Last lecture

- Beginning of CH8: Resistive sensing elements:
  - Displacement sensors (potentiometers).
  - Temperature sensors
  - Strain gauges.
- Beginning of CH9: Signal Conditioning Elements:
  - Deflection bridges.

Today's menu

- Introduction to model building and parameter estimation
  - Definitions and naming conventions.
  - Problem formulation.
  - The least-squares principle.
- Models linear in the parameters:
  - Polynomial models.
  - Response surface models.
  - Least-squares parameter estimation.
- Non-linear models:
  - Model example.
  - The Gauss-Newton Linearization Method

Introduction to model building

First of all, what do we mean?

- In the context of measurement systems engineering we use models in order to:
  - monitor,
  - control/optimize,
  - understand a process or system...
- We differ between hard models (e.g. physical, chemical) and soft models (empirical models).
- The purpose of a hard model is to gain understanding of the underlying mechanisms of the process.
- An empirical model does not automatically provide such insight, and the goal is primarily to explain the observed data in terms of some variables of interest. The actual model structure, however, is not based on fundamental physical principles.

Introduction to model building

Linear or non-linear?

- A model can be linear with respect to the input and output, i.e. a linear system.
- Even a linear model can be *non-linear* in the parameters.
- Even a non-linear input-output relationship can be linear in the model parameters.
- Of course, the problem can also be non-linear in both variables and parameters.

How we attack the problems depend on which of the above cases we have.

Model examples

Soft (empirical) model

Consider a platinum temperature sensor with the resistance

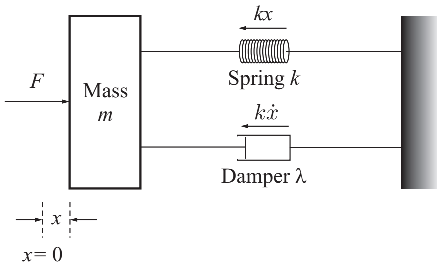
$$R = R_0(\alpha + \beta T + \gamma T^2).$$

This is an empirical relationship and the coefficients are normally estimated from calibration experiments.

The model of R/R_0 is linear in the parameters α, β, γ but non-linear with respect to the input (temperature).

Model examples (cont'd...)

Hard (physical) model



The equation relating force to displacement of this simple force sensor can be derived from first principles, and becomes

$$F - kx - \lambda \frac{d}{dt}x = m\frac{d^2}{dt^2}x.$$

Model examples (cont'd...)

Combined systems

Often hard and soft models are combined in order to describe a complete system. A simple example is when the platinum resistance sensor (soft model) is incorporated in a deflection bridge. The deflection bridge is described by a hard physical (electrical) model.

Definitions and notations

- **Response(s):** The observed or measured data, generated by a process. Usually denoted y_i, for $i = 1, 2, \ldots, N_y$.
- **Variables or factors:** Variables that can be tuned to affect the response, usually denoted x_i, for $i = 1, 2, \ldots, N_x$.
- **Parameters:** Implicit properties of the model, i.e. parameters we can not change. They do however significantly affect the response, and they could be unknown. Parameters that we need to estimate are usually denoted θ_i, for $i = 1, 2, \ldots, N_\theta$.

The model can then be written as

$$y_i = f(x_1, x_2, \ldots, x_{N_x}; \theta_1, \theta_2, \theta_{N_\theta}),$$

or, in vector notation, as

$$y_i = f(\mathbf{x}; \boldsymbol{\theta})$$

Problem formulation

The work-flow can be described as:

- Determine a class of models (soft or hard) that is likely to describe your system.
- Identify what variables you can change.
- Identify the unknown (and possibly known) parameters.
- Design an experiment series.
- Perform measurements.
- Estimate the model parameters.
- Evaluate the model.
- Iterate!