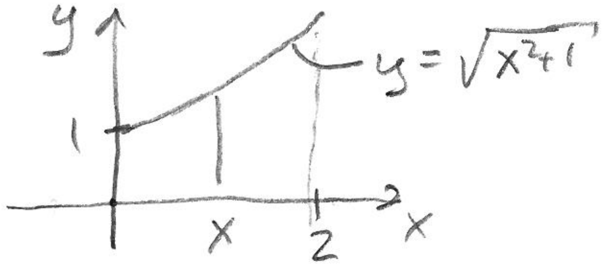
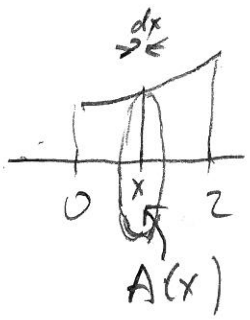
$\boxed{09\text{-}12\text{-}18}$ Uppgift 4



(a) Rotera runt x-axeln

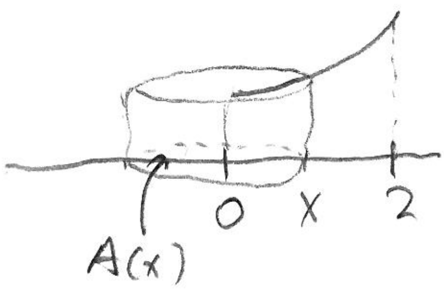


$$A(x)=\pi y^2=\pi\left(\sqrt{x^2+1}\right)^2=\pi(x^2+1)$$

$$V=\int_0^2 A(x)\cdot dx=\int_0^2 \pi(x^2+1)dx=\left[\pi\left(\tfrac{1}{3}x^3+x\right)\right]_0^2$$

$$=\pi\left(\tfrac{1}{3}\cdot 2^3+2\right)=\pi\,\frac{8+6}{3}=\frac{14\pi}{3} \text{ v.e.}$$

(b) Rotera runt y-axeln



$$A(x)=2\pi x\cdot y=2\pi x\sqrt{x^2+1}$$

$$V=\int_0^2 A(x)\cdot dx=\int_0^2 2\pi x\sqrt{x^2+1}\,dx=\left[\begin{array}{l|l} u=x^2+1 & x=0 \\ \frac{du}{dx}=2x & \Downarrow\ u=1 \\ \frac{1}{2}du=x\,dx & x=2 \\ & \Downarrow\ u=5 \end{array}\right]$$

$$=2\pi\int_1^5 \sqrt{u}\cdot\frac{1}{2}du=\pi\int_1^5 u^{1/2}du=\pi\left[\frac{1}{3/2}u^{3/2}\right]_1^5$$

$$=\pi\left(\frac{2}{3}\underbrace{5^{3/2}}_{5\sqrt{5}}-\frac{2}{3}1^{3/2}\right)=\frac{2\pi}{3}(5\sqrt{5}-1)$$

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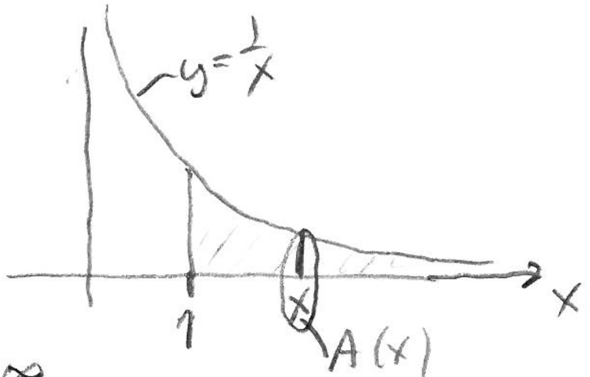
$\boxed{5a}$



$A(x) = \pi \cdot y^2 = \pi \frac{1}{x^2}$

$$V = \int_1^\infty A(x)\cdot dx = \lim_{R\to\infty} \int_1^R \pi \underbrace{\frac{1}{x^2}}_{x^{-2}} dx = \lim_{R\to\infty} \left[\pi \frac{1}{-1} x^{-1}\right]_1^R$$

$$= \lim_{R\to\infty} \left(-\pi R^{-1} + \pi \cdot 1\right) = \lim_{R\to\infty} \left(\pi - \pi \underbrace{\frac{1}{R}}_{\to 0}\right) = \pi \text{ v.e.}$$

$\boxed{5b}$

$$s = \int_{x=\pi/6}^{x=\pi/4} ds = \int_{\pi/6}^{\pi/4} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_{\pi/6}^{\pi/4} \sqrt{1+\left(\frac{1}{\cos x}(-\sin x)\right)^2}\, dx$$

$$= \int_{\pi/6}^{\pi/4} \sqrt{1+\tan^2 x}\, dx = \int_{\pi/6}^{\pi/4} \sqrt{\frac{1}{\cos^2 x}}\, dx = \int_{\pi/6}^{\pi/4} \frac{1}{\cos x}\, dx$$

$$= \left[\ln\left|\frac{1}{\cos x} + \tan x\right|\right]_{\pi/6}^{\pi/4} = \ln\left|\frac{1}{\underbrace{\cos\frac{\pi}{4}}_{=1/\sqrt{2}}} + \underbrace{\tan\frac{\pi}{4}}_{=1}\right| - \ln\left|\frac{1}{\underbrace{\cos\frac{\pi}{6}}_{\sqrt{3}/2}} + \overbrace{\tan\frac{\pi}{6}}^{=\frac{1}{\sqrt{3}}}\right|$$

$$= \ln\left(\sqrt{2}+1\right) - \ln\left(\underbrace{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}_{\frac{3}{\sqrt{3}} = \sqrt{3}}\right) = \ln\left(\sqrt{2}+1\right) - \ln\sqrt{3} = \ln\frac{\sqrt{2}+1}{\sqrt{3}}$$

08-03-18

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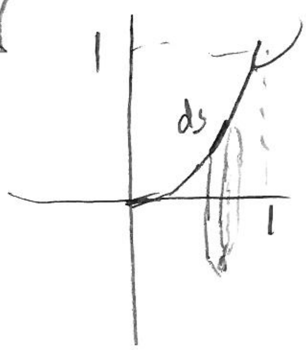

$$A = \int_{x=0}^{x=1} \overbrace{2\pi y}^{\text{omkrets}} \cdot \overbrace{ds}^{\text{bredd}} = \int_0^1 2\pi \cdot x^3 \cdot \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{1+(3x^2)^2}\, dx$$

$$= 2\pi \int_0^1 x^3 \sqrt{1+9x^4}\, dx = \left[\begin{array}{l|l} u = 1+9x^4 & x=0 \\ \frac{du}{dx} = 36x^3 & \Downarrow\ u=1 \\ & x=1 \\ \frac{1}{36}du = x^3\, dx & \Downarrow\ u=10 \end{array}\right]$$

$$= 2\pi \int_1^{10} \sqrt{u} \cdot \frac{1}{36}\, du = \frac{\pi}{18}\int_1^{10} u^{1/2}\, du = \frac{\pi}{18}\left[\frac{1}{3/2}u^{3/2}\right]_1^{10}$$

$$= \frac{\pi}{18}\left(\frac{2}{3}10^{3/2} - \frac{2}{3}1^{3/2}\right) = \frac{\pi}{18}\cdot\frac{2}{3}\left(10\cdot\sqrt{10}-1\right) = \frac{\pi}{27}\left(10\sqrt{10}-1\right)$$