Electromagnetic Modeling

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1. **EMC at EISLAB**

1.1. **Group**

1. Jerker Delsing, Professor  
2. Jonas Ekman, Professor  
3. Giulio Antonini, Guest Professor  
5. Maria De Lauretis, PhD Student  
6. Andreas Hartman, PhD Student  
7. Ake Wisten, PhD Student  
8. Albert Ruehli, Honorary doctor  
9. Sohrab Safavi, PhD alumni  
10. Danesh Daroui, PhD alumni  
12. Tore Lindgren, PhD alumni  
13. Mathias Enohnyakhet, PhD alumni  
14. Urban Lundgren, PhD alumni

1.2. **Facilities**

The EMC lab has a large, fully attenuated, electromagnetically shielded chamber. The chamber is equipped with antennas, transmission and receiving equipment covering the frequency range 30 MHz-6 GHz.
2. EMC Lab

The EMC Lab at EISLAB has two main purposes:

- Research and teaching facility.
- A resource for the electronic industry in the region.

Research is performed on computational electromagnetics using the PEEC (Partial Element Equivalent Circuit) approach.

The EMC lab has a large, fully attenuated, electromagnetically shielded chamber. The chamber is equipped with antennas, transmission and receiving equipment covering the frequency range 30 MHz - 18 GHz.

Examples of tests performed at the EMC Lab:

- Emission and immunity testing, both conducted and radiated.
- Transient testing (ESD, EFT, surge, burst).
- Electromagnetic shielding efficiency (SE) measurements.
- Measurements of constitutive parameters (permeability, permittivity, electrical conductivity)
EMC Lab, cont.

We offer the following courses to undergraduate students:

- EMC Technology, E7031E.
- Waves and Antennas, E7026E.

Driving licence course

We offer a basic (mandatory) EMC course for companies who wish to use our lab for (pre-compliance) testing.

Courses for PhD students

EMC technology, PEEC Modeling, basic and advanced.
3. EMC Research

- Electromagnetic modeling (integral equation/equivalent circuit):
  - high performance/parallel computing (Jonas, Danesh),
  - for power electronic system modeling (Danesh, ABB),
  - variable frequency drives (Maria),
  - circuit and EM co-simulation (Sohrab).

- Railroad EMC problem
  - bearing currents (mitigation techniques),
  - coupling currents (mitigation techniques).
3.1. Power electronic system modeling

Together with ABB CRC modeling of bus bar structures

Funded by: Elforsk/ELEKTRA
Period: 2008 to 2012

Contact: Dr. Jonas Ekman
PhD Student: Danesh Daroui
Resulted in an internal ABB tool: BusBar Tool
3.2. Circuit and EM co-simulation

Project goal is to develop a full-wave, full spectrum PEEC solver with full support for SPICE components.

Funded by: EU-RDF
Period: 2009 to 2013

Contact: Dr. Jonas Ekman
Student: Sohrab Safavi
3.3. EMC for variable frequency drive systems

Project goal is to develop PEEC capabilities in system simulations for VFDs.

Funded by: Svenska Kraftnät
Period: 2014 to 2018

Contact: Dr. Jonas Ekman
Student: Maria DeLauretis
4. **EM Modeling**

This talk will give an introduction to the field of electromagnetic (EM) modeling. For this purpose I will talk about:

1. the basis of EM modeling,

2. available modeling techniques,

3. differential equation based technique = FDTD,

4. integral equation based technique = PEEC.
5. Solving Maxwell’s Equations

**Differential form**

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]

**Integral form**

\[ \oint_L \vec{H} \cdot d\vec{l} = \int_S (\vec{J} + \frac{\partial \vec{D}}{\partial t}) \cdot d\vec{S} \]  

\[ \oint_L \vec{E} \cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \]  

\[ \nabla \cdot \vec{D} = \rho_v \]

\[ \oint_S \vec{D} \cdot d\vec{S} = \int_v \rho_v \, dv \]  

\[ \nabla \cdot \vec{B} = 0 \]

\[ \oint_S \vec{B} \cdot d\vec{S} = 0 \]
5.1. Techniques for Solving Maxwell’s Equations

The techniques for solving field problems, Maxwell’s equations, can be classified as:

- experimental,
- analytical (exact), or
- numerical (approximate).

The experimental techniques are expensive and time consuming but are still widely used.

The analytical solution of Maxwell’s equations involve, among others, Separation of variables and Series expansions, but are not applicable in a general case.

The numerical solution of field problems became possible with the availability of high performance computers.
5.2. Techniques for Solving Maxwell’s Equations

The most popular numerical techniques are:

- Finite difference methods (FDM).
- Finite element methods (FEM).
- The method of moments (MoM).
- Finite integration technique (FIT).
- The partial element equivalent circuit (PEEC) method.
5.3. **Classes of Problems within EM Modeling**

The differences in the numerical techniques have its origin in the basic mathematical approach and therefore make one technique more suitable for a specific *class of problem* compared to the others. Typical classes of problems, with the suitable modeling techniques indicated in parenthesis, in the area of EM modeling are:

- Heterogeneous materials, compact structures (FEM, FDM).
- Antenna design (MoM).
- Mixed electric functionality and EM problem (PEEC).
5.4. Solution Domain/Variables

Further, the problems presented above require different kinds of analysis in terms of:

- Requested solution domain:
  - Time domain.
  - Frequency domain.

- Requested solution variables:
  - Circuit variables (currents and/or voltages).
  - Field variables (electric and/or magnetic fields).
5.5. Differential or Integral Formulation

The numerical techniques used for EM simulations can be classified depending on which formulation of Maxwell’s equations are solved numerically. The two formulations are displayed in parallel in (1) - (4) as Differential form and Integral form. The main differences between the two formulations are:

- The discretization of the structure. For the differential formulation the complete structure, including the air needs to be discretized. For the integral formulation only the materials needs to be discretized.

- The solution variables. The differential based techniques where the discretization of the complete computational domain is performed, delivers predominantly the solution in field variables. For the integral based techniques the solution is expressed in circuit variables, i.e. currents and voltages.
### 5.6. Summary

<table>
<thead>
<tr>
<th>Method</th>
<th>FDM</th>
<th>FEM</th>
<th>MoM</th>
<th>PEEC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Formulation</strong></td>
<td>Differential</td>
<td>Differential</td>
<td>Integral</td>
<td>Integral</td>
</tr>
<tr>
<td><strong>Solution variables</strong></td>
<td>Field</td>
<td>Field</td>
<td>Circuit</td>
<td>Circuit</td>
</tr>
<tr>
<td><strong>Solution domain</strong></td>
<td>TD or FD</td>
<td>TD or FD</td>
<td>TD or FD</td>
<td>TD and FD</td>
</tr>
<tr>
<td><strong>Drawbacks</strong></td>
<td>Cell nonflex. Memory</td>
<td>Large lin.syst. FD-(O(n^3))</td>
<td>Green’s fun.</td>
<td>Green’s fun. FD-(O(n^3))</td>
</tr>
</tbody>
</table>
6. **Differential - FDTD**

FDTD = Finite Difference Time Domain method. The method is widely used within EM modeling mainly due to its *simplicity*. The FDTD method can be used to model arbitrary heterogeneous structures, for instance, PCBs and the human body.

![Model for a human body used in EM simulations.](image)

Figure 1: Model for a human body used in EM simulations.
6.1. FDTD Basics

- Grid all Computational Domain
  - Select Air, Metal, Dielectric, Other
  - Cell size small vs wavelength
- Time Domain Technique
  - Wide Frequency Response with FFT
- E and H fields found directly

6.2. Maxwell’s Equations are NOT Hard

For example....

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]  \tag{5} \]

changing to difference equations for (5) results in

\[ \frac{E_i^n - E_i^{n-1}}{\Delta x} = -\mu \frac{H_i^{n+1} - H_i^n}{\Delta t} \]  \tag{6} \]
6.3. **FDTD Discretization**

In the FDTD method, finite difference equations are used to solve Maxwell’s equations for a restricted computational domain. The method requires the whole computational domain to be divided, or discretized, into volume elements for which Maxwell’s equations have to be solved. The volume element sizes are determined by considering two main factors:

1. **Frequency.** The cell size should not exceed \( \frac{\lambda_{\text{min}}}{10} \).

2. **Structure.** The cell sizes must allow the discretization of thin structures.

The volume elements are not restricted to cubical cells, parallelepiped cells can also be used with a side to side ratio not exceeding 1:3, mainly to avoid numerical problems.
6.4. FDTD Unit Cell

After discretizing the structure, the electromagnetic field components, \( E_X \), \( E_Y \), \( E_Z \), \( H_X \), \( H_Y \), and \( H_Z \), are defined for the cells, for example as shown in Fig. 2.

![FDTD cell with indicated field components in (a) and human body model created from FDTD cells in (b).](image-url)
6.5. **FDTD Model Setup**

To be able to solve the discretized Maxwell’s equations in the FDTD method the following must be specified:

1. *Initial conditions and excitation.* The initial electromagnetic field components for each discrete point in the discretized structure must be specified.

2. *Boundary conditions.* Open region problems can be solved using mathematical formulations, absorbing boundary conditions, or absorbing material at the computational boundary.

3. *Time step, $\Delta t$.* To ensure that the electromagnetic wave propagation between the nodes does not exceed the speed of light a time step condition has to be fulfilled.

6.6. **FDTD Model Solution**

The equations are then solved by:

1. Calculating the electric field components for the complete structure.

2. Advance time by $\frac{\Delta t}{2}$.

3. Calculate the magnetic field components for the complete structure based on the electric field components calculated in 1.

4. Advance time by $\frac{\Delta t}{2}$ and continue to 1.
7. Integral - PEEC

7.1. PEEC theory - Starting point

The theoretical derivation starts from the expression of the total electric field in free space, $\vec{E}^T$, by using the magnetic vector and electric scalar potentials, $\vec{A}$ and $\phi$ respectively

$$\vec{E}^T(\vec{r},t) = \vec{E}^i(\vec{r},t) - \frac{\partial \vec{A}(\vec{r},t)}{\partial t} - \nabla \phi(\vec{r},t) \quad (7)$$

where $\vec{E}^i$ is a potential applied external electric field. If the observation point, $\vec{r}$, is on the surface of a conductor, the total electric field can be written as $\vec{E}^T = \frac{\vec{J}(\vec{r},t)}{\sigma}$, in which $\vec{J}(\vec{r},t)$ is the current density in a conductor and $\sigma$ is the conductivity of the conductor. To transform into the electric field integral equation (EFIE), the definitions of the electromagnetic potentials, $\vec{A}$ and $\phi$ are used. The magnetic vector potential, $\vec{A}$, at the observation point $\vec{r}$ is given by

$$\vec{A}(\vec{r},t) = \sum_{k=1}^{K} \mu \int_{\nu_k} G(\vec{r},\vec{\nu}) \vec{J}(\vec{\nu},t_d) d\nu_k \quad (8)$$

in which the summation is over $K$ conductors and $\mu$ is the permeability of the medium. Since no magnetic material medium are considered in this thesis $\mu = \mu_0$. In (8) the free space Green’s function is used and is defined as $G(\vec{r},\vec{\nu}) = \frac{1}{4\pi |\vec{r} - \vec{\nu}|}$. $\vec{J}$ is the current density at a source point $\vec{\nu}$ and $t_d$ is the retardation
time between the observation point, \( \vec{r} \), and the source point given by \( t_d = t - \frac{|\vec{r} - \vec{r}'|}{c} \), where \( c = 3 \cdot 10^8 \text{m/s} \). The electric scalar potential, \( \phi \), at the observation point \( \vec{r}' \) is given by

\[
\phi(\vec{r}, t) = \sum_{k=1}^{K} \frac{1}{\epsilon_0} \int_{v_k} G(\vec{r}, \vec{r}'); q(\vec{r}', t_d) \, dv_k
\]

in which \( \epsilon_0 \) is the permittivity of free space and \( q \) is the charge density at the source point. Combining results in the well known electric field integral equation (EFIE) or mixed potential integral equation (MPIE) that is to be solved according to

\[
\hat{n} \times \vec{E}^i(\vec{r}, t) = \hat{n} \times \left[ \frac{\vec{J}(\vec{r}, t)}{\sigma} \right] + \hat{n} \times \left[ \sum_{k=1}^{K} \mu \int_{v_k} G(\vec{r}, \vec{r}'); \frac{\partial \vec{J}(\vec{r}', t_d)}{\partial t} \right] \quad  \text{(10)}
\]

\[
+ \hat{n} \times \left[ \sum_{k=1}^{K} \nabla \left( \epsilon_0 \int_{v_k} G(\vec{r}, \vec{r}'); q(\vec{r}', t_d) \right) \right]
\]

where \( \hat{n} \) is the surface normal to the body surfaces.
7.2. Interpreting the EFIE as KVL

Repeating the EFIE from (10) for simplicity.

\[ \vec{E}_i(\vec{r}, t) = \frac{\vec{J}(\vec{r}, t)}{\sigma} + \mu \int_{v'} G(\vec{r}, \vec{r}') \frac{\partial \vec{J}(\vec{r}', t_d)}{\partial t} dv' + \nabla \frac{\nabla}{\epsilon_0} \int_{v'} G(\vec{r}, \vec{r}') q(\vec{r}', t_d) dv' \quad (11) \]

It is possible to interpret each term in the above equation as KVL since

\[ V = RI + sLpI + Q/C \quad (12) \]

This results in interpreting:

- The first RHS term as Resistance,
- The second RHS term as Partial inductance
- The third RHS term as Coefficient of potential
7.3. The resulting equivalent circuit

Figure 3: One way of visualizing the interpretation of the EFIE as an equivalent circuit.

- The current controlled current sources $I_P^n$ are used to model the electric field couplings,

- The current controlled voltage sources $V_{m}^{L,m} \beta$ are used to model the magnetic field couplings.
7.4. Simple PEEC model for two wires in free space

Figure 4: PEEC model for two conducting wires (a) using controlled voltage- and current- sources to account for the electromagnetic couplings (b).
7.5. PEEC model for nonorthogonal patch in free space

(a) Nonorthogonal metal patch

(b) PEEC model

Figure 5: Nonorthogonal metal patch (a) and PEEC model (b).
7.6. Summary

• Partial Element Equivalent Circuit (PEEC)
  – Solve Maxwells equations completely (full-wave),
  – Similar to Method of Moments,
  – Transformation from the field to circuit domain (E/H → V/I),

• Background.
  – Albert Ruehli, IBM
  – From VLSI inductance calculations performed in the early 70s.

• Present.
  – Full-wave method,
  – Time- and frequency- domain,
  – Dispersive dielectrics,
  – Nonorthogonal structures.
7.7. Practical PEEC Modeling

Figure 6: Basic structure of PEEC based EM solver.
7.8. Graphical user interface
EMC at EISLAB
EMC Lab
EMC Research
EM Modeling
Solving Maxwell's...
Differential - FDTD
Integral - PEEC
7.9. **Meshing**

Mesh the structure into volume (current) and surface (charge) cells.
3.4. Discretization

Figure 3.5: Cube with divisions \(x, y, z = \{2, 2, 1\}\) resulting in a 3D PEEC model with 35 volume cells and 42 surface cells. Rings indicate nodes. a) X-directed volume cell partition. b) Y-directed volume cell partition. c) Z-directed volume cell partition. d) Surface cell partition.
7.10. Partial element calculation

In this stage the following is calculated:

1. Partial inductances, $L_p$, from the volume mesh.
2. Coefficient of potentials, $P$, from the surface mesh.
3. Conductive losses, $R$, from the volume mesh.
4. Excess capacitance, $C_E$, from the volume mesh.
5. Connectivity information (surfaces-to-volumes).
7.11. Matrix formulation

7.11.1. Time domain analysis

\[
\begin{bmatrix}
    -A & -(R + L_S \frac{d}{dt}) \\
    F \frac{d}{dt} + S_S Y_L & -S_S A^T
\end{bmatrix}
\begin{bmatrix}
    V(t) \\
    I_L(t)
\end{bmatrix} = \begin{bmatrix}
    V_S + L_M \frac{d}{dt} I_L(t - \tau) \\
    S_S I_S(t) + S_M I_S(t - \tau) - S_M Y_L V(t - \tau) + S_M A^T I_L(t - \tau)
\end{bmatrix}
\]

- \(L_S\) and \(L_M\) contain partial inductances,
- \(F, S_S, S_M\) contain coefficient of potentials,
- \(A\) is connectivity information while \(V_S\) and \(I_S\) are voltage and current sources,
- \(V\) and \(I_L\) are the unknown volume cell currents and surface cell voltages.
7.11.2. Frequency domain analysis

\[
\begin{bmatrix}
-\textbf{A} & -(\textbf{R} + j\omega\textbf{L}) \\
-j\omega\textbf{P}^{-1} + \textbf{Y}_L & -\textbf{A}^T
\end{bmatrix}
\begin{bmatrix}
\textbf{V} \\
\textbf{I}_L
\end{bmatrix}
= 
\begin{bmatrix}
\textbf{V}_S \\
\textbf{I}_S
\end{bmatrix}
\] (14)

In this case, \(\textbf{L}\) contains the partial inductances (complex values) and \(\textbf{P}\) contains the coefficients of potential (complex values).
7.12. Matrix solution

We have the following cases:

1. Time domain
   (a) Quasi-static assumption\(^*\).
   (b) Full-wave solution\(^\diamond\). Fast! Late time instability.

2. Frequency domain
   (a) Quasi-static assumption\(^*\).
   (b) Full-wave solution\(^\diamond\). Computationally demanding!
7.13. Post processing in PEEC

1. Calculate radiated fields from PEEC model results

\[
E_r = \eta \frac{I_z l_z \cos \theta}{2\pi R^2} [1 + \frac{1}{jkR}] e^{-jkR} \tag{15}\n\]

\[
E_\theta = j\eta \frac{k I_z l_z \sin \theta}{4\pi R} [1 + \frac{1}{jkR kR^2}] e^{-jkR} \n\]

\[
E_\phi = 0 \n\]

2. Calculate currents when using the Nodal Analysis formulation

\[
I_L = (R + j\omega L)^{-1}(-AV - V_S) \tag{16}\n\]

3. Input impedance, impedance matrices, inductance calculations, capacitance calculations, ..... 

4. Visualization of currents, voltages, fields etc.
7.14. LTU/AUq PEEC Solver

- Website: http://www.csee.ltu.se/peec

- General Features:
  - Orthogonal and nonorthogonal structures
  - Parallel computing
  - Quasi-static and full-wave analysis
  - Inductance and capacitance matrix extraction
  - Transmission line support
  - Nodal analysis & Modified nodal analysis (MNA)
  - Lumped elements (one port, two ports, n ports)
  - Post processing (current- and voltage- distribution)

- Frequency Domain:
  - Ideal, lossy, and dispersive dielectrics
  - Skin effect

- Time Domain:
  - Different order of integration schemes
  - Stability check and improvements
7.15. PEEC Modeling Examples

7.15.1. RFID System

For the company, the challenge is to manufacture an antenna and driving circuitry. The prerequisites change for each job (size, position, cost, performance). We analyze this problem through the three different ways to solve EM problems:

1. **Experimental techniques**
   Involves manufacturing of several antenna prototypes, several driving circuits (modeled in SPICE first), resonance circuits (modeled in SPICE first), and then hope for the best.

2. **Analytical techniques**
   Works pretty good in this case. Possible to do analytical calculations on simple antenna structures. However, SPICE-like solvers are needed for the modeling of driving and resonance circuitry.

3. **Numerical modeling**
   By using PEEC, different antenna structures are included with the driving and resonance circuitry enabling a full system simulation without any experimental work. All modeling performed in the PEEC solver - even SPICE-solver can be omitted.
The challenge is to model driving circuit, resonance circuits, and decoupling capacitors together with a complete electromagnetic model for the antenna.

Figure 7: Schematic for simple RFID antenna system.
7.15.2. Automotive chassis

- Nonorthogonal PEEC
- 2,862 surfaces and 3,816 volumes
- Front bumper is excited using a 1 V, Gaussian pulse with ‘rise time’ 50 ns.
- The front bumper grounded by a 100 Ω resistor. Back bumper grounded by a 50 Ω resistor.
- FW, TD analysis, 200 points: 4 h, 10 m.
- FW, FD analysis: 50 freqs.: 5 h.
- Regular Linux server with 4 Gb Ram
- Code by LTU, UAq, and IBM.
- Example can be downloaded from: http://www.csee.ltu.se/peec
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Solving Maxwell's . . .
Differential - FDTD
Integral - PEEC

Graphical User Interface (GUI)

Meshing → Partial Element Calculation → Matrix Formulation → Matrix Solution

Post-processor
Figure 8: Frequency domain analysis for 30 MHz (a) and 90 MHz (b).
7.15.3. Aperture in groundplane

- Orthogonal PEEC
- 724 surfaces and 1 244 volumes
- Near-end differentially excited with 1 A, Gaussian pulse with 'rise time' 1 ns.
- Far-end grounded by 50 Ω resistor.
- FW, TD analysis, 200 points: 4 h, 10 m.
- FW, FD analysis: 250 freqs.: 5 h.
- Regular Linux server with 4 Gb Ram
- Code by LTU, UAq, and IBM.
- Example can be downloaded from: http://www.csee.ltu.se/peec
7.15.4. Metallic case

A 19x43x38 cm (LxWxT) case with one opening (19x10) in the front is modeled in the time- and frequency domain.

- Orthogonal PEEC
- 1 470 surfaces and 2 703 volumes
- Front excited with 1 A, pulse with rise time 1 ns.
- Case grounded at 2 locations.
- FW, TD analysis, 250 points: ~ 1 m
- FW, FD analysis: 200 freqs.: 30 mins.
- Regular Linux server with 4 Gb Ram
- Code by LTU, UAq, and IBM.

Example can be downloaded from:
http://www.csee.ltu.se/peec
7.15.5. Reactor

- 1.8 m high free-space reactor, 1 m sides
- Orthogonal PEEC
- 1 200 surfaces and 800 volumes
- Near-ends excited with 1 A, 'Gaussian' pulse with different rise times.
- Near and far-ends grounded.
- QS, TD analysis, 300 points: \( \sim \) 50 s.
- Regular Linux server with 4 Gb Ram
- Code by LTU, UAq, and IBM.

Example can be downloaded from: [http://www.csee.ltu.se/peec](http://www.csee.ltu.se/peec)
7.16. PEEC Research Project

7.16.1. Reactor PEEC Modeling

Project goals are to (1) use PEEC to construct high frequency (up to 10 MHz) models for reactors, (2) synthesize into (reduced) equivalent circuits, and (3) exported to SPICE-like solvers for use in system studies.

Funded by: Elforsk/ELEKTRA
Industry partners: ABB, Banverket, STRI
Period: May 2005 to Dec. 2007

Contact: Dr. Jonas Ekman
PhD Student: Mathias Enohnyaket
7.16.2. PEEC for High Performance Computing

Project goal is to develop PEEC solvers for high performance/parallel computing to enable studies of systems with a large number of unknowns.

Funded by: CASTT
Period: May 2006 to Dec. 2008

Contact: Dr. Jonas Ekman
PhD Student: Peter Anttu
7.16.3. **EM modeling for automotive applications**

Project goals are to develop PEEC for combined analysis of:

- chassis and multi-conductor transmission lines,
- chassis and antennas.

Funded by: CASTT  
Period: May 2006 to Dec. 2008

Contact: Dr. Jonas Ekman  
PhD Student: Peter Anttu
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Solving Maxwell’s…
Differential - FDTD
Integral - PEEC

Multic.-transmission lines
Chassis/body
Antennas
Two-band
Three-band
Four-band
One-band
7.16.4. **PEEC for antenna analysis**

Project goals are to develop extension to PEEC to enable antenna analysis. For example:

- radiation diagram visualization,
- gain calculations.

Funded by: LTU
Industry partners: Perlos, Sweden
Period: May 2005 to Dec. 2007

Contact: Dr. Jonas Ekman
PhD Student: Sofia Sundberg