Report on the evaluation of partial coefficients of potential and partial inductances using the contour integral formulation

Report nr.1, L'Aquila

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1. Introduction

The following paragraph introduces the abbreviations used in the text.

lp1 (LpSelfRect)

The partial self inductance for a rectangular bar (BM J Sept 1972, Ruehli, Inductance Calculations, p475, eq (15)).

lp2 (LpSelfZero)

The partial self inductance of a rectangular and zero thickness conductor (IBM J Sept 1972, Ruehli, Inductance Calculations, p475 eq (16)).

lpmno_cont (The contour formulation)

Calculation of the partial mutual inductances, based on the transformation of the surface integral in eq (1.1) to a 4 by 4 summation of contour integrals, eq (1.2).

$$I_T = \int\limits_{Q'} \int\limits_{Q} \frac{1}{R} \, dS \; dS' \tag{1.1}$$

$$I_T = -\sum_{i=1}^{4} \sum_{j=1}^{4} \int\limits_{l'_i} \int\limits_{l_j} R \; (\hat{u}_j \bullet \hat{u}'_i) \; dl_j \; dl'_i \tag{1.2}$$

lpmno ('Old' surface formulation)

Partial mutual inductance calculation.

$$Lp_{ij} = \frac{\mu}{a_{ci} \cdot a_{cj}} \int_{\alpha_i=-1}^{1} \int_{\beta_i=0}^{1} \int_{\gamma_i=-1}^{1} \left(\frac{\partial r}{\partial \alpha}\right)^i \left(\frac{\partial r}{\partial \beta}\right)^i \left(\frac{\partial r}{\partial \gamma}\right)^i \cdot Sin\Theta^i_{\alpha\beta}$$

$$\int_{\alpha_j=-1}^{1} \int_{\beta_j=0}^{1} \int_{\gamma_j=-1}^{1} (\widehat{\alpha}_i \bullet \widehat{\alpha}_j) \cdot Sin\Theta^j_{\alpha\beta} \cdot G \cdot \left(\frac{\partial r}{\partial \alpha}\right)^j \left(\frac{\partial r}{\partial \beta}\right)^j \left(\frac{\partial r}{\partial \gamma}\right)^j d\alpha_i \; d\beta_i \; d\gamma_i \; d\alpha_j \; d\beta_j \; d\gamma_j$$

lpmno3d (Volume formulation) Partial mutual inductance calculation. Based on *lpmno*, but here for a constant thickness.

lpmno2

Partial mutual inductance calculation by the filament approximation technique.

pmon (PSelfZero)

Partial self coefficients of potentials calculated using eq (16) from IBM J Sept 1972, Ruehli, Inductance Calculations, p475.

Pp_cont

Same as for *Lp_cont*.

2. Orthogonal geometries

2.1. Test case 1

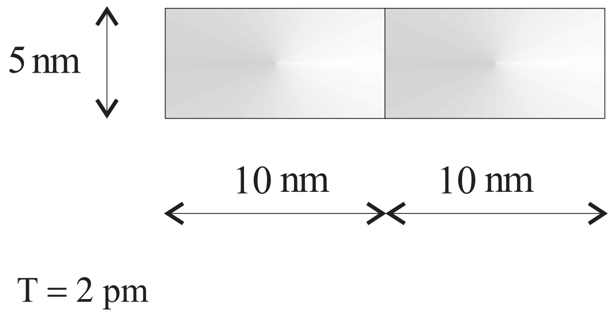

Figure 2.1: The geometry for test case 1 (orthogonal quad)

2.1.1. Partial inductances

Partial inductances for test case 1.
Self term / nH

$$
\begin{aligned}
lp1 &= 4.0845 \\
lp2 &= 4.08534 \\
lpmno\_cont &= 4.16708
\end{aligned}
$$

Mutual term / nH

$$
\begin{aligned}
lpmno &= 1.22371 \\
lpmno3d &= 1.23531 \\
lpmno2 &= 1.24629 \\
lpmno\_cont &= 1.23048
\end{aligned}
$$

2.1.2. Partial coefficients of potential / pF^{-1}

$$Pmon = \begin{bmatrix} 5.34449e+018 & 1.99756e+018 & 9.18049e+017 & 6.04796e+017 \\ 1.99756e+018 & 5.34449e+018 & 1.99756e+018 & 9.18049e+017 \\ 9.18049e+017 & 1.99756e+018 & 5.34449e+018 & 1.99756e+018 \\ 6.04796e+017 & 9.18049e+017 & 1.99756e+018 & 5.34449e+018 \end{bmatrix}$$

$$Pp\_cont = \begin{bmatrix} 5.50647e+018 & 1.95865e+018 & 9.18038e+017 & 6.04795e+017 \\ 1.95865e+018 & 5.50647e+018 & 1.95865e+018 & 9.18038e+017 \\ 9.18038e+017 & 1.95865e+018 & 5.50647e+018 & 1.95865e+018 \\ 6.04795e+017 & 9.18038e+017 & 1.95865e+018 & 5.50647e+018 \end{bmatrix}$$

2.2. Test case 2

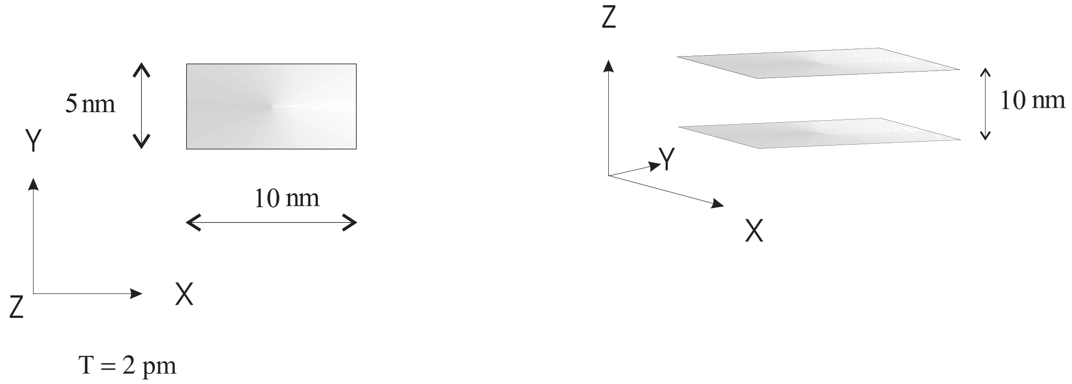

Figure 2.2: The geometry for test case 2 (orthogonal quad)

2.2.1. Partial inductances

Partial inductances for test case 2.

Self term / nH

$$
\begin{aligned}
lp1 &= 4.0845 \\
lp2 &= 4.08534 \\
lpmno\_cont &= 4.16708
\end{aligned}
$$

Mutual term / nH

$$
\begin{aligned}
lpmno &= 0.918152 \\
lpmno3d &= 0.918123 \\
lpmno2 &= 0.918124 \\
lpmno\_cont &= 1.70451
\end{aligned}
$$

2.2.2. Partial coefficients of potential / pF^{-1}

$$Pmon = \begin{bmatrix} 5.34449e+018 & 1.99756e+018 & 8.64795e+017 & 7.85579e+017 \\ 1.99756e+018 & 5.34449e+018 & 7.85579e+017 & 8.64795e+017 \\ 8.64795e+017 & 7.85579e+017 & 5.34449e+018 & 1.99756e+018 \\ 7.85579e+017 & 8.64795e+017 & 1.99756e+018 & 5.34449e+018 \end{bmatrix}$$

$$Pp\_cont = \begin{bmatrix} 5.50647e+018 & 1.95865e+018 & 1.66769e+018 & 1.39625e+018 \\ 1.95865e+018 & 5.50647e+018 & 1.39625e+018 & 1.66769e+018 \\ 1.66769e+018 & 1.39625e+018 & 5.50647e+018 & 1.95865e+018 \\ 1.39625e+018 & 1.66769e+018 & 1.95865e+018 & 5.50647e+018 \end{bmatrix}$$

2.3. Test case 3, 45 deg rotation of test case 1

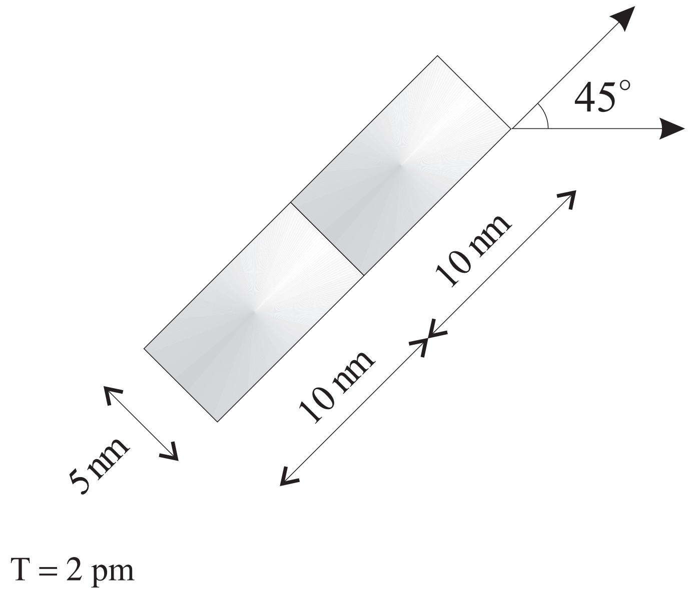

Figure 2.3: The geometry for test case 3 (orthogonal quad)

2.3.1. Partial inductances

Partial inductances for test case 3.

Self term / nH

$$
\begin{aligned}
lp1 &= 2.88793 \\
lp2 &= 2.88877 \\
lpmno\_cont &= 4.16708
\end{aligned}
$$

Mutual term / nH

$$
\begin{aligned}
lpmno &= 1.22371 \\
lpmno3d &= 1.23531 \\
lpmno2 &= 1.00218 \\
lpmno\_cont &= 1.23048
\end{aligned}
$$

2.3.2. Partial coefficients of potential / pF^{-1}

$$Pmon = \begin{bmatrix} 5.34449e+018 & 1.99756e+018 & 9.18049e+017 & 6.04796e+017 \\ 1.99756e+018 & 5.34449e+018 & 1.99756e+018 & 9.18049e+017 \\ 9.18049e+017 & 1.99756e+018 & 5.34449e+018 & 1.99756e+018 \\ 6.04796e+017 & 9.18049e+017 & 1.99756e+018 & 5.34449e+018 \end{bmatrix}$$

$$Pp\_cont = \begin{bmatrix} 5.50647e+018 & 1.95865e+018 & 9.18038e+017 & 6.04795e+017 \\ 1.95865e+018 & 5.50647e+018 & 1.95865e+018 & 9.18038e+017 \\ 9.18038e+017 & 1.95865e+018 & 5.50647e+018 & 1.95865e+018 \\ 6.04795e+017 & 9.18038e+017 & 1.95865e+018 & 5.50647e+018 \end{bmatrix}$$

2.4. Conclusions - Orthogonal geometries

- Good agreement, max 2% off compared to correct solution, is obtained when using the contour formulation to calculate self and mutual parital inductances.
- The calculation of the partial coefficients of potential is done with good agreement, max 3% off, using the contour formulation.
- Test 2 shows, as already know, that the contour formulation require the patches to be in the same plane when calculating mutual partial elements.
- In test 3 (45 deg rotation of test case 1), the contour formulation is the only method that is sucessful in calculating the self partial inductance. For the mutual partial inductance, same agreement is obtained as earlier (max 2% off).
- The accuracy of the calculated partial coefficients of potential is not affected by the rotation (in test case 3) when using the contour formulation. And, same agreement is obtained as earlier i.e. max 3% off.

3. Non orthogonal geometries

3.1. Test case 1

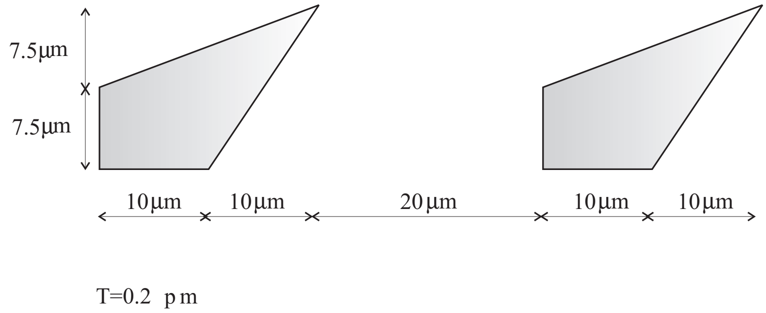

Figure 3.1: The geometry for test case 1 (non-orthogonal quad)

3.1.1. Partial inductances

Partial inductances for test case 1.

Self term / μH

$$\begin{aligned} lp1 &= 3.41330 \\ lp2 &= 3.41599 \\ lpmno\_cont &= 5.58571 \end{aligned}$$

Mutual term / μH

$$\begin{aligned} lpmno &= 0.590947 \\ lpno3d &= 0.590939 \\ lpmno2 &= 0.264365 \\ lpmno\_cont &= 0.596417 \end{aligned}$$

3.1.2. Partial coefficients of potential / pF^{-1}

$$Pmon = \begin{bmatrix} 3.07581e+015 & 1.26913e+015 & 2.25165e+014 & 1.89009e+014 \\ 1.26913e+015 & 2.40610e+015 & 2.77496e+014 & 2.25194e+014 \\ 2.25165e+014 & 2.77496e+014 & 3.07581e+015 & 1.26913e+015 \\ 1.89009e+014 & 2.25194e+014 & 1.26913e+015 & 2.40610e+015 \end{bmatrix}$$

$$Pp\_cont = \begin{bmatrix} 3.37915e+015 & 1.19343e+015 & 2.25184e+014 & 1.89029e+014 \\ 1.19343e+015 & 2.94611e+015 & 2.77508e+014 & 2.25228e+014 \\ 2.25184e+014 & 2.77508e+014 & 3.37915e+015 & 1.19343e+015 \\ 1.89029e+014 & 2.25228e+014 & 1.19343e+015 & 2.94611e+015 \end{bmatrix}$$

3.2. Test case 2

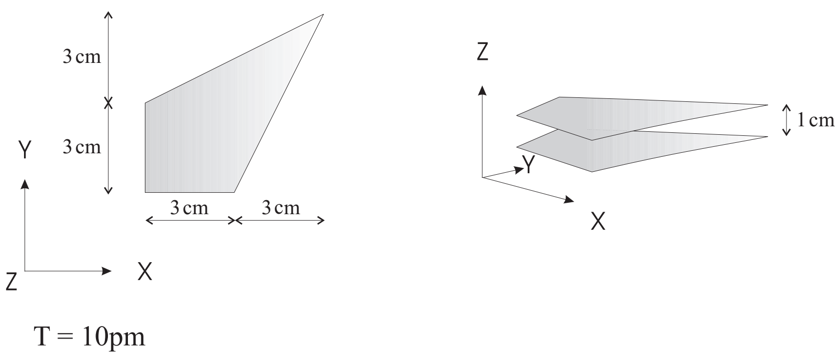

Figure 3.2: The geometry for test case 2 (non orthogonal quad)

3.2.1. Partial inductances

Partial inductances for test case 2.

Self term / mH

$$\begin{aligned} lp1 &= 667.407 \\ lp2 &= 8.91963 \\ lpmno\_cont &= 15.186 \end{aligned}$$

Mutual term / mH

$$\begin{aligned} lpmno &= 9.98069 \\ lpmno3d &= 9.75202 \\ lpmno2 &= 5.4697 \\ lpmno\_cont &= 13.2494 \end{aligned}$$

3.2.2. Partial coefficients of potential / pF^{-1}

$$Pmon = \begin{bmatrix} 8.75159e+011 & 3.86218e+011 & 5.25584e+011 & 3.29946e+011 \\ 3.86218e+011 & 6.93412e+011 & 3.22245e+011 & 4.63486e+011 \\ 5.25584e+011 & 3.22245e+011 & 8.75159e+011 & 3.86218e+011 \\ 3.29946e+011 & 4.63486e+011 & 3.86218e+011 & 6.93412e+011 \end{bmatrix}$$

$$Pp\_cont = \begin{bmatrix} 9.73136e+011 & 3.55775e+011 & 7.57191e+011 & 3.94752e+011 \\ 3.55775e+011 & 8.53721e+011 & 3.94752e+011 & 6.49711e+011 \\ 7.57191e+011 & 3.94752e+011 & 9.73136e+011 & 3.55775e+011 \\ 3.94752e+011 & 6.49711e+011 & 3.55775e+011 & 8.53721e+011 \end{bmatrix}$$

3.3. Conclusions - Non orthogonal geometries

- No comparison can be made, so far, when calculating the self partial inductance for non orthogonal geometries. For the mutual partial inductance the agreement against the surface and volume formulation is good, within 1%.
- For the self partial coefficients of potential no good comparison can be made, so far. The mutual coefficients of potential are max. 6% off.
- Test 2 shows as for the orthogonal case that the contour formulation require the patches to be in the same plane when calculating mutual partial elements.