Part B

Typical Measurement System Elements (CH8→CH11).

Recap of part A

General Principles (CH2→CH7):

Recap of part A

General Principles (CH2→CH7):

- Mathematical and Statistical Tools.
- Definitions.
- Notations.

Recap of part A: CH2

Static Elements:

Recap of part A: CH2

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$$O = \tilde{K} I + \tilde{a}$$

- Ideal (linear) output.

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- Resolution, wear and aging...

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- Predict the output given I, I_M, I_I, and your estimated parameter...

Recap of part A: CH3

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- Error propagation for n elements in series:

$$\sigma_{O_n}^2 = \left(\frac{\partial O_n}{\partial I_n}\right)^2 \sigma_{I_n}^2 + \left(\frac{\partial O_n}{\partial K_n}\right)^2 \sigma_{K_n}^2 + \cdots,$$

where

$$\sigma_{I_n}^2 = \sigma_{O_{n-1}}^2 .$$

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  - Differential systems (reduce interfering inputs).
  - High-gain negative feedback (reduce modifying inputs), problem 3.4.

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- Basically, all elements are more or less dynamic.
- However, for "slow" changing inputs (low-frequency $s = j\omega \rightarrow 0$), a dynamic element gives ($G(0) = 1$) a static behavior.

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- Perfect for describing *inter-element loading*, where i:th element changes the characteristics of the $i-1$:th (for example: by eating current, adding a mass, or preventing a flow).
- Basically, it makes life easier...

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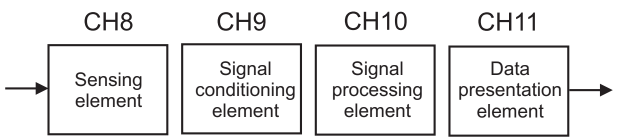
- Methods to reduce noise and interference.

Part B

Typical Measurement System Elements:

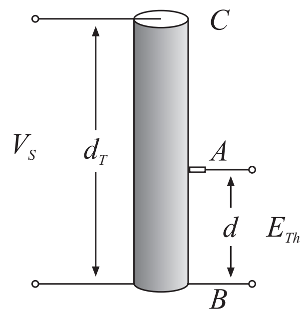


Today's menu

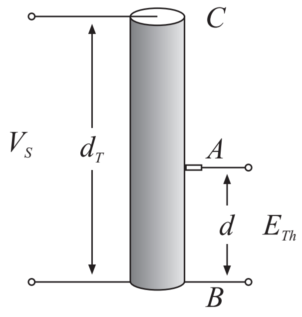
- Beginning of CH8: Resistive sensing elements:
  - Displacement sensors (potentiometers).
  - Temperature sensors
  - Strain gauges.
- Beginning of CH9: Signal Conditioning Elements:
  - Deflection bridges.

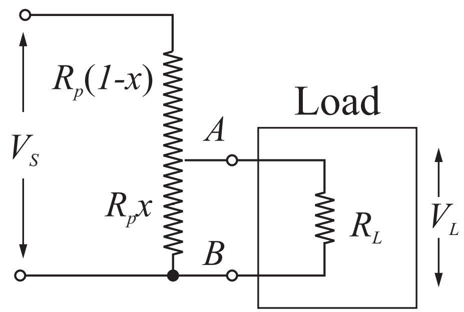
What you need to work with LAB2!

Displacement sensors



Displacement sensors



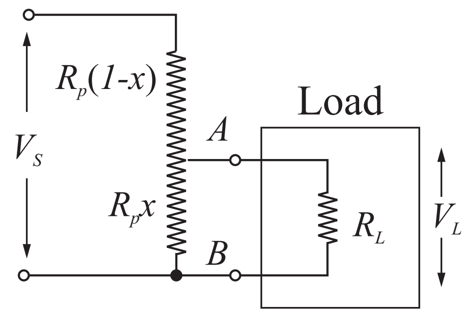


for $x = d/d_T$

Displacement sensors

From the previous lecture:

$$\begin{aligned}
E_{Th} &= V_s x \\
R_{Th} &= R_p x\,(1-x) \\
V_L &= \frac{V_s x}{x\,(1-x)\,R_p/R_L + 1} \\
N(x) &= \frac{V_s R_p/R_L\,\left(x^2 - x^3\right)}{R_p/R_L x\,(1-x) + 1}
\end{aligned}$$

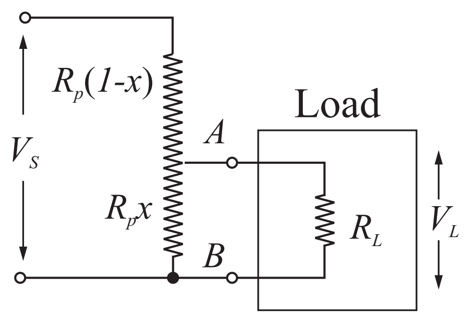


for $x = d/d_T$

Displacement sensors

This leads to some design choices:

- Maximum displacement d_T.
- Supply voltage V_S.
- Resistance R_p ($R_L >> R_p$).
- Power rating W_{MAX} ($W_{\text{MAX}} > V_S^2/R_p$).



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Potentiometers (cont'd...)

Types of potentiometers

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- **Wire wound:** The resistive track consists of n discrete turns of wire. This means that the output can only take on n different discrete values, even though the displacement is continuous. The results in a resolution (or quantization) error.

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- **Hybrid:** Combination of the two above. Consists of a plastic film deposited on a wire.

Temperature sensors

The resistance of most **metals** increases reasonably linear with temperature, in a wide temperature range.

The general relationship between resistance and temperature is of the form

$$R_T = R_0\left(1 + \alpha T + \beta T^2 + \gamma T^3 + \dots\right),$$

where R_0 is the resistance at $0\ ^\circ$C and α, β, and γ are temperature coefficients.

Temperature sensors (cont'd...)

Metals used in temperature sensors

- Platinum is the most commonly used metal. The temperature/resistance characteristics are linear and highly repeatable. However, platinum is expensive.
- For less demanding applications, copper or nickel can be used.

Temperature sensors (cont'd...)

Other types of temperature sensors

- Thermistor: Resistive element made of semiconductor materials. Highly non-linear temperature/resistance characteristics.
- Thick film polymer: Can be used for both temperature and humidity measurements. The polymer absorbs water vapor, which causes the volume to change. This increases the resistance of the material.

Resistive strain gauges

Some definitions

- **Stress:** Defined as *force/area*. Positive values is called **tensile stress** and causes the length of the body to increase. **Compressive stress** has negative.
- **Strain:** The resulting change in length of a body, caused by applying stress, i.e. $e = \Delta l/l$.
- **Elastic modulus:** The relationship between strain and stress is linear over a certain range. The slope of this line is the elastic modulus, i.e.

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}}$$

Resistive strain gauges (cont'd...)

Some more definitions

- For linear tensile or compressive stress, the elastic modulus is called **Young's modulus**, E.
- For shear stress, the elastic modulus is called the **shear modulus**, S.
- If the length of a body increases, then the thickness and width must decrease. Thus, a longitudinal tensile strain e_L is accompanied by a transverse compressive strain e_T. The relationship is

$$e_T = -\nu e_L,$$

where ν is the **Poisson's ratio** (a material constant).

Resistive strain gauges (cont'd...)

A **strain gauge** is a metal or semiconductor whose resistance changes with strain.

The **gauge factor**, G is defined as

$$G = \frac{\Delta R/R_0}{e},$$

where R is the resistance when the strain e is applied, and R_0 is the unstrained resistance.

Resistive strain gauges (cont'd...)

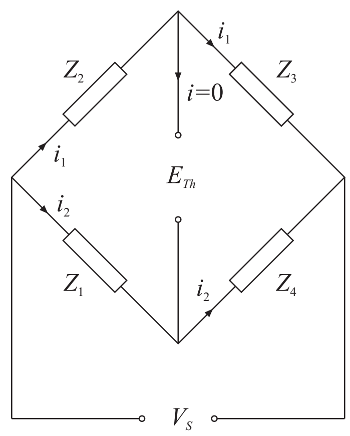
A typical gauge has:

- Gauge factor 2.0 to 2.2
- Unstrained resistance $120 \pm 1\ \Omega$.
- Linearity within $\pm 0.3\%$.
- Maximum tensile strain $+2 \times 10^{-2}$.
- Maximum compressive strain -1×10^{-2}.
- Maximum operating temperature 150 °C.

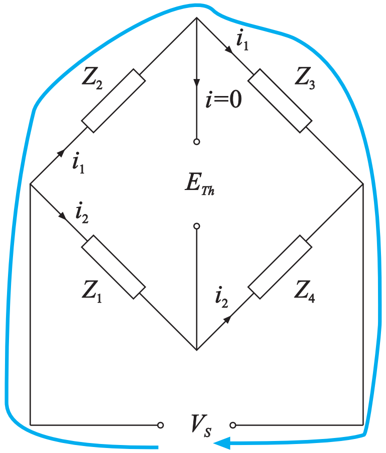
Deflection bridges

Deflection bridges are used to convert the output of resistive, capacitive, and inductive sensors into a voltage signal.

Deflection bridges (cont'd...)



Deflection bridges (cont'd...)



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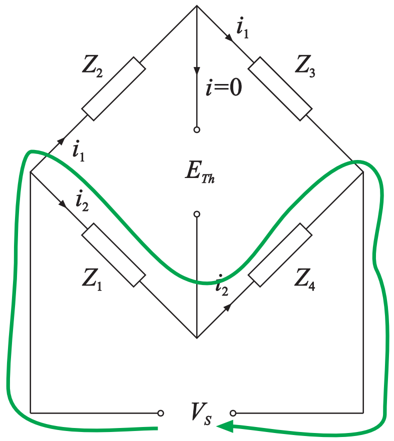

Deflection bridges (cont'd...)

After some calculations we get

$$
\begin{aligned}
E_{Th} &= V_S \left(\frac{Z_1}{Z_1 + Z_4} - \frac{Z_2}{Z_2 + Z_3} \right) \\
Z_{Th} &= \frac{Z_2 Z_3}{Z_2 + Z_3} + \frac{Z_1 Z_4}{Z_1 + Z_4}
\end{aligned}
$$

Deflection bridges (cont'd...)

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- Constraints and limitations from the sensing elements.
- Constraints and limitations from the signal processing element.

Summary

- Resistive sensing elements:
  - Displacement sensors (potentiometers).
  - Temperature sensors.
  - Strain gauges.
- Deflection bridges.

Next lecture

- The remainder of chapter 8.

Recommended exercises

- Homework assignments in Lab 2.