

# Implementation of complex partial elements in C++

Report nr.6, L'Aquila

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# INTRODUCTION

In the PEEC method the static partial elements are often used (1) to simplify the preparation of the complete PEEC model and (2) to speed up the solution. In eq (1) the surface formulation, including the complex part, for the partial coefficients of potential is shown. For 'low' frequencies or for simplified PEEC models the complex part is excluded resulting in the expression in eq (2).

$$p_{ij} = \frac{1}{4\pi\epsilon s_i s_j} \iint_{S_j} \iint_{S_i} \frac{e^{-j\beta|r_i-r_j|}}{|r_i-r_j|} ds_i ds_j \quad (1)$$

$$p_{ij} = \frac{1}{4\pi\epsilon s_i s_j} \iint_{S_j} \iint_{S_i} \frac{1}{|r_i-r_j|} ds_i ds_j \quad (2)$$

To be able to develop a 'full-wave PEEC solver' in C++ the complex part in eq (1) has to be included in the evaluation of the partial elements, ie inductances and capacitances. The complex part has been added to four different routines in the existing C++ code. First, the surface formulation from eq. (1) and (2) is completed resulting in a *Pmnoc* and a *Lpmnoc* routine. Second, the contour formulation that is explained in report nr. 1 entitled *Report on the evaluation of partial coefficients of potential and partial inductances using the contour integral formulation*, L'Aquila nov, 2001, has been completed with the complex part. To keep the speed, and thus the advantages, of the contour formulation the center to center distance between the patches is used when calculating the complex part. The calculation takes part in the beginning of the routine and is based on the 4 or 8 corner coordinates describing the capacitive or inductive patches, respectively. The new complex contour routines are named, *Pp\_cont\_c* and *Lp\_cont\_c*.

In this report, the mutual partial elements for different patch-configurations are calculated and compared using the complex formulations implemented in C++ and the surface formulation implemented in matlab code, *Matlab*.

# COMPARISON FOR ORTHOGONAL GEOMETRIES

## Test case 1

10 x 10 cm patches, shifted 30 cm in the 'x-direction'.

### Mutual Partial inductances

Mutual term /  $nH$

$$\begin{aligned} Matlab &= 3.3645626 - i2.09582e - 15 \\ Lpmnoc &= 3.36456 - i2.09582e - 15 \quad frequency = 100Hz \\ Lp\_cont\_c &= 3.36456 - i2.11546e - 15 \end{aligned}$$

$$\begin{aligned} Matlab &= 3.35791476 - i2.09439e - 10 \\ Lpmnoc &= 3.35791 - i2.09439e - 10 \quad frequency = 10MHz \\ Lp\_cont\_c &= 3.35791 - i2.11406e - 10 \end{aligned}$$

$$\begin{aligned} Matlab &= 2.29957718 - i1.463644e - 10 \\ Lpmnoc &= 2.29958 - i1.46364e - 10 \quad frequency = 1GHz \\ Lp\_cont\_c &= 3.35453 - i0.144106e - 10 \end{aligned}$$

$$\begin{aligned} Matlab &= 0.0936706 - i7.572838e - 12 \\ Lpmnoc &= 0.0936707 - i7.57284e - 12 \quad frequency = 5GHz \\ Lp\_cont\_c &= 3.36379 - i7.20478e - 11 \end{aligned}$$

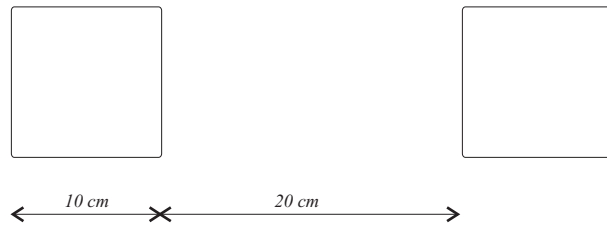


Figure 1: Test case 1

### Mutual partial coefficients of potential

Mutual term /  $pF^{-1}$

$$\begin{aligned}
 Matlab &= 0.03023980e12 + i18836.7 \\
 Pmnoc &= 0.0302398e12 + i18836.7 \quad frequency = 100Hz \\
 Pp\_cont\_c &= 0.0302398e12 + i19013.2
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= 0.03018005e12 + i1.8823e9 \\
 Pmnoc &= 0.0301801e12 + i1.8823e9 \quad frequency = 10MHz \\
 Pp\_cont\_c &= 0.0301801e12 + i1.9001e9
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= 0.02066884e12 + i1.31428e9 \\
 Pmnoc &= 0.020668e12 + i1.31549e9 \quad frequency = 1GHz \\
 Pp\_cont\_c &= 0.0302395e12 + i1.29519e8
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= 0.000841887e12 + i6.806272e7 \\
 Pmnoc &= 0.000841827e12 + i6.80246e7 \quad frequency = 5GHz \\
 Pp\_cont\_c &= 0.030239e12 + i6.47547e8
 \end{aligned}$$

## Test case 2

1 x 1 cm patches, shifted 5 cm in the 'x-direction'.

### Partial inductances

Mutual term /  $nH$

$$\begin{aligned} \text{Matlab} &= 0.2006693 - i2.0958e - 17 \\ \text{Lpmnoc} &= 0.200669 - i2.0958e - 17 \quad \text{frequency} = 100Hz \\ \text{Lp\_cont\_c} &= 0.200669 - i2.10284e - 17 \end{aligned}$$

$$\begin{aligned} \text{Matlab} &= 0.0891792 + j1.59793e - 10 \\ \text{Lpmnoc} &= 0.0891792 + j1.59793e - 10 \quad \text{frequency} = 5GHz \\ \text{Lp\_cont\_c} &= 0.100954 + j1.73426e - 10 \end{aligned}$$

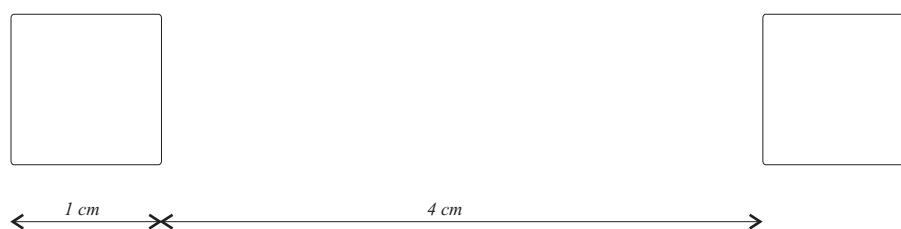


Figure 2: Test case 2

### Partial coefficients of potential

Mutual term /  $pF^{-1}$

$$\begin{aligned} \text{Matlab} &= 0.18035647e12 + i18836.72 \\ \text{Pmnoc} &= 0.180356e12 - i18836.7 \quad \text{frequency} = 100Hz \\ \text{Pp\_cont\_c} &= 0.180356e12 - i18899.8 \end{aligned}$$

$$\begin{aligned} \text{Matlab} &= 0.0801519e12 - j1.43618e11 \\ \text{Pmnoc} &= 0.080152e12 + i1.43618e11 \quad \text{frequency} = 5GHz \\ \text{Pp\_cont\_c} &= 0.0907352e12 + j1.5587e11 \end{aligned}$$

### Test case 3

1 x 5 cm patches, shifted 10 cm in the 'x-direction'.

#### Partial inductances

Mutual term /  $nH$

$$\begin{aligned} Matlab &= 0.098207125 - i2.095823e - 17 \\ L_{pmnoc} &= 0.0982071 - i2.09582e - 17 \quad frequency = 100Hz \\ L_{p\_cont\_c} &= 0.0982071 - i2.05646e - 17 \end{aligned}$$

$$\begin{aligned} Matlab &= 0.05558293 + j7.7458304e - 11 \\ L_{pmnoc} &= 0.0555829 + j7.74583e - 11 \quad frequency = 2.5GHz \\ L_{p\_cont\_c} &= 0.0494068 + j8.48741e - 11 \end{aligned}$$

$$\begin{aligned} Matlab &= -0.028108318 + j8.252131e - 11 \\ L_{pmnoc} &= -0.0281083 + j8.25213e - 11 \quad frequency = 5GHz \\ L_{p\_cont\_c} &= -0.0484952 + j8.53982e - 11 \end{aligned}$$

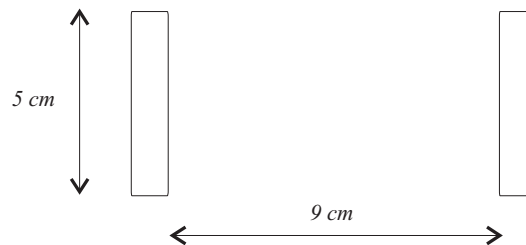


Figure 3: Test case 3

## Partial coefficients of potential

Mutual term /  $pF^{-1}$

$$\begin{aligned} Matlab &= 0.088266217e12 + i18836.7 \\ Pmnoc &= 0.088266e12 + i18836.7 \quad frequency = 100Hz \\ Pp\_cont\_c &= 0.088266e12 - i18482.9 \end{aligned}$$

$$\begin{aligned} Matlab &= 0.0499565e12 - j6.961752e + 10 \\ Pmnoc &= 0.0499565e12 + j6.96175e + 010 \quad frequency = 2.5GHz \\ Pp\_cont\_c &= 0.0444056e12 + j7.62826e + 010 \end{aligned}$$

$$\begin{aligned} Matlab &= -0.025263032e12 - j7.416803e10 \\ Pmnoc &= -0.025263e12 + j7.4168e10 \quad frequency = 5GHz \\ Pp\_cont\_c &= -0.0435862e12 + j7.67537e10 \end{aligned}$$

# COMPARISON FOR NON-ORTHOGONAL GEOMETRIES

## Test case

Patches, base 10 mm, top 7mm, height 7mm, shifted 20 mm in the 'x-direction'.

## Mutual Partial inductances

Mutual term /  $nH$

$$\begin{aligned} Matlab &= 0.347426 - j1.42612e - 17 \\ Lpmnoc &= 0.369142 - j1.51423e - 17 \text{ frequency} = 100Hz \\ Lp\_cont\_c &= 0.369142 - j1.54731e - 17 \end{aligned}$$

$$\begin{aligned} Matlab &= 0.347125 - j1.42568e - 11 \\ Lpmnoc &= 0.368822 - j1.51377e - 011 \text{ frequency} = 100MHz \\ Lp\_cont\_c &= 0.368818 - j1.54686e - 011 \end{aligned}$$

$$\begin{aligned} Matlab &= -0.121726 - j6.10483e - 11 \\ Lpmnoc &= -0.158979 - j30.5847e - 11 \text{ frequency} = 5GHz \\ Lp\_cont\_c &= -0.185027 - j31.9423e - 11 \end{aligned}$$

$$\begin{aligned} Matlab &= -0.1523396 + j2.15993e - 10 \\ Lpmnoc &= -0.161642 + j2.27541e - 10 \text{ frequency} = 10GHz \\ Lp\_cont\_c &= -0.183658 + j3.20212e - 10 \end{aligned}$$



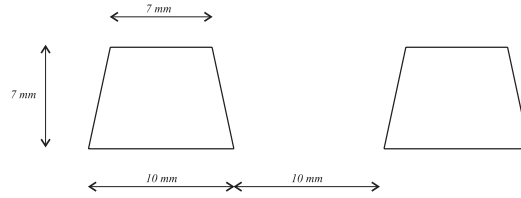


Figure 4: Test case 1

### Mutual partial coefficients of potential

Mutual term /  $pF^{-1}$

$$\begin{aligned}
 Matlab &= 0.317210 + j13012.0 \\
 Pmnoc &= 0.458864 - j18836.7 \text{ frequency} = 100Hz \\
 Pp\_cont\_c &= 0.459205 - j19248.2
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= 0.316934 + j1.30080e10 \\
 Pmnoc &= 0.458466 - j1.88309e10 \text{ frequency} = 100MHz \\
 Pp\_cont\_c &= 0.458802 - j1.92426e10
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= -0.136614 + j2.62818e11 \\
 Pmnoc &= -0.198495 - j3.80465e11 \text{ frequency} = 5GHz \\
 Pp\_cont\_c &= -0.230170 - j3.97355e11
 \end{aligned}$$

$$\begin{aligned}
 Matlab &= -0.138901 - j1.95529e11 \\
 Pmnoc &= -0.201343 + j2.85218e11 \text{ frequency} = 10GHz \\
 Pp\_cont\_c &= -0.228466 + j3.98337e11
 \end{aligned}$$

## Concluding remarks

- The comparison of the surface based routines, *pmnoc* & *lpmnoc*, implemented in C++ and Matlab show exact agreement for orthogonal geometries. For the non orthogonal case the agreement is not good.
- The test shows that the complex contour implementation, *Lp\_cont\_c* & *Pp\_cont\_c*, offer acceptable agreement for non orthogonal geometries, compared with the surface formulation, for frequencies up to 2.5GHz under certain discretization and distance constraints. The computation time is considerably less than for the *Pmnoc* and *Lpmno* routines. To be remembered, the contour formulation require the patches to be located in the same plane.