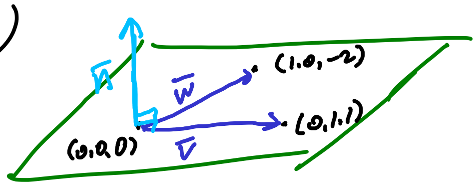
07-01-09

1a) $1(x-2)+(-1)(y-1)+0(z-3)=0$

$x-2-y+1=0$

$$\boxed{x-y=1}$$

1b)



$$\bar{v} = \begin{bmatrix} 0-0 \\ 1-0 \\ 1-0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\bar{w} = \begin{bmatrix} 1-0 \\ 0-0 \\ -2-0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$$\bar{n} = \bar{v} \times \bar{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1\cdot(-2) - 1\cdot 0 \\ -(0\cdot(-2) - 1\cdot 1) \\ 0\cdot 0 - 1\cdot 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

ger planet $-2(x-0)+1(y-0)+(-1)(z-0)=0$

$$\boxed{-2x+y-z=0}$$

1c) Vinkeln mellan planen är vinkeln mellan normalvektorerna

$$\bar{n}_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \bar{n}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$$

vinkeln mellan $\bar{n}_1$ & $\bar{n}_2$

$$\bar{n}_1 \bullet \bar{n}_2 = \|\bar{n}_1\| \|\bar{n}_2\| \cos\theta$$

$$1\cdot(-2)+(-1)\cdot 1+0\cdot(-1) = \sqrt{1^2+(-1)^2+0^2}\cdot\sqrt{(-2)^2+1^2+(-1)^2}\cdot\cos\theta$$

$$-3 = \sqrt{2}\cdot\sqrt{6}\cdot\cos\theta$$

$$-3 = \sqrt{2\cdot 2\cdot 3}\cdot\cos\theta$$

$$-3 = 2\sqrt{3}\cdot\cos\theta$$

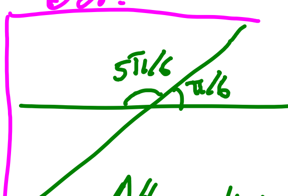
$$-\frac{3}{2\sqrt{3}} = \cos\theta$$

$$\cos\theta = -\frac{\sqrt{3}}{2}$$

$$\boxed{\theta = \frac{5\pi}{6}} \cancel{+n2\pi}$$

$$\theta = \cancel{\frac{2\pi}{6}+n2\pi}$$

Obs!



Alternativt $\frac{\pi}{6}$

07-01-09, ~~forts~~

1d) $\begin{cases} \Pi_1: & x-y = 1 \\ \Pi_2: & -2x+y-z=0 \end{cases}$ (2) $\quad \begin{cases} x-y = 1 \\ -y-z = 2 \end{cases}$

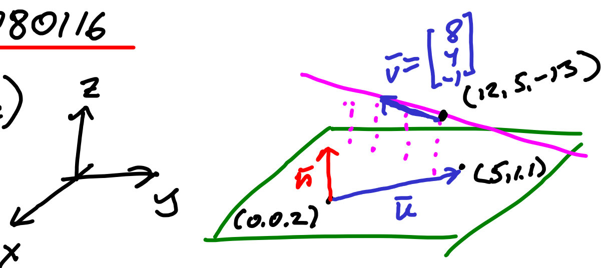
$z=t, \quad -y-t=2 \qquad\qquad x-(-2-t)=1$

$y = \frac{2+t}{-1} = -2-t \qquad x = -1-t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1-t \\ -2-t \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix} + t\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$ ← Linjens ekvation med riktning: $\bar{v} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

080116

3a)



$$\bar{u} = \begin{bmatrix} 5-0 \\ 1-0 \\ 1-2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

$$\bar{n} = \bar{u} \times \bar{v} = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} \times \begin{bmatrix} 8 \\ 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 1(-1)-(-1)\cdot 4 \\ -(5(-1)-(-1)8) \\ 5\cdot 4 - 1\cdot 8 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -3 \\ 12 \end{bmatrix}$$

Ger planets ekvation

$$3(x-0)+(-3)(y-0)+12(z-2)=0$$

$$3x-3y+12z-24=0 \qquad \text{(dela med 3)}$$

$$x-y+4z-8=0$$

$$\boxed{x-y+4z=8}$$

3b) Vektor från planet till punkten $\bar{w} = \begin{bmatrix} 2-0 \\ 1-0 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Skalärprojektion av $\bar{w}$ på $\bar{n}$

$$s = \frac{\bar{w}\bullet\bar{n}}{\|\bar{n}\|} = \frac{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}\bullet\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}}{\sqrt{1^2+(-1)^2+4^2}} = \frac{2\cdot 1+1(-1)+0\cdot 4}{\sqrt{18}} = \frac{1}{\sqrt{2\cdot 9}} = \frac{1}{3\sqrt{2}}$$

ger avståndet $\|\bar{w}_{\bar{n}}\| = |s| = \frac{1}{3\sqrt{2}}$

09-01-13 uppg. 2

1. Linjens ekvation

$$\begin{cases} x-y+z=1 \\ x-y \quad\; =2 \end{cases} \Leftrightarrow \begin{cases} x-y+z=1 \\ \quad\quad -z=1 \end{cases}$$

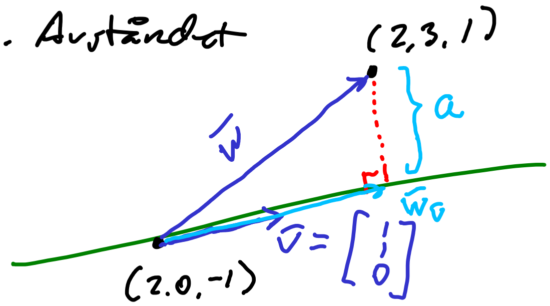
(-1)

$z=-1$ | $x-t-1=1$

$y=t$ | $x=2+t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2+t \\ t \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

2. Avståndet



$$\bar{w} = \begin{bmatrix} 2-2 \\ 3-0 \\ 1-(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

skalärprojektion av $\bar{w}$ på $\bar{v}$

$$s = \frac{\bar{w} \bullet \bar{v}}{\|\bar{v}\|} = \frac{\begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}{\sqrt{1^2+1^2+0^2}} = \frac{0\cdot 1 + 3\cdot 1 + 2\cdot 0}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

så $\|\bar{w}_{\bar{v}}\| = \frac{3}{\sqrt{2}}$ och $\|\bar{w}\| = \sqrt{0^2+3^2+2^2} = \sqrt{13}$

Avståndet a med pythagoras sats

$$a^2 + \|\bar{w}_{\bar{v}}\|^2 = \|\bar{w}\|^2$$

$$a^2 + \frac{9}{2} = 13$$

$$a^2 = 13 - \frac{9}{2} = \frac{26}{2} - \frac{9}{2} = \frac{17}{2}$$

Svar:

Avståndet $a = \sqrt{\frac{17}{2}}$

09-12-18 uppg. 3

1. Linjen $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + t \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

ligger i planet, dvs $(2,3,0)$ är en punkt i planet och $\bar{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ pekar i planets riktning.

2. Vinkelrätt mot $2x - 3y + z = 0$, ger att normalen $\bar{w} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

pekar i riktningen för planet som söks.

3. Därmed kan vi bilda normalen $\bar{n} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} (-1)\cdot 1 - 2(-3) \\ -(1\cdot 1 - 2\cdot 2) \\ 1\cdot(-3) - (-1)\cdot 2 \end{bmatrix}$

$$= \begin{bmatrix} 5 \\ 3 \\ -1 \end{bmatrix}$$

och planets ekvation

$$5(x-2) + 3(y-3) + (-1)(z-0) = 0$$

$$5x - 10 + 3y - 9 - z = 0$$

$$\boxed{5x + 3y - z = 19}$$