LULEÅ UNIVERSITY OF TECHNOLOGY
Dept. of Computer Science and Electrical Engineering

Course	E7021E
Date	2009-10-27
Time	15:00-19:00

Exam in:	**Measurement Technology & Uncertainty Analysis**
Attending teacher:	Johan Carlson (070-580 82 52)
Problems:	5 (5 points per problem)
Tools allowed:	BETA (Mathematics Handbook), Physics handbook, Language dictionary, calculator Text books: Principles of Measurement Systems, by John Bentley Introduction to Empirical Model Building and Parameter Estimation, by Johan E. Carlson (including the Errata sheet)

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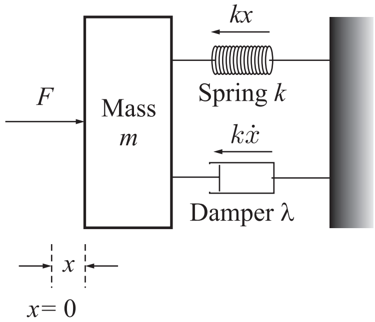
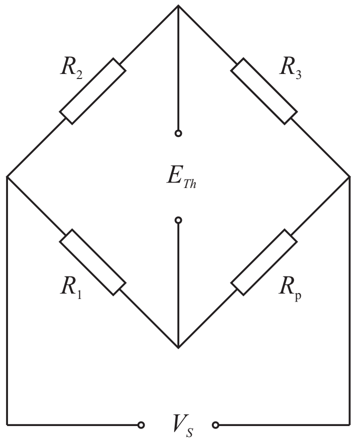
1. The mechanical system in the figure below could be simulated using an electronics simulation package (e.g. Orcad or Spice), provided that we can find an analogous electrical system.



   (a) Draw the circuit of the analogous electrical system, explain how the electrical quantities should be chosen in order to model the mechanical system. (2p)

   (b) Derive an expression for the transfer function of the electrical system, showing how voltages are related to the force and displacement in the mechanical system. (3p)

2. A resistive sensing element with resistance R_p is connected to a deflection bridge according to the figure below. The supply voltage was measured 10 times and stored in the vector



$$\mathbf{V}_s = \begin{bmatrix} 5.02 & 4.99 & 5.0 & 4.98 & 5.01 & 4.99 & 5.01 & 5.0 & 5.01 & 4.98 \end{bmatrix}^T.$$

Assume that $\sigma_{R_p} = 0.1$ and $\sigma_{R_1} = \sigma_{R_2} = \sigma_{R_3} = 0.05$, and that $\overline{R}_1 = \overline{R}_2 = \overline{R}_3 = 200\ \Omega$.

(a) Estimate the mean $\overline{V}_s$ and the standard deviation σ_{V_s} of the supply voltage. (1p)

(b) Derive an expression for the total variance, σ_E^2 of E, as a function of the resistance R_p, where (3p)

$$E = V_s \left(\frac{R_1}{R_1 + R_p} - \frac{R_2}{R_2 + R_3} \right).$$

(c) Looking at a $2\sigma_E$, how much of the total variation does the supply voltage variation account for (in percent of the full-scale deflection), when $\overline{R}_p = 150\ \Omega$? (1p)

3. A temperature system incorporating a PT100 platinum resistance sensor, a resistive deflection bridge and a recorder is used to measure temperatures in the range 0–100 °C. The PT100 sensor has a resistance of 100 Ω at 0 °C and and a sensitivity of 0.385 Ω/°C

(a) Design the resistive deflection bridge according to the following specifications: (3p)

- It is balanced (or at least approximately so),
- The current through the sensor is no larger than 1 mA.

Motivate your choices clearly.

(b) The model equation for the recorder is

$$T_{\text{meas}} = K_2 V_o + a_2,$$

where V_o is the voltage output from the deflection bridge and T_{meas} is the output (measured temperature). Determine the coefficients K_2 and a_2 so that the sum of squares of the error between the true temperature and the measured temperature is minimized.
Hint: Use the input/output relationship of the sensor and deflection bridge to calculate a couple of points and then use the principle of least-squares to fit the best straight line for the recorder. If you did not solve (a), assume the resistances of the bridge, $R_2 = R_3 = R_4 = 10\ \text{k}\Omega$, and the supply voltage $V_s = 3$ V. (2p)

4. A temperature sensor, modeled as a first order system (as in the figure) with a time constant $\tau = 10$ s is connected first to a perfectly linear, balanced deflection bridge, that gives a voltage output as

$$\Delta V(t) = K \cdot \Delta T_t(t) \text{ volts.}$$

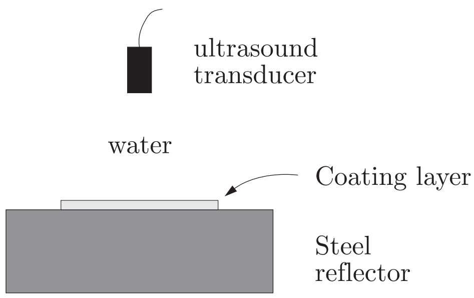
This is then followed by a compensating element, with the purpose of compensating for the large time constant of the sensor.

(a) If $\tau = 0$, what should K and $G_2(s)$ be in order for the system to have a zero steady-state bias? (1p)

(b) Determine the dynamic error of the system when $\tau = 10$ s, and $G_2(s) = 1$, i.e. for the un-compensated system, when the input is a sudden change of the temperature of 20 °C. (2p)

(c) Consider $G_2(s)$ to be a lead-lag compensating circuit. Give an expression for the transfer function and calculate the dynamic error of the total, compensated system. (2p)

5. An ultrasound pulse-echo setup is being used to measure the thickness of a thin coating layer, bonded to a thick steel layer (see figure).



Assuming there is a total reflection of the sound at the coating/steel interface and that the attenuation inside the coating layer is negligible, this can be modeled as the linear system $g(\omega)$, given below. In the frequency domain the relationship between the Fourier transform of the transmitted pulse and the received echo from the coating layer can be written as

$$y(\omega) = u(\omega)\, g(\omega),$$

where $y(\omega)$ is the measured signal, $u(\omega)$ is the transmitted pulse, and $g(\omega)$ is the transfer function of the layer, given by

$$g(\omega) = \frac{(1-R)^2 e^{-j\omega 2\tau}}{1 + R e^{-j\omega 2\tau}},$$

where τ is the time-of-flight of the pulse through the coating layer, and R is the reflection coefficient of the interface between the water and the coating. Given that we can estimate R and τ, and that the speed of sound within the coating layer is known, we can compute the coating thickness and the coating density. We also assume all properties of the water buffer region is known.

Consider the case where we know the input signal $u(\omega)$, for a number of different frequencies, ω_k, $k = 1, 2, \ldots K$. and that the received signal $y(\omega)$ is measured for the same frequencies. We can then use the Gauss-Newton linearization method to iteratively estimate the parameters τ and R.

Explain how this is done by deriving the model gradient with respect to the parameters, and stating the resulting equations, necessary for implementing the algorithm. (5p)