FINITE ELEMENT MODELING OF A TRANSFORMER FEEDING A RECTIFIED LOAD: THE COUPLED POWER ELECTRONICS AND NONLINEAR MAGNETIC FIELD PROBLEM

R. H. Vander Heiden, A. A. Arkadan
EECE Department
Marquette University
Milwaukee, WI 53233 USA

J. R. Brauer, G. T. Hummert
The MacNeal-Schwendler Corp.
4300 W. Brown Deer Road
Milwaukee, WI 53223 USA

***Abstract*—Zero-dimensional finite elements are used to help directly incorporate models of nonlinear power electronic switching devices into nonlinear transient magnetic finite element analysis. Two applications are considered: a buck regulator circuit and a transformer feeding a rectified load.**

I. INTRODUCTION

Magnetic devices with nonlinear B-H curves are often analyzed using the finite element method, usually with the excitation current assumed known. If the excitation current is a known DC value, then a nonlinear magnetostatic analysis is made. If the excitation current is time-varying with a known waveform (including AC) and induced currents are significant, then a nonlinear transient finite element analysis is required [1].

Oftentimes in practice, the current exciting a nonlinear magnetic device is unknown. If the excitation is a linear circuit, this circuit can be modeled by two basic methods. One method consists of adding the unknown circuit currents as unknowns to the finite element matrix equation [2],[3]. The other method consists of modeling the circuit with zero-dimensional finite elements [4].

In many cases, however, a nonlinear active electronic device, such as a switching diode or switching transistor, is present in the excitation circuit or in another circuit attached to the magnetic device. Recent work has described modeling such coupled nonlinear problems by adding the unknown circuit currents to the finite element matrix equation [5].

This paper describes a new method which uses zero-dimensional finite elements to directly analyze the coupled problem of nonlinear electronic circuits connected to nonlinear magnetic devices. In this paper a brief summary of the formulations used in the finite element program MSC/EMAS™ is presented. In addition, the analysis results from two systems are presented: the switching of a buck regulator circuit, and a transformer feeding a rectified load.

II. THEORY

The matrix equation derived and solved by the MSC/EMAS computer program is based on a three-dimensional formulation of Maxwell's equations [6]. Maxwell's equations are solved by determining four unknown degrees of freedom (DOFs) at each node i (the corners of the elements); three components of the magnetic vector potential A_{xi}, A_{yi}, A_{zi}, and the time-integrated electric scalar potential ψ_i.

The formulation is based on the principle of virtual work [6]. By summing the virtual work associated with all of the finite elements during variations of the potentials, the following matrix equation is obtained:

$$[M]\{\ddot{u}\} + [B]\{\dot{u}\} + [K]\{u\} = \{P\} \qquad (1)$$

where the matrix $[M]$ is directly related to the permittivity of the materials modeled, the matrix $[B]$ is related to the conductivity, the matrix $[K]$ represents the reluctivity of the system, and the excitation applied to the system is represented by the vector $\{P\}$. Eq. 1 above can be rewritten in the following partitioned form [6]:

$$\begin{bmatrix} M^{AA} & M^{A\psi} \\ M^{\psi A} & M^{\psi\psi} \end{bmatrix} \begin{Bmatrix} \ddot{\vec{A}} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} B^{AA} & B^{A\psi} \\ B^{\psi A} & B^{\psi\psi} \end{bmatrix} \begin{Bmatrix} \dot{\vec{A}} \\ \dot{\psi} \end{Bmatrix} + \begin{bmatrix} K^{AA} & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \vec{A} \\ \psi \end{Bmatrix} = \begin{Bmatrix} \vec{M}_o \\ 0 \end{Bmatrix}_{vol} + \begin{Bmatrix} \vec{H}_{\tan} \\ \vec{J}_{nor} \end{Bmatrix}_{surf} + \begin{Bmatrix} \vec{J} \\ 0 \end{Bmatrix}_{node} \qquad (2)$$

The submatrices with the superscript (AA) in Eq. 2 above, represent the curl-curl equation of the vector potential. The submatrices with the superscript ($\psi\psi$) represent the charge continuity condition. The right-hand side of Eq. 2 represents the excitations applied to the system. Volume excitations from remanent magnetization apply to $\vec{A}$ DOFs only. Direct node point excitations on $\vec{A}$ DOFs are also allowed in the form of node point current segments and constrained current densities. Surface excitations on $\vec{A}$ DOFs come from constrained values of $\vec{H}$. The only excitations on ψ come from surface excitations due to constrained values of $\vec{J}$.

Again, Eq. 2 is the general form of Maxwell's equations. Based on the type of solution needed, some of the unknowns and therefore corresponding partitioned matricies will be dropped. Since this paper deals with the coupled nonlinear electromagnetic field problem, the full matrix equation above is used for the transformer example.

Zero-dimensional finite elements attach to the ψ degrees of freedom of the [M], [B], and [K] submatrices [4]. The zero-dimensional elements used in this analysis include resistors, capacitors, and inductors. Each of these elements has only two node points. A capacitor element contributes to the permittivity matrix [M] of Eq. 1 as follows:

$$[M] = \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} C \qquad (3)$$

where C is the capacitance. A resistor element contributes to the conductivity matrix [B]:

$$[B] = \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} 1/R \qquad (4)$$

where R is the resistance, and the inductor element with inductance L contributes:

$$[K] = \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} 1/L \qquad (5)$$

to the reluctivity matrix [K]. Substituting Equations 3, 4, and 5 into Eq. 1 gives the familiar circuit equation:

$$C\ \partial V/\partial t + V/R + 1/L \int V\ dt = I \qquad (6)$$

where V is the difference between $\dot{\psi}$ at the two node points.

The formulations summarized above can be used to model coupled electromagnetic-electronic systems. Often-

Manuscript received July 12, 1991.

0018-9464/91$01.00 © 1991 IEEE

times such systems cannot be modeled directly in terms of the material properties: permittivity, conductivity, and permeability alone. For instance, in situations where loads are handled as nonlinear functions of DOFs or their first-time derivatives, nonlinear loading (NOLIN1) capabilities are required. On the other hand, when DOFs are linear functions of other DOFs and their first- and second-time derivatives, these cases are handled using transfer functions (TF).

In dynamic analysis, nonlinear loading effects are treated by MSC/EMAS as an additional applied load vector {N}, which is included in the excitation vector {P} and whose values are functions of DOFs. Normally, nonlinear effects are modeled as nonlinear material properties, e.g., magnetic B-H curves. At times, however, it is useful to model additional nonlinear effects by applying a nonlinear load. In general, these loads are nonlinear functions of the potential DOFs and their first-time derivatives. Therefore, they can only be used in a transient analysis.

The nonlinear loading capability used here is represented by:

$$N_i(t) = S_i F(u_j) \qquad (7)$$

where $N_i(t)$ is the load applied to the dependent DOF u_i, S_i is a scaling factor, $F(u_j)$ is a tabulated function, and u_j is the independent DOF, or its first-time derivative.

A transfer function represents a general nodal constraint between a designated dependent DOF u_d (and its time derivatives) and any number of other independent DOFs (and their time derivatives). This constraint takes the following form:

$$\left(b_o + b_1 p + b_2 p^2\right) u_d = \sum_{i \neq d} \left(a_o^i + a_1^i p + a_2^i p^2\right) u_i \qquad (8)$$

where any of the coefficients may be zero. Here, the time derivative operator, $\partial/\partial t$, is represented by p; thus, p^2 is the second-time derivative. This above relationship is interpreted as a differential equation with terms in the row of the dynamic matrices corresponding to the dependent DOFs. These capabilities will be illustrated in the two examples that follow.

III. BUCK REGULATOR

In the first example, a buck regulator (DC-DC converter) is used to verify the circuit modeling capability of MSC/EMAS. The steady-state operation of a buck regulator circuit consisting of a BJT power transistor, a diode, an inductor, a capacitor, and a resistor is predicted. A diagram of the circuit is displayed in Fig. 1.

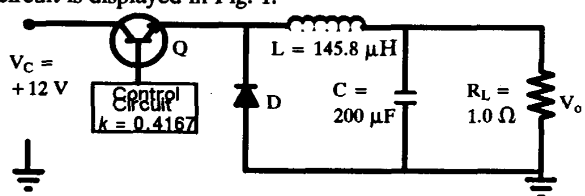

Fig. 1. Buck Regulator Circuit

The principal function of the circuit is to step down the voltage from an input DC source (V_c) to a desired voltage (V_o) across a load (R_L). The input and the output voltages are related through the duty cycle *(k)* such that $V_o = k V_c$; $0 \leq k \leq 1$. For the circuit under study, Vc = 12 V DC and k = 0.4167. The values for the rest of the circuit elements are indicated in Fig. 1 [7]. The circuit operation can be divided into two modes. Mode 1 starts at time t = 0 when the transistor Q is turned on. The rising input current passes through L, C, and R_L. Mode 2 starts at time t when Q is switched off. In this mode, the freewheeling diode, D, starts conducting due to the energy stored in the inductor L.

The buck regulator circuit model is shown in Fig. 2. The model was constructed from three zero-dimensional resistor elements, a capacitor element, an inductor element, and a current source. The current source and the two resistors R_1 and R_2 are used to simulate the effect of the transistor and its base control circuit as well as the switching of the diode. The rest of the circuit is the same as shown.

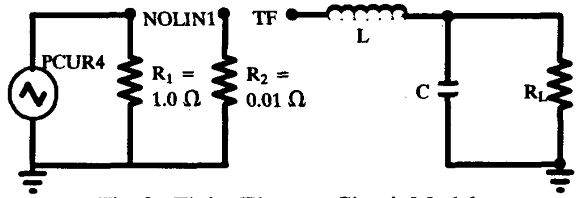

Fig. 2. Finite Element Circuit Model

The boundary conditions applied to the model are as follows: all three components of the magnetic vector potential (A_x, A_y, and A_z) have been constrained to zero in the model. Therefore, the only potential left unconstrained is the electric scalar potential ψ. The units of ψ are volt-seconds; thus, taking the derivative of ψ with respect to time yields the voltage.

The nonlinear loading capability (NOLIN1) is used in the analysis to simulate the on/off states of the transistor and the diode. The transfer function (TF) is used to apply the switched on/off states to the rest of the circuit.

The analysis was carried out over 125 cycles (5 ms) to ensure that steady state conditions were reached. The analysis resulted in all of the currents and voltages in the network; however, only the load voltage is shown here. The voltage obtained across the load resistor is plotted in Fig. 3. The load voltage is 4.46 volts DC with a 20 millivolt peak-to-peak ripple.

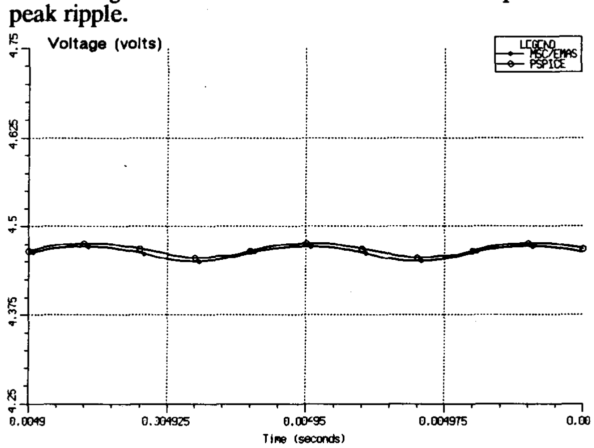

Fig. 3. Buck Regulator Load Voltage

The circuit shown in Fig. 1 was also analyzed using the circuit analysis program PSPICE. Here the transistor switching was controlled using a train of pulses at its base. For comparison purposes, Fig. 3 shows the PSPICE load voltage in addition to the MSC/EMAS results. The PSPICE voltage is comprised of an average DC value of 4.47 volts with a 20 millivolt ripple.

A close inspection of the waveforms given in Fig. 3 reveals excellent agreement between the two sets of data. As a matter of fact, the differences are within 1%.

IV. TRANSFORMER

In this second example, a system consisting of a transformer feeding a rectified load is analyzed. A simple diagram of the circuit configuration is displayed in Fig. 4 below.

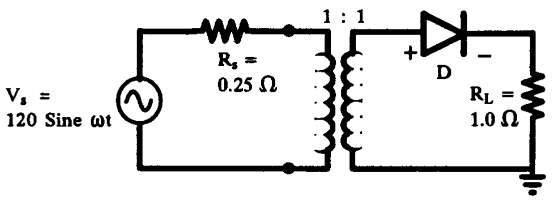

Fig. 4. Transformer Circuit Model

The finite element model of the transformer is given here in Fig. 5. The model consists of zero–, one–, and two–dimensional finite elements. Three zero–dimensional resistor elements are used in the finite element model. Two resistors are used at the primary, and a load resistor is connected to the secondary. Primary and secondary windings are each modeled using one–dimensional line elements. The steel core is modeled with nonlinear B–H material characteristics. The core is laminated to eliminate eddy currents; therefore, a conductivity of zero is included in the analysis for the core steel.

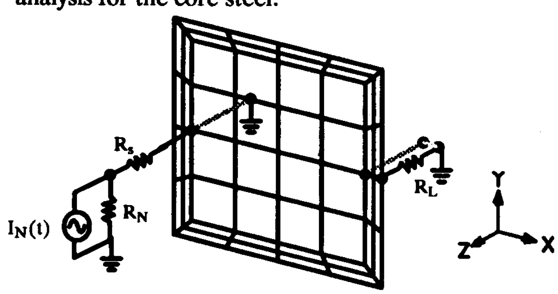

Fig. 5. Finite Element Transformer Model

A transient sinusoidal current source excitation is applied in parallel with a small resistor R_N to create a Norton equivalent voltage source at the primary. The current source waveform oscillates sinusoidally with a frequency of 60 cycles per second. Two cycles of the excitation are analyzed with time steps of 0.2083 milliseconds (4.5 degrees).

The nonlinear effects of the diode attached to the secondary coil of the transformer is again simulated using the NOLIN1 capability. Its function is to rectify the voltage and associated current established at the secondary coil.

As is usually the case in a two–dimensional model, the X and Y components of the magnetic vector potential are constrained to zero at every node point. The Z component of the magnetic vector potential is constrained to zero along the four exterior boundaries of the steel core establishing a Dirichlet boundary condition there. The fourth potential, the electric scalar potential ψ, is constrained to zero at all node points except at the nodes which make up the electronic circuit and the nodes which are indicated in the Fig. 5 with a ground symbol. A periodic boundary condition is modeled between the ends of the line elements to establish the correct flux linkage and therefore the desired primary and secondary voltages.

The MSC/EMAS analysis of the model of Fig. 5 resulted in the various current and voltage waveforms in the electric circuit along with the field values developed in the magnetic circuit. The transient voltages obtained across the primary and the secondary coils are plotted in Fig. 6. Note the primary voltage is sinusoidal; but the secondary voltage is peaked, which is expected due to the nonlinearity of the magnetic core material, and unidirectional as a result of the diode.

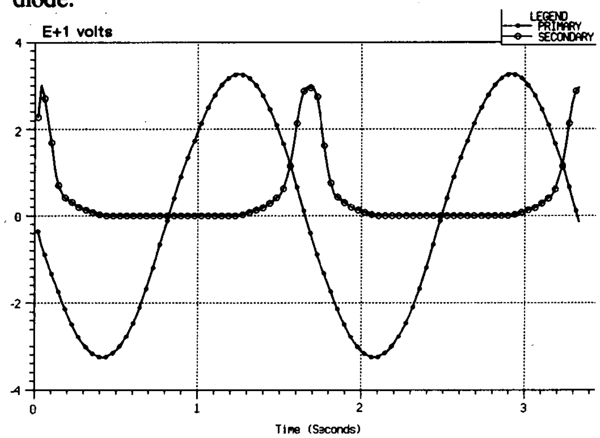

Fig. 7. Primary Voltage and Secondary Load Voltage

V. CONCLUSIONS

A new method which uses zero–dimensional finite elements to directly analyze the coupled problem of switching power electronics and nonlinear magnetic devices was presented. The method was used in a nonlinear transient analysis of two systems. In the first example, the method was used to predict the performance of a buck regulator circuit. The results of the analysis compared favorably with corresponding results obtained using the network analysis program PSPICE. In the second example, the coupled power electronics and nonlinear magnetic field problem was considered through the analysis of a transformer feeding a rectified load. As expected, the load voltage was unidirectional as compared to the bidirectional primary voltage. In a future paper, this work will be extended to include the effects of turns ratio in a transformer system.

REFERENCES

[1] John R. Brauer (ed.), *What Every Engineer Should Know About Finite Element Analysis*, New York: Marcel Dekker, Inc., 1988.

[2] T. Nakata, N. Takahashi, K. Fujiwara, and A. Ahagon, "3D Finite Element Method for Analyzing Magnetic Fields in Electrical Machines Excited from Voltage Sources," *IEEE Trans. Magn.*, Vol. 24, No. 6, Nov. 1988, pp. 2582–2584.

[3] S. J. Salon, M. J. DeBortoli, and R. Palma, "Coupling of Transisent Fields, Circuits, and Motion Using Finite Element Analysis," *Journal of Electromagnetic Waves and Applications*, Vol. 4, No. 11, Nov. 1990, pp. 1077–1106.

[4] J. R. Brauer, B. E. MacNeal, L. A. Larkin, and V. D. Overbye, "New Method of Modeling Electronic Circuits Coupled with 3D Electromagnetic Finite Element Models," *IEEE Trans. Magn.*, Vol. 27, No. 5, Sept. 1991.

[5] Francis Piriou and Adel Razek, "Coupling of Saturated Electromagnetic Systems to Non–Linear Power Electronic Devices," *IEEE Trans. Magn.*, Vol. 24, No. 1, Jan. 1988, pp. 274–277.

[6] B. E. MacNeal, J. R. Brauer, and R. N. Coppolino, "A General Finite Element Vector Potential Formulation of Electromagnetics Using a Time–Integrated Scalar Potential," *IEEE Trans. Magn.*, Vol. 26, No. 5, Sept. 1990, pp. 1768–1770.

[7] M. H. Rashid, *Power Electronics: Circuits, Devices, and Applications*, Prentice Hall, New Jersey, 1988.