## M0030M

# Föreläsning 7 - Mapleintroduktion

Ove Edlund

> *restart*

Aritmetiska operatorer: ^, *, /, +, -
Inbyggda funktioner: sin(), cos(), ln(), exp(), arctan(), sqrt(), abs()

> $\frac{5 \cdot 2^3}{3} + \frac{2+3}{5}$

$$\frac{43}{3} \quad (1.1)$$

> sqrt(7)

$$\sqrt{7} \quad (1.2)$$

> sqrt(9)

$$3 \quad (1.3)$$

> ln(1)

$$0 \quad (1.4)$$

> cos(Pi)

$$-1 \quad (1.5)$$

> Pi

$$\pi \quad (1.6)$$

Uttryck formuleras mha någon oberoende variabel, tex "x"

> $\frac{\sin(x)}{x}$

$$\frac{\sin(x)}{x} \quad (2.1)$$

> $\text{sqrt}(x^2 - x) - \text{sqrt}(x^2 + 2)$

$$\sqrt{x^2 - x} - \sqrt{x^2 + 2} \quad (2.2)$$

> $\frac{x^{20}}{\exp(x)}$

$$\frac{x^{20}}{e^x} \quad \textbf{(2.3)}$$

Uttryck kan namnges och sparas med "tilldelning", dvs med ":="

> $sinckvot := \frac{\sin(x)}{x}$

$$sinckvot := \frac{\sin(x)}{x} \quad \textbf{(3.1)}$$

> $subs(x = 0, sinckvot)$

Error, numeric exception: division by zero

> $limit(sinckvot, x = 0)$

$$1 \quad \textbf{(3.2)}$$

> $subs(x = 1, sinckvot)$

$$\sin(1) \quad \textbf{(3.3)}$$

> $eval(sinckvot, x = 1)$

$$\sin(1) \quad \textbf{(3.4)}$$

Överkurs:
Definition av funktioner görs med "->"

> $sinc := x \rightarrow \frac{\sin(x)}{x}$

$$sinc := x \rightarrow \frac{\sin(x)}{x} \quad \textbf{(3.5)}$$

> $sinc(3)$

$$\frac{1}{3} \sin(3) \quad \textbf{(3.6)}$$

> $sinc\left(\frac{Pi}{2}\right)$

$$\frac{2}{\pi} \quad \textbf{(3.7)}$$

> $sinc(0)$

Error, (in sinc) numeric exception: division by zero

> $limit(sinc(t), t = 0)$

$$1 \quad \textbf{(3.8)}$$

Derivera uttryck med *diff* (och funktioner med "D").
Adressera föregående resultat med "%".
Omvandla till decimalform med *evalf*.
Förenkla uttryck med *simplify* (och *expand*)

> $diff(x \cdot \arctan(x^2), x)$

$$\arctan(x^2) + \frac{2x^2}{x^4+1} \tag{4.1}$$

> $sinckvotdiff := diff(sinckvot, x)$

$$sinckvotdiff := \frac{\cos(x)}{x} - \frac{\sin(x)}{x^2} \tag{4.2}$$

> $subs(x = 1, sinckvotdiff)$

$$\cos(1) - \sin(1) \tag{4.3}$$

> $eval(sinckvotdiff, x = 1)$

$$\cos(1) - \sin(1) \tag{4.4}$$

> $evalf(\%)$

$$-0.3011686789 \tag{4.5}$$

> $Dsinc := D(sinc)$

$$Dsinc := x \to \frac{\cos(x)}{x} - \frac{\sin(x)}{x^2} \tag{4.6}$$

> $Dsinc(1)$

$$\cos(1) - \sin(1) \tag{4.7}$$

> $p1 := (x-1) \cdot (x-3) \cdot (x+2) \cdot (x+4)$

$$p1 := (x-1)(x-3)(x+2)(x+4) \tag{4.8}$$

> $simplify(p1)$

$$(x-1)(x-3)(x+2)(x+4) \tag{4.9}$$

> $expand(p1)$

$$x^4 + 2x^3 - 13x^2 - 14x + 24 \tag{4.10}$$

> $dp1dx := diff(p1, x)$

$$dp1dx := (x-3)(x+2)(x+4) + (x-1)(x+2)(x+4) + (x-1)(x-3)(x+4) + (x-1)(x-3)(x+2) \tag{4.11}$$

> $simplify(dp1dx)$

$$4x^3 + 6x^2 - 26x - 14 \tag{4.12}$$

>

>

Lös ut "x" symboliskt med *solve*, (eller numeriskt med *fsolve)*

> $solve(x + 2 = \mathrm{sqrt}(x + 3))$

$$\frac{1}{2}\sqrt{5} - \frac{3}{2} \tag{5.1}$$

> $evalf(\%)$

$$-0.381966012 \tag{5.2}$$

> $solve(dp1dx = 0)$

$$-\frac{1}{2}, -\frac{1}{2} - \frac{1}{2}\sqrt{29}, -\frac{1}{2} + \frac{1}{2}\sqrt{29} \tag{5.3}$$

> $solve\left(\left\{x^2 + y\cdot x = 5, \arctan(y) = \frac{\mathrm{Pi}}{3}\right\}\right)$

$$\left\{x = RootOf\left(\_Z^2 + \sqrt{3}\,\_Z - 5\right), y = \sqrt{3}\right\} \tag{5.4}$$

> $allvalues(\%)$

$$\left\{x = -\frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{23}, y = \sqrt{3}\right\}, \left\{x = -\frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{23}, y = \sqrt{3}\right\} \tag{5.5}$$

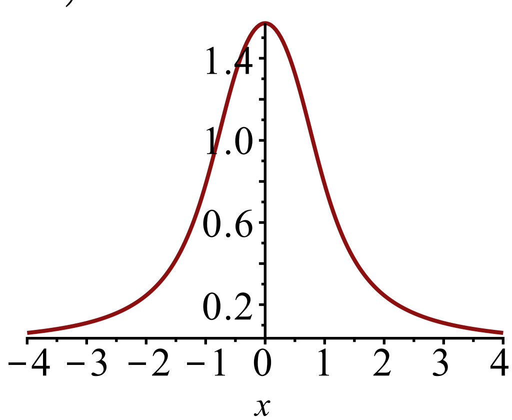
> $evalf(\%)$

$$\{x = 1.531890358, y = 1.732050808\}, \{x = -3.263941166, y = 1.732050808\} \tag{5.6}$$

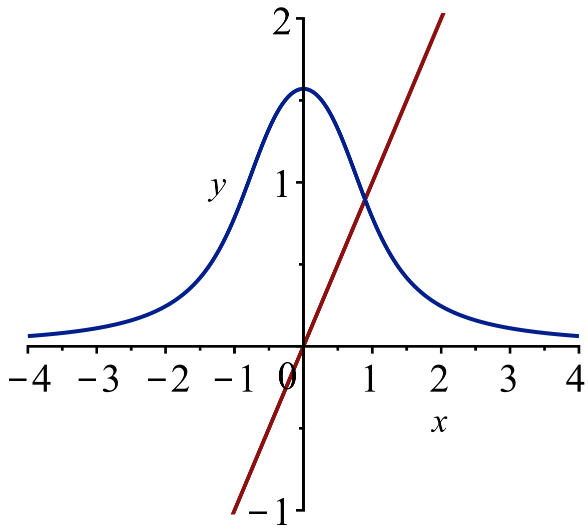
>

Rita grafer i 2D med *plot* och i 3D med *plot3d*

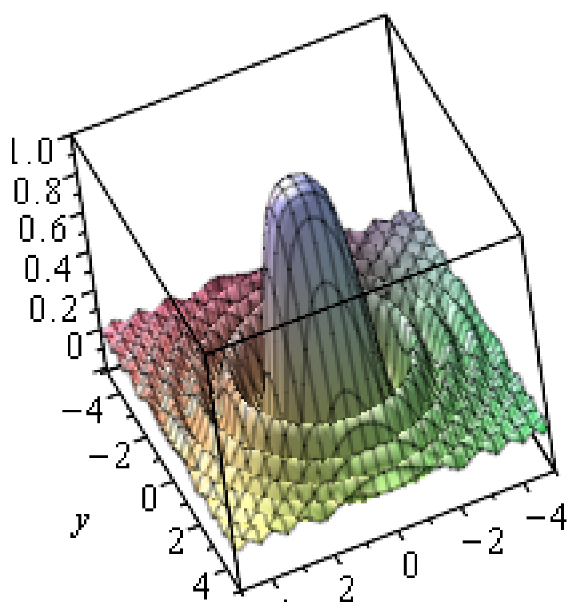
> $plot\left(\arctan\left(\frac{1}{x^2}\right), x = -4..4\right)$



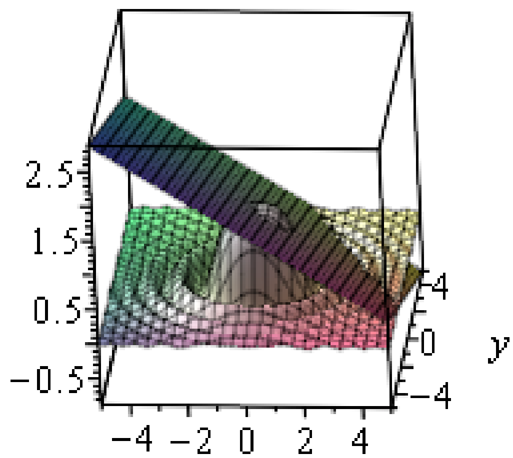
> $plot\left(\left\{\arctan\left(\frac{1}{x^2}\right), x\right\}, x = -4..4, y = -1..2\right)$




> $plot3d\left(\frac{\sin(x^2 + y^2)}{x^2 + y^2}, x = -5..5, y = -5..5\right)$



> $plot3d\left(\left\{\frac{\sin(x^2+y^2)}{x^2+y^2}, 1-\frac{1}{4}\cdot x-\frac{1}{8}\cdot y\right\}, x=-5..5, y=-5..5\right)$

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## *Linjär algebra*

Läs in biblioteket för linjär algebra med *with*(*LinearAlgebra*)
Skapa matriser och vektorer med *Matrix* och *Vector*, eller med "<", "|", ">" - notationen

Addera/subtrahera matriser och vektorer med +, -
Matrismultiplikation med "."

Skalärprodukt med *DotProduct*
Vektorprodukt med *CrossProduct* och
längd (= norm) av vektor med *Norm*( "vektor" , 2)

> $with(LinearAlgebra):$

> $u := \langle 4, -1, -1\rangle$

$$u := \begin{bmatrix} 4 \\ -1 \\ -1 \end{bmatrix} \quad \textbf{(7.1)}$$

> $v := \langle 1, 0, 1\rangle$

$$v := \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \textbf{(7.2)}$$

> $u + 2 \cdot v$

$$\begin{bmatrix} 6 \\ -1 \\ 1 \end{bmatrix} \quad (7.3)$$

> $u.v$

$$3 \quad (7.4)$$

> $DotProduct(u, v)$

$$3 \quad (7.5)$$

> $CrossProduct(u, v)$

$$\begin{bmatrix} -1 \\ -5 \\ 1 \end{bmatrix} \quad (7.6)$$

> $norm(u, 2)$

$$3\sqrt{2} \quad (7.7)$$

> $norm(v, 2)$

$$\sqrt{2} \quad (7.8)$$

> $\arccos\left(\frac{DotProduct(u, v)}{norm(u, 2) \cdot norm(v, 2)}\right)$

$$\frac{1}{3}\pi \quad (7.9)$$

Lös system med *LinearSolve*
Trappsegsform med *GaussianElimination*
Reducerad trappstegsform med *ReducedRowEchelonForm*

Inversmatris med *MatrixInverse*
Deteriminant med *Determininant*

> $A := \langle 1, -3, 5 \mid 2, -4, 2 \mid -1, 2, 3 \rangle$

$$A := \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix} \quad (7.10)$$

> $A1 := Matrix(3, 3, [1, 2, -3, -3, -4, 2, 5, 2, 3])$

$$A1 := \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 2 \\ 5 & 2 & 3 \end{bmatrix} \quad (7.11)$$

> $b := \langle 1, 2, -3 \rangle$

$$b := \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \textbf{(7.12)}$$

> $x := LinearSolve(A, b)$

$$x := \begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix} \quad \textbf{(7.13)}$$

> $A.x$

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \quad \textbf{(7.14)}$$

> $Aaug := \langle A \mid b \rangle$

$$Aaug := \begin{bmatrix} 1 & 2 & -1 & 1 \\ -3 & -4 & 2 & 2 \\ 5 & 2 & 3 & -3 \end{bmatrix} \quad \textbf{(7.15)}$$

> $GaussianElimination(Aaug)$

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & -1 & 5 \\ 0 & 0 & 4 & 12 \end{bmatrix} \quad \textbf{(7.16)}$$

> $ReducedRowEchelonForm(Aaug)$

$$\begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \textbf{(7.17)}$$

> $B := \langle 1, -3, 5 | 2, -4, 2 | -1, 1, 3 \rangle$

$$B := \begin{bmatrix} 1 & 2 & -1 \\ -3 & -4 & 1 \\ 5 & 2 & 3 \end{bmatrix} \quad \textbf{(7.18)}$$

> $b2 := \langle 0, -2, 8 \rangle$

$$b2 := \begin{bmatrix} 0 \\ -2 \\ 8 \end{bmatrix} \quad \textbf{(7.19)}$$

> $LinearSolve(B, b2)$

$$\begin{bmatrix} 2 - \_t0_3 \\ -1 + \_t0_3 \\ \_t0_3 \end{bmatrix} \tag{7.20}$$

> $Baug := \langle B \mid b2 \rangle$

$$Baug := \begin{bmatrix} 1 & 2 & -1 & 0 \\ -3 & -4 & 1 & -2 \\ 5 & 2 & 3 & 8 \end{bmatrix} \tag{7.21}$$

> $ReducedRowEchelonForm(Baug)$

$$\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7.22}$$

> $Ainv := MatrixInverse(A)$

$$Ainv := \begin{bmatrix} -2 & -1 & 0 \\ \frac{19}{8} & 1 & \frac{1}{8} \\ \frac{7}{4} & 1 & \frac{1}{4} \end{bmatrix} \tag{7.23}$$

> $Ainv.b$

$$\begin{bmatrix} -4 \\ 4 \\ 3 \end{bmatrix} \tag{7.24}$$

> $MatrixInverse(B)$

Error, (in MatrixInverse) singular matrix

> $Determinant(A)$

$$8 \tag{7.25}$$

> $Determinant(B)$

$$0 \tag{7.26}$$

>

**Exempel**

5. a) Låt

$$A = \begin{pmatrix} 1 & 2 & a \\ 0 & a & 2 \\ 1 & -2 & -a \end{pmatrix}.$$ För vilka värden på $a$ har ekvationen $A\mathbf{x} = \mathbf{b}$, $\mathbf{b} \in \mathbb{R}^3$,

en entydig lösning? (2p)

b) Beräkna $A^{-1}$ för ovanstående matris då $a = 1$. (3p)

> $A := \langle 1, 0, 1 \mid 2, a, -2 \mid a, 2, -a \rangle$

$$A := \begin{bmatrix} 1 & 2 & a \\ 0 & a & 2 \\ 1 & -2 & -a \end{bmatrix} \tag{8.1}$$

> $Determinant(A)$

$$-2\, a^2 + 8 \tag{8.2}$$

> $solve(-2 \cdot a^2 + 8 = 0)$

$$-2, 2 \tag{8.3}$$

>

Dvs entydig lösning då a ≠ -2 och a ≠ 2

> $Aspec := subs(a = 1, A)$

$$Aspec := \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 1 & -2 & -1 \end{bmatrix} \tag{8.4}$$

> $Ainv := MatrixInverse(Aspec)$

$$Ainv := \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{6} & \frac{2}{3} & \frac{1}{6} \end{bmatrix} \tag{8.5}$$

**Exempel**

Bestäm *a*, *b*, *c*, *d* i

$$A = \begin{bmatrix} a\,c & b & a \\ b\,c & -a & b \\ 1 & 0 & -c \end{bmatrix}$$

så att $A.\langle 1, 2, 1\rangle = \langle 1, 1, 2\rangle$

```
> b :='b'
```
*# För att b ska vara en variabel, och ej associeras med ett specifikt "värde" (en specifik vektor i detta fall)*

$$b := b \quad \textbf{(8.6)}$$

```
> A := ⟨a·c, b·c, 1 | b, -a, 0 | a, b, -c⟩
```

$$A := \begin{bmatrix} a\,c & b & a \\ b\,c & -a & b \\ 1 & 0 & -c \end{bmatrix} \quad \textbf{(8.7)}$$

```
> v := A.⟨1, 2, 1⟩
```

$$v := \begin{bmatrix} a\,c + a + 2\,b \\ b\,c - 2\,a + b \\ 1 - c \end{bmatrix} \quad \textbf{(8.8)}$$

```
> v[1]
```

$$a\,c + a + 2\,b \quad \textbf{(8.9)}$$

```
> solve({v[1] = 1, v[2] = 1, v[3] = 2})
```

$$\left\{a = -\frac{1}{2}, b = \frac{1}{2}, c = -1\right\} \quad \textbf{(8.10)}$$

```
> Aspec := subs(%, A)
```

$$Aspec := \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 1 \end{bmatrix} \quad \textbf{(8.11)}$$

```
> Aspec.⟨1, 2, 1⟩
```

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \textbf{(8.12)}$$