Report on the evaluation of partial coefficients of potential and partial inductances using the contour integral formulation

Report nr.4, L'Aquila

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Introduction

The following paragraph introduces the abbreviations used in the text.

IEEE 1995

A. E. Ruehli, U. Miekkala, H. Heeb, "Stability of Discretized Partial Element Equivalent EFIE Circuit Models", IEEE Transactions on Antennas and Propagation, Vol. 43, No. 6. June 1995, pages 553-559.

IEEE 1995 - no rotation

Same reference as above, but here used for the comparision of the non-orthogonal evaluations using *lpmno_cont*, *lpmno*, *Pp_cont* and *pmno*.

lpmno_cont (The contour formulation)

Calculation of the partial mutual inductances, based on the transformation of the surface integral in eq (1) to a 4 by 4 summation of contour integrals, eq (2).

$$I_T = \int_{Q'} \int_{Q} \frac{1}{R} \, dS \, dS' \tag{1}$$

$$I_T = -\sum_{i=1}^{4} \sum_{j=1}^{4} \int_{l_i'} \int_{l_j} R \, (\widehat{u}_j \bullet \widehat{u}_i') \, dl_j \, dl_i' \tag{2}$$

lpmno ('Old' surface formulation)

Partial mutual inductance calculation.

$$Lp_{ij} = \frac{\mu}{a_{ci} \cdot a_{cj}} \int\limits_{\alpha_i=-1}^{1} \int\limits_{\beta_i=0}^{1} \int\limits_{\gamma_i=-1}^{1} \left(\frac{\partial r}{\partial \alpha}\right)^i \left(\frac{\partial r}{\partial \beta}\right)^i \left(\frac{\partial r}{\partial \gamma}\right)^i \cdot Sin\Theta^i_{\alpha\beta}$$

$$\int\limits_{\alpha_j=-1}^{1} \int\limits_{\beta_j=0}^{1} \int\limits_{\gamma_j=-1}^{1} (\widehat{\alpha}_i \bullet \widehat{\alpha}_j) \cdot Sin\Theta^j_{\alpha\beta} \cdot G \cdot \left(\frac{\partial r}{\partial \alpha}\right)^j \left(\frac{\partial r}{\partial \beta}\right)^j \left(\frac{\partial r}{\partial \gamma}\right)^j d\alpha_i \; d\beta_i \; d\gamma_i \; d\alpha_j \; d\beta_j \; d\gamma_j$$

lpmno3d (Volume formulation) Partial mutual inductance calculation. Based on *lpmno*, but here for a constant thickness.

lpmno2

Partial mutual inductance calculation by the filament approximation technique.

pmno (PSelfZero)

Partial self coefficients of potentials calculated using eq (16) from IBM J Sept 1972, Ruehli, Inductance Calculations, p475.

Pp_cont

Same as for *Lp_cont*.

Test geometries

Orthogonal case

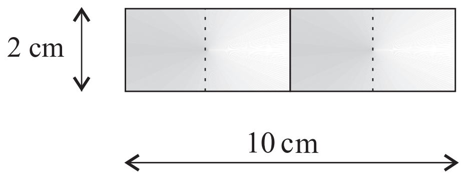

Figure 1: The geometry for test 1

Non-orthogonal case

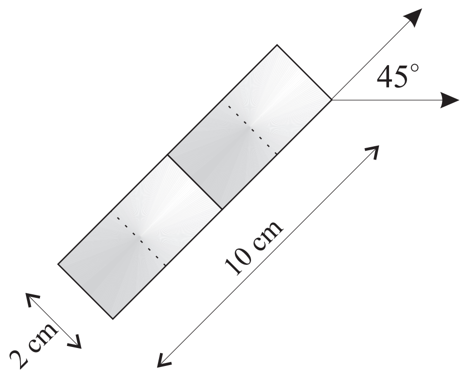

Figure 2: The geometry for test 2

Partial elements

$\mu H, \frac{1}{pF}$	Lp or p	Min. τ	Center τ	Max τ	
Lp_{11}	0.022362				IEEE 1995
	0.022965				*lpmno_cont*
Lp_{12}, τ_L	0.006314	0.0	0.167	0.333	IEEE 1995
	0.006296				*lpmno_cont*
	0.006305				*lpmno*
	0.006314				*lpmno3d*
	0.006359				*lpmno2*
p_{11}=p_{33}	1.19143				IEEE 1995
	1.22972				*Pp_cont*
	1.19146				*pmno*
p_{22}	0.80392				IEEE 1995
	0.85752				*Pp_cont*
	0.80392				*pmno*
p_{12}=p_{23}, τ_1	0.300756	0.0	0.167	0.25	IEEE 1995
	0.297492				*Pp_cont*
	0.300782				*pmno*
p_{13}, τ_2	0.121378	0.0	0.333	0.333	IEEE 1995
	0.121381				*Pp_cont*
	0.121381				*pmno*

Figure 3: Element values for test 1, edge-based nodes

$\mu H, \frac{1}{pF}$	Lp or p	Min. τ	Center τ	Max τ	
Lp_{11}	0.022362				IEEE 1995 - no rotation
	0.022965				*lpmno_cont*
Lp_{12}, τ_L	0.006314	0.0	0.167	0.333	IEEE 1995 - no rotation
	0.006296				*lpmno_cont*
	0.006305				*lpmno*
$p_{11}=p_{33}$	1.19143				IEEE 1995 - no rotation
	1.22972				*Pp_cont*
	1.19146				*pmno*
p_{22}	0.80392				IEEE 1995 - no rotation
	0.85752				*Pp_cont*
	0.80392				*pmno*
$p_{12}=p_{23}$, τ_1	0.300756	0.0	0.167	0.25	IEEE 1995 - no rotation
	0.297492				*Pp_cont*
	0.300782				*pmno*
p_{13}, τ_2	0.121378	0.0	0.333	0.333	IEEE 1995 - no rotation
	0.121381				*Pp_cont*
	0.121381				*pmno*

Figure 4: Element values for test 2, edge-based nodes

Finally

- The contour formulation offer good agreement compared to the computation time required.
- In test 2 (45 deg rotation of test case 1), the contour formulation is the only method that is sucessful in calculating the self partial inductance. For the mutual partial inductance, same agreement is obtained as earlier.