COMPARISON BETWEEN FINITE ELEMENT AND INTEGRAL EQUATION MODELING OF POWER BUSBAR SYSTEMS

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ABSTRACT

The paper deals with the modelling of power busbar systems comparing two different numerical techniques. The first model develops a two-dimensional finite element procedure in the frequency domain and it can be generalized to the transient behaviour by means of the modified Fourier transform. The second model is based on a three-dimensional integral approach developed in the time domain. The methods have been applied to the steady state and transient analysis of busbar systems and the comparison of the results obtained by the two models has been carried out. In spite of the different features of the two models, the predictions are found to be in good agreement.

INTRODUCTION

The increasing power of electrical installations is causing a large diffusion of busbar systems having several paralleled subconductors. These equipments require expensive experimental verifications and related certifications and classifications. Therefore computer simulation of the test provided in the experimental verifications can avoid an annoying and expensive unsuccessful testing of these devices.

An accurate model of these devices requires the solution of an electromagnetic field problem which is completely defined only if the external circuit connections are taken into account, together with the eddy current effects within the conductors. Different approaches have been proposed in literature: some of them utilize integro-differential techniques, as finite element method, while others are based only on integral equations. Moreover, the interest can be devoted to both periodic and transient behaviour.

This paper proposes a comparison between two completely different approaches for the modelling of power busbars operating in steady-state and transient conditions.

One model employs a 2-D finite element technique in the frequency domain; it uses the linear field solution for deducing an impedance matrix which is included in the circuit equations. Of course, such an approach, based on the definition of constant impedances, is limited to steady-state conditions; however, it can be generalized to the transient behaviour through the modified Fourier transform provided that the dependence of matrix parameters on frequency has been deduced.

The other model is formulated directly in the time domain by means of the 3-D volume integral solution obtained imposing the Lorentz Gauge, and leads to the solution of an equivalent electric network.

MODELS

Both the Finite Elements (F.E.) and the Integral Equation (I.E.) models start from the numerical solution of the field problem governed by the well known equation $\nabla^2 A=\mu J$. Even though the models develop two alternative solutions, both of them lead to the definition of an impedance matrix that can be easily included in the circuit equations involving external electric elements. Nevertheless the elements of these matrixes represent completely different electrical parameters.

Numerical solution of $\nabla^2 A=\mu J$.

F.E. model

Provided that the busbars are sufficiently long, a 2-D field analysis can be performed considering that end effects are negligible. The magnetic field is assumed to lay in the *(y,z)* plane and the current is supposed to flow along the x-axis. The electromagnetic problem is expressed in terms of a vector potential A $(B = curl\,A)$ by the equation:

$$curl\,\nu\,curl A = \sum_{n=1}^{M} \chi_n \left[-\sigma_n \frac{dA}{dt} + \frac{I_n}{S_n} + \frac{\sigma_n}{S_n} \int_{\Omega_n} \frac{dA}{dt} dS \right]$$

where ν and σ are the magnetic reluctivity and the electrical conductivity respectively, χ_n is the characteristic function of the *n*-th conductor having section S_n and supply current I_n , M is the number of the conductors and Ω_n is the trace in the *y,z* plane of the *n-th* conductor. Under periodic supply conditions the use of a truncated Fourier series allows one to remove the time derivative and it leads to a problem expressed in terms of complex variables. The field problem is then solved through a finite element method using piecewise linear shape functions in the frequency domain [1].

I.E. model

The whole volume of conductive regions is subdivided in N parallelepipeds, having faces parallel to the unit vectors of the coordinate axis, where only one component of current density is assumed [2]. The magnetic vector potential A_p at the center P of a generic parallelepiped is given by the superposition of the A_j contributions, obtained imposing the Lorentz Gauge, due to every elementary parallelepipeds with current density J_j :

$$Ak\,(t) = \frac{\mu_0}{4\pi}\left[\iiint_{\Gamma_s} \frac{Js\,(t,x')}{|xk - x'|} d^3x' + \sum_{j=1}^{3N} \iiint_{\Gamma_j} \frac{Jij\,(t,x')}{|xk - x'|} d^3x'\right]$$

where Γ_j is the j-th parallelepiped, J_s are the current sources, J_{ij} the induced currents.

Furthermore assuming an uniform distribution of the current density $J_{ij}(t,x) = J_{ij}(t)$ and $J_s(t,x) = J_s(t)$ we obtain that the magnetic vector potential A, and consequently the flux density B, are proportional to the currents I_{ij} and I_s.

Electrical Parameters

F.E. Model

Under linear hypothesis, the superposition principle is invoked in order to deduce an impedance matrix which can be included in the network equations. In particular, by imposing a unitary current in one conductor at a time, a row of the impedance matrix can be evaluated; thus M field solutions provide the whole matrix [1]. This impedance matrix enables the analysis of the busbar behaviour under any periodic operating conditions by simply modifying the equations of the external circuit. When the supply shows harmonic contents, an impedance matrix is computed for each frequency and the results are finally superposed.

I.E. Model

The Ohm's law inside every volume element is:

$$\rho_k J_k(t) = -\nabla V_k(t) - \frac{\partial A_k(t)}{\partial t} \tag{1}$$

where Ak is the magnetic vector potential in the k-th elementary parallelepiped, ∇V_k is the irrotational component of the electric field E_k and J_k is the current density in the *k-th* volume.

Integrating eq. (1) in each volume element Γ_k and averaging the result on the surface S_k eq. (1) becomes:

$$RkIk + \sum_{j=1}^{3N} Lkj \frac{\partial Ij}{\partial t} = Uk \tag{2}$$

where U_k is the electric potential difference between the centres of two nearby parallelepipeds, R_k is the electric resistance of the parallelepipeds Γ_k, I_k is the current in Γ_k , the L_{kj} are the inductances between Γ_k and Γ_j and L_{ksj} are the inductances between the volume Γ_k and the current sources volumes Γ_{sj}.

This equation represents the electric equilibrium equation of a branch of a network constituted by a resistance, a self inductance, a mutual inductance with all other network's branches. Writing equation (2) inside every parallelepiped the following system is obtained:

$$[R][I] + [L]\left[\frac{\partial}{\partial t}\right] = [V_g]$$

$$[N][I] = [I_g]$$

where $[R]$ and $[L]$ are respectively the matrices of the resistances and of the self and mutual inductances of the system, $[M]$ is the nodal incidence matrix of the system, and $[V_g]$ and $[I_g]$ are the known values of the voltage and current generators applied to the bodies. A quick and accurate evaluation of the L_{kj} can be obtained by means of analytical expressions [3,4] developed in previous works. These parameters are not frequency dependent, as for the F.E. model, but represent the electrical parameters of every parallelepipeds in which the conductive region have been subdivided. Given the initial conditions, the differential equations system can be integrated with classical numerical methods, and gives the current distribution.

Transient Analysis

F.E. Model

The analysis performed through the impedance matrix allows the steady-state simulation under different frequencies. The knowledge of the behaviour of the matrix elements versus frequency enables the transient analysis using the modified Fourier transform. This transform is applied to the ordinary differential equations which govern the transient state of the circuit; the transform of a generic function *f(t)* is:

$$f(a + j\omega) = \int_0^\infty f(t) e^{-(a+j\omega)t} dt$$

where a is an arbitrary coefficient, sufficiently large to ensure that the integral converges. This transform allows one to study the problem in the frequency domain taking into account the dependence of the parameters on frequency. After the solution of the algebraic system in the frequency domain, the behaviour of the electrical quantities versus time can be deduced by applying the antitransform. In particular, the instantaneous values of the current in each conductor are obtained by using the method proposed in [5]:

$$i(t) = \frac{2e^{at}}{\pi} \int_0^\Omega \mathrm{Re}\left[i(a + j\omega)\right] \cos\omega t\ \varepsilon\, d\omega$$

where $\varepsilon = [\sin(\pi\omega/\Omega)]/(\pi\omega/\Omega)$ is a suitable correction function used to remove the oscillations which occur in the Fourier transform [4] and Ω is a fixed limit of the numerical integration.

I.E. Model

The model is formulated in the time domain, therefore it is directly suitable for the transient analysis.

RESULTS

The proposed models are applied to the analysis of a simple busbar system constituted by three 10 m long copper conductors whose cross-section is shown in Fig. 1, considering 45 unknowns for the I.E. model and 7000 unknowns for the F.E. model.

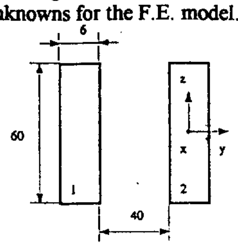

Fig. 1 Cross section of the busbar system , dimensions are in mm.

Two different configurations (a) and (b) are considered: in the first one (Fig. 2a) a three-phase voltage source is imposed; the second one includes a single voltage source paralleling conductors 1 and 2 (Fig. 2b).

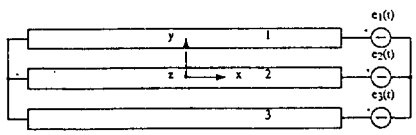

Fig. 2a Three phase supply

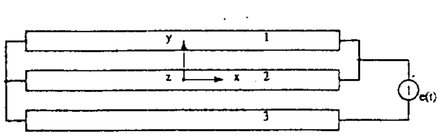

Fig. 2b Single voltage supply

It is worth noting that these model problems, in spite of their simplicity, involve the same computational difficulties which occur in actual operating conditions (e.g. three-phase fault and current sharing between subconductors of the same phase).

Preliminary computations performed through the I.E. method have shown that the end-effects are negligible for the considered duct length; therefore, the two-dimensional simplifying assumption does not introduce any errors in the F.E. analysis.

In the first comparison the steady state behaviour is analyzed. The results are summarized in tables I and II, which show the real and imaginary parts of the currents computed by F.E. and I.E. methods. The agreement is satisfactory: discrepancies of a few percents are always found.

Table I. Short circuit currents [kA] for configuration (a)

	F.E.		I.E.	
Duct	Real	Imag.	Real	Imag.
1	215.4	-496.1	198.2	-494.5
2	201.1	590.4	202.2	589.9
3	-416.5	-94.3	-400.5	-95.4

Table II. Short circuit currents [kA] for configuration (b)

	F.E.		I.E.	
Duct	Real	Imaginary	Real	Imaginary
1	-82.5	-84.0	-81.5	-74.5
2	-67.0	-196.8	-67.4	-196.6
3	149.5	280.8	148.9	271.1

The results of a transient analysis are presented in Figs. 3 and 4 for configuration (a) and (b) respectively.

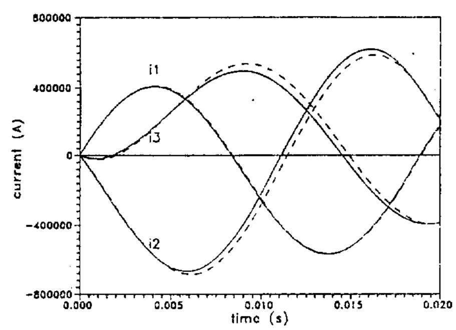

Fig. 3 Transient currents for configuration (a). Solid lines: I.E.; dashed lines F.E.

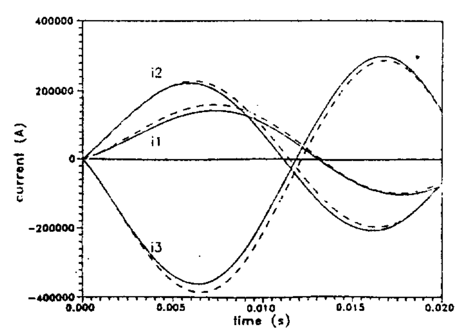

Fig. 4 Transient currents for configuration (b). Solid lines: I.E.; dashed lines F.E.

These diagrams show that the waveform predicted by the two models are very similar. The transient analysis has been carried out through the usual circuital procedure which assumes the circuit parameters to be independent of frequency: the currents obtained under this simplifying assumption are significantly different (up to 30%) from those reported in the previous figure.

Eventually, a comparison between the magnetic flux density computed by the two methods in the surrounding of the conductors is performed for configuration (a). The results are reported in Figs. 5 and 6 where the z component of the magnetic flux density is computed along the y and z axes, respectively.

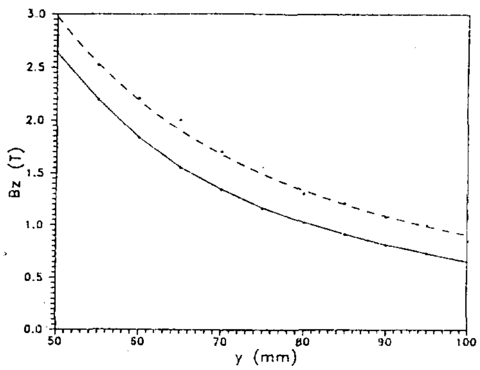

Fig. 5 Component z of the magnetic flux density vs. y (at x=0, z=0) computed by the I.E. (solid line) and F.E. (dashed line).

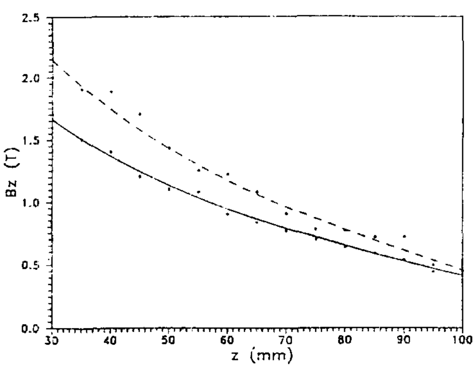

Fig. 6 Component z of the magnetic flux density vs. z (at x=0, y=0) computed by the I.E. (solid line) and F.E. (dashed line).

It is worth remarking that this analysis is purely devoted to the comparison of the two methods, so that it does not correspond to an actual situation. In this case the agreement between the two method is less good mainly because the I.E. method operates in an open boundary domain, while the F.E. method assumes homogeneous Dirichlet conditions at a stated distance from the conductors.

CONCLUSIONS

Two different numerical methods are used for analyzing busbar systems. Even if the models employ completely different formulations, the results are found in good agreement.

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