LULEÅ UNIVERSITY OF TECHNOLOGY
Dept. of Computer Science and Electrical Engineering

Course	E7021E
Date	2009-10-20
Time	15:00-19:00

Exam in:	**Measurement Technology & Uncertainty Analysis**
Attending teacher:	Johan Carlson (070-580 82 52)
Problems:	5 (5 points per problem)
Tools allowed:	BETA (Mathematics Handbook), Physics handbook, Language dictionary, calculator Text books: Principles of Measurement Systems, by John Bentley Introduction to Empirical Model Building and Parameter Estimation, by Johan E. Carlson

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1. A displacement sensor has an input range of 0.0 to 3.0 cm and a supply voltage $V_s = 0.5$ V. Results from a calibration experiments are given in the table below

Displacement x (cm)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
Output voltage (mV)	0.0	16.5	32.0	44.0	51.5	55.5	58.0

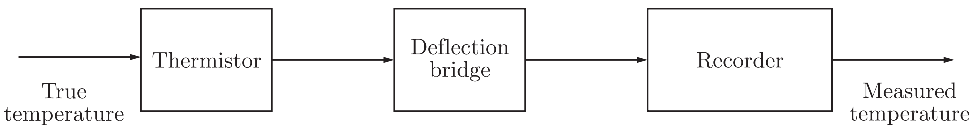
   (a) Calculate the maximum non-linearity as a percentage of the full-scale deflection (f.s.d.), assuming the steady state sensitivity is calculated as in the book, i.e. (2p)

$$K = \frac{O_{MAX} - O_{MIN}}{I_{MAX} - I_{MIN}}$$

   (b) The performance of the system can easily be improved by instead fitting the *optimal* straight line, using the principle of least-squares. Give the necessary equations for finding the slope K and the intersection a of the straight line using the data in the table above.
   Note: You do not need to solve for the actual numerical values of K and a. (2p)

   (c) Explain why the method in (b) gives a better average result. (1p)

2. A temperature measurement system consists of the following elements



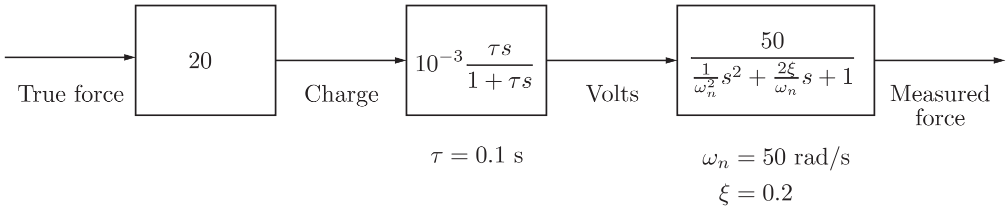
where θ is the true temperature and θ_M is the measured temperature (in Kelvin). The model equations and the corresponding uncertainties are given in the table below.

	Thermistor	Bridge	Recorder
Model equations	$R_\theta = K_1 \exp\left(\frac{\beta}{\theta}\right)$	$V_O = V_S\left(\frac{1}{1+\frac{3.3}{R_\theta}} - a_1\right)$	$\theta_M = K_2 V_O + a_2$
Mean values	$\overline{K}_1 = 5\times 10^{-4}$ kΩ	$\overline{V}_S = -3.00$ V	$\overline{K}_2 = 50.0$ K/V
	$\overline{\beta} = 3\times 10^3$ K	$\overline{a}_1 = 0.77$	$\overline{a}_2 = 300$ K
Standard deviations	$\sigma_{K_1} = 0.5\times 10^{-4}$	$\sigma_{V_S} = 0.03$	$\sigma_{K_2} = 0$
	$\sigma_\beta = 0$	$\sigma_{a_1} = 0.01$	$\sigma_{a_2} = 3.0$

(a) Calculate the mean output $\overline{\theta}_M$ and the mean error $\overline{E} = \overline{\theta} - \overline{\theta}_M$ for an input temperature of 320 K. (2p)

(b) Calculate the standard deviation of the output error $\overline{E}$ for an input of 320 K. (3p)

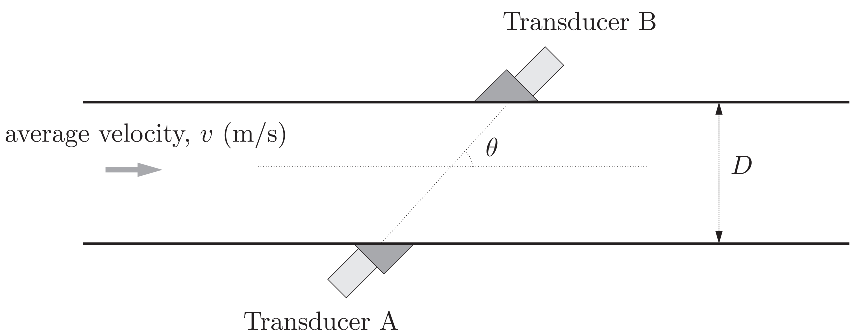
3. A force measurement system consisting of a piezoelectric crystal, charge amplifier and recorder is shown in the figure below.



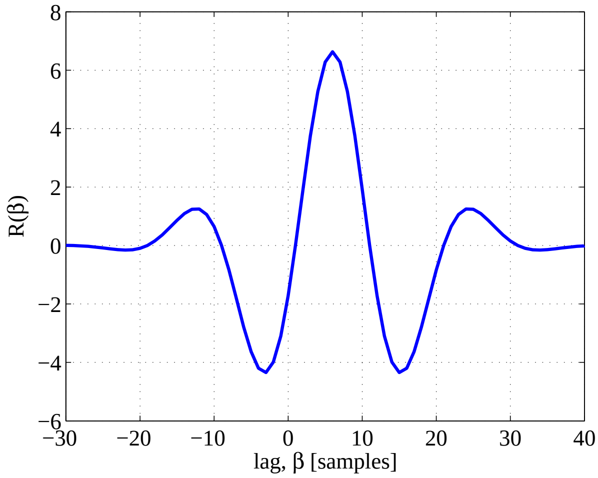
Calculate the system output and the corresponding dynamic error when the force input signal is (5p)

$$F(t) = 50\left[\sin 10t + \frac{1}{3}\sin 30t + \frac{1}{5}\sin 50t\right].$$

4. An ultrasonic transit-time flowmeter is used to measure the average flow velocity v (m/s) through a pipe.



A short ultrasonic pulse is first transmitted from Transducer A to Transducer B and then from Transducer B to Transducer A. The cross-correlation function $R(\beta)$ between the two pulses is shown in the figure below.



The flow velocity is given by

$$v = \frac{\Delta T \cdot c^2}{2D \cot\theta},$$

where θ is the angle between the transducers and the center axis of the flow, c is the speed of sound through the fluid, and D is the pipe diameter.

Assuming that $\theta = \pi/6$ rad, $c = 1480$ m/s, $D = 10$ cm, and the sampling frequency of the A/D converter used to measure the pulses is $f_s = 10$ MHz:

(a) Determine the average flow velocity, v. (2p)

(b) In reality, the speed of sound, c, is temperature dependent. Assuming it can be modeled as a second-order polynomial function of temperature, make the necessary modifications to the equation for the flow velocity. (1p)

(c) Assuming there is some uncertainty regarding the mounting of the transducer, modeled as $\sigma_\theta = 0.01$ rad, what is the effect on the overall uncertainty of the measured flow velocity?
Note: You do not need to estimate the total uncertainty, only the contribution of the transducer angle uncertainty. (2p)

5. We have a differential pressure flowmeter setup designed for incompressible fluids. The volume flow rate is given by

$$Q = \frac{A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2\,(P_1 - P_2)}{\rho}},$$

where A_1 and A_2 are the cross-sectional areas of the pipe where the pressures P_1 and P_2 are measured, respectively. The problem now is that we do not know the fluid density. Assuming we can control the volume flow rate, Q to within some uncertainty, i.e. the flow rate can be said to be normally distributed as $N(Q, \sigma_Q)$, derive and expression for the least-squares estimator of the flow rate, using the Gauss-Newton linearization method, that based on the calibration measurements also estimates the unknown fluid density ρ. (5p)