LULEÅ UNIVERSITY OF TECHNOLOGY
Dept. of Computer Science and Electrical Engineering

Course	E7021E
Date	2009-12-18
Time	09:00-13:00

Exam in:	**Measurement Technology & Uncertainty Analysis**
Attending teacher:	Johan Carlson (070-580 82 52)
Problems:	5 (5 points per problem)
Tools allowed:	BETA (Mathematics Handbook), Physics handbook, Language dictionary, calculator Text books: Principles of Measurement Systems, by John Bentley Introduction to Empirical Model Building and Parameter Estimation, by Johan E. Carlson

---

1. A displacement sensor has an input range of 0.0 to 3.0 cm and a supply voltage $V_s = 0.5$ V. Results from a calibration experiments are given in the table below

Displacement x (cm)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Output voltage (mV)	0.0	16.5	32.0	44.0	51.5	55.5	58.0

   (a) Calculate the maximum non-linearity as a percentage of the full-scale deflection (f.s.d.), assuming the steady state sensitivity is calculated as in the book, i.e. (2p)

$$K = \frac{O_{MAX} - O_{MIN}}{I_{MAX} - I_{MIN}}$$

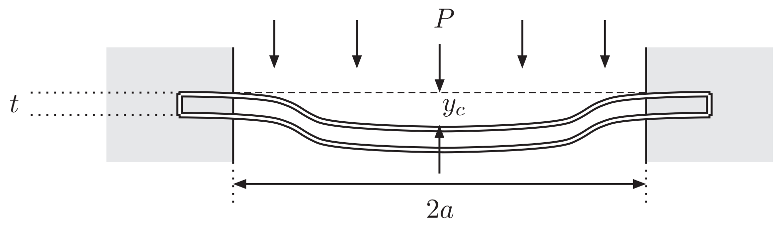
   (b) The performance of the system can easily be improved by instead fitting the *optimal* straight line, using the principle of least-squares. Give the necessary equations for finding the slope K and the intersection a of the straight line using the data in the table above.
   Note: You do not need to solve for the actual numerical values of K and a. (2p)

   (c) Explain why the method in (b) gives a better average result. (1p)

2. A flat circular diaphragm of density 6×10^3 kg/m^3 is to be used as a pressure sensor. The element should fulfill the following specification:

Input range = 0 to 10^4 Pa
Maximum non-linearity = 1 % of the full-scale deflection
Amplitude ratio to be flat with ± 3 % up to 100 Hz

   Using the equations given in the figure below, and assuming a damping ratio of 0.01, calculate:

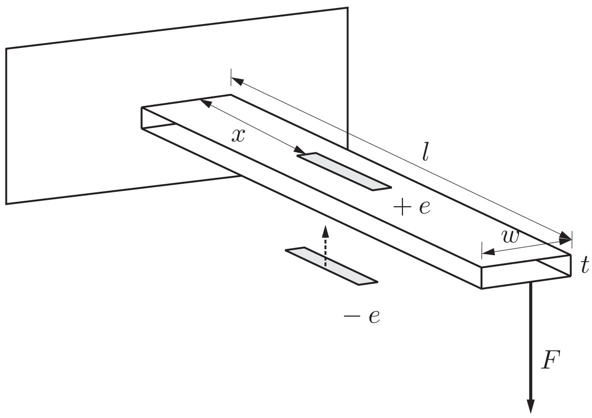


Accurate steady-state relation	$P = \frac{16Et^3}{3a^4(1-\nu^2)} y_c \left[1 + 0.5\left(\frac{y_c}{t}\right)^2\right]$	E = Young's modulus
if $\frac{y_c}{t} \ll 1$ then		ν = Poisson's ratio
Steady-state sensitivity	$K = \frac{y_c}{P} \approx \frac{3a^4(1-\nu^2)}{16Et^3} = \frac{1}{16t\gamma}$	ρ = density
and		$\gamma = \frac{Et^2}{3a^4(1-\nu^2)}$
Natural frequency	$f_n = \frac{2.56}{\pi}\sqrt{\frac{Et^2}{3a^4(1-\nu^2)\rho}} = \frac{2.56}{\pi}\sqrt{\frac{\gamma}{\rho}}$ Hz	

(a) The thickness, t, of the diaphragm. (2p)

(b) The output displacement range of the sensor. (3p)

3. Two strain gauges are bonded onto a cantilever as shown in the figure below. Given that the gauges are placed halfway along the cantilever and the cantilever is subject to a downward force *F*. Use the tabulated data below to:



Cantilever data		*Strain gauge data*	
Length	$l = 25$ cm	Gauge factor	$G = 2.1$
Width	$w = 6$ cm	Unstrained resistance	$R_0 = 120\ \Omega$
Thickness	$t = 3$ mm		
Young's modulus	$E = 70 \times 10^9$ Pa		

(a) Calculate the resistance of each strain gauge for $F = 0.5\ N$ and $F = 10\ N$. (2p)

(b) Design a resistive deflection bridge suitable for force measurements in the interval in (a). Motivate your design choices clearly. (3p)

4. A measurement system consists of a chromel-alumel thermocouple (with cold-junction compensation), a millivolt-to-current converter and a recorder. The table below gives the model equations and parameters for each element. Assuming that all probability density functions are normal, calculate the mean and standard deviation of the error probability distribution, when the input temperature is 117 °C. (5p)

	Chromel-alumel thermocouple	e.m.f.-to current converter	Recorder
Model equation	$E = C_0 + C_1 T + C_2 T^2$	$i = K_1 E + K_M E \Delta T_a + K_I \Delta T_a + a_1$	$T_M = K_2 i + a_2$
Mean values	$\overline{C}_0 = 0.00$ $\overline{C}_1 = 4.017 \times 10^{-2}$ $\overline{C}_2 = 4.66 \times 10^{-6}$	$\overline{K}_1 = 3.893$ $\overline{\Delta T}_a = -10$ $\overline{a}_1 = -3.864$ $\overline{K}_M = 1.95 \times 10^{-4}$ $\overline{K}_I = 2.00 \times 10^{-3}$	$\overline{K}_2 = 6.25$ $\overline{a}_2 = 25.0$
Standard deviations	$\sigma_{C_0} = 6.93 \times 10^{-2}$ $\sigma_{C_1} = \sigma_{C_2} = 0$	$\sigma_{a_1} = 0.14$, $\sigma_{\Delta T_a} = 10$ $\sigma_{K_1} = \sigma_{K_M} = \sigma_{K_I} = 0$	$\sigma_{a_2} = 0.30$ $\sigma_{K_2} = 0$

5. We have a differential pressure flowmeter setup designed for incompressible fluids. The volume flow rate is given by

$$Q = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\left(P_1 - P_2\right)}{\rho}},$$

where A_1 and A_2 are the cross-sectional areas of the pipe where the pressures P_1 and P_2 are measured, respectively. The problem now is that we do not know the fluid density. Assuming we can control the volume flow rate, Q to within some uncertainty, i.e. the flow rate can be said to be normally distributed as $N\left(Q, \sigma_Q\right)$, derive and expression for the least-squares estimator of the flow rate, using the Gauss-Newton linearization method, that based on the calibration measurements also estimates the unknown fluid density ρ. (5p)