and Propagation.*

Dr. Brauer received the Memorial Award from the Milwaukee Section of the IEEE in 1988 for his work in finite element analysis of electromagnetic devices. He is chair of the Milwaukee Chapter of the IEEE Magnetics Society and serves on two subcommittees of the IEEE Power Engineering Society. In 1996 he was elected to a three year term on the Board of Directors of the Applied Computational Electromagnetics Society.

Jeffrey J. Ruehl received the B.S. and M.S. degrees in engineering from Marquette University, Milwaukee, WI, in 1976 and 1978, respectively.

From 1978 to 1988 he worked for the A. O. Smith Corporation developing finite element software for electromagnetic analysis. From 1988 through July 1996 he was employed by the MacNeal–Schwendler Corporation's Electronics Business Unit where he served as Section Manager for Product Development. Since July 1996 he has worked for Ansoft Corporation, where he currently serves as Development Manager for the Milwaukee Development Branch. He is responsible for the commerical electromagnetic analysis products EMAS and MicroWaveLab and integrating their capabilities into other Ansoft products including Eminence and EMSS.