
Rapid and Robust Fitting of Cole-Cole Models to Electrical Permittivity Spectra

Master's project in Scientific Computing (30 ECTS)

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Background

Dielectric properties, such as relative permittivity, are used in combination with other metrics when characterizing crude oils. Current research aims at developing methods for automatic measurement of dielectric parameters, using a wide range of frequencies.

The relative permittivity, ε , is a function of frequency ω (rad/s), and can be described by the Cole-Cole model in Eq. (1) as

$$\varepsilon = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + (j\omega\tau)^{1-\alpha}} - j \frac{\sigma}{\omega\varepsilon_0}, \quad (1)$$

where ε_∞ is the infinite frequency permittivity, ε_s is the static permittivity, τ is the macroscopic relaxation time, α is an empirical factor ($0 \leq \alpha < 1$), ω is the angular frequency, σ is the finite conductivity, and $\varepsilon_0 = 8.854187817$ F/m is the permittivity of vacuum.

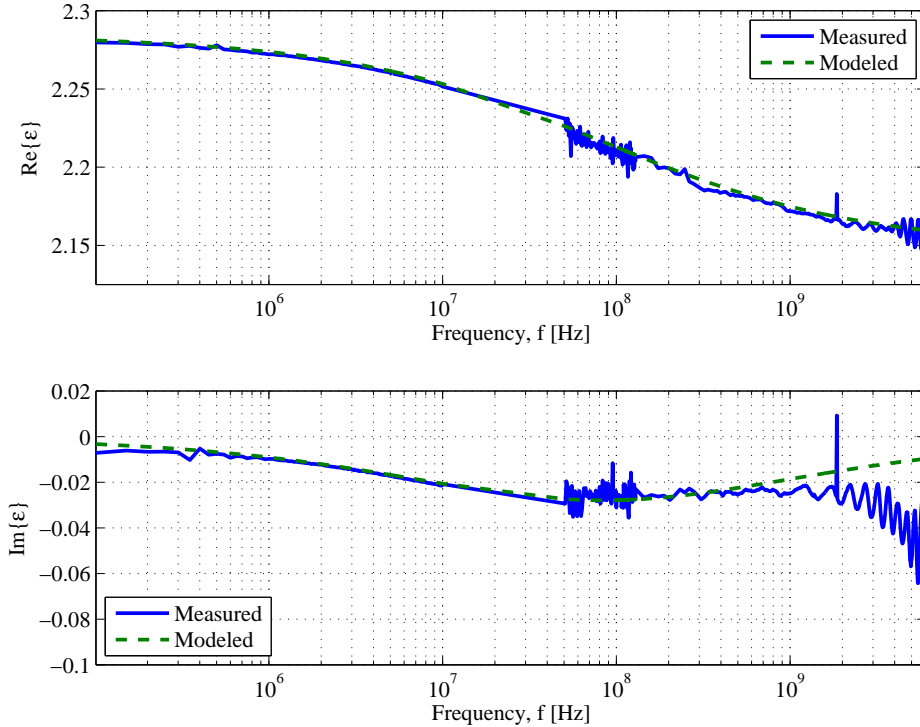


Figure 1: Example of measured relative permittivity as function of frequency. Top: Real part of ε . Bottom: Imaginary part of ε .

The measured permittivity is a complex quantity which depends on frequency (see Fig. 1 for an example). The measurements are corrupted by noise, which statistical properties vary over the frequency range. The task of the model-fitting is to adjust the unknown parameters, α , τ , ε_∞ , ε_s , and σ so that the model in Eq. (1) fits the measured spectrum. Furthermore, the estimation problem is ill-conditioned, since the size of the parameters varies by several orders of magnitude. The estimated parameters in Fig. 1 were: $\varepsilon_s = 2.2845$, $\varepsilon_\infty = 2.1481$, $\tau = 1.9 \times 10^{-9}$, $\alpha = 0.5053$, and $\sigma = 0$. As the figure shows, model fitting was successful, but in this case, manual adjustments were required in order for the algorithm to converge. In an

on-line setup, the process must be automated and robust. In order to be of practical use, the algorithm should also be reasonably fast.

Project description

Main objectives

- Develop and implement a rapid and robust estimation technique for fitting of Cole-Cole models to electrical permittivity measurements.
- The algorithm should be fully automated, i.e. no manual adjustments or initialization should be required.

Tasks

- Literature review of existing techniques.
- Selection of candidate algorithms.
- Implementation and evaluation, with respect to:
 1. Robustness,
 2. Computational speed.
- Selection of final algorithm.
- Implementation as flexible and easy-to-use MATLAB toolbox, including documentation.
- Oral and written presentation of Master's thesis.

Deliverables

- A Master's thesis, written in English.
- MATLAB source code for the final algorithm.
- Documentation of the source code.

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