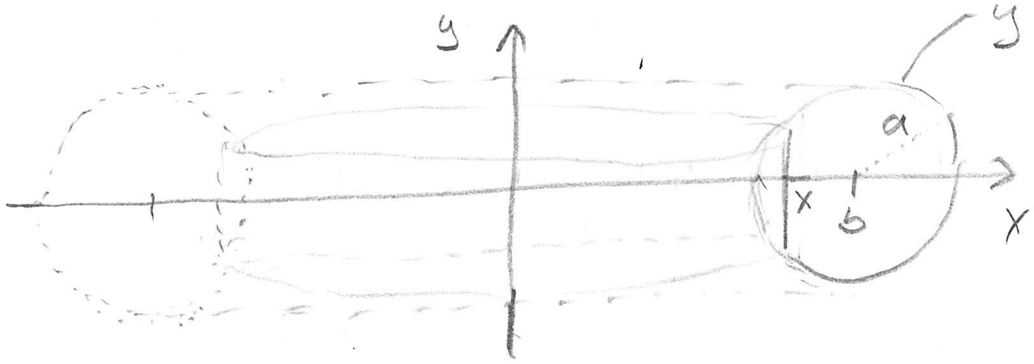
Exempel: Volym för "munk" med cylindriska skal



$$y = \sqrt{a^2-(x-b)^2}$$
$$\Downarrow$$
$$(x-b)^2+y^2=a^2$$

ger $\quad A(x) = 2\pi x \cdot 2\sqrt{a^2-(x-b)^2} \quad$ dvs

$$V = \int_{b-a}^{b+a} A(x)\,dx = 4\pi \int_{b-a}^{b+a} \underset{\uparrow\, =(x-b+b)}{x}\sqrt{a^2-(x-b)^2}\,dx$$

$$= 4\pi \underbrace{\int_{b-a}^{b+a} (x-b)\sqrt{a^2-(x-b)^2}\,dx}_{1} + 4\pi \underbrace{\int_{b-a}^{b+a} b\sqrt{a^2-(x-b)^2}\,dx}_{2}$$

$$1 = \left[\begin{array}{l|l} u = (x-b)^2 & x = b-a \\ \frac{du}{dx} = 2(x-b) & \Downarrow u = a^2 \\ \frac{1}{2}du = (x-b)dx & x = b+a \\ & \Downarrow u = a^2 \end{array}\right] = 4\pi \int_{a^2}^{a^2} \sqrt{a^2-u}\,\frac{1}{2}\,du = 0$$

$$2 = \left[\begin{array}{l|l} x-b = a\cdot\sin\theta & x = b-a \\ dx = a\cdot\cos\theta\,d\theta & \Downarrow \theta = -\frac{\pi}{2} \\ & x = b+a \\ & \Downarrow \theta = \frac{\pi}{2} \end{array}\right] = 4\pi b \int_{-\pi/2}^{\pi/2} \sqrt{\underbrace{\underbrace{a^2 - a^2\sin^2\theta}_{a^2(1-\sin^2\theta)}}_{a^2\cos^2\theta}}\; a\cos\theta\,d\theta$$

$$= 4\pi b \int_{-\pi/2}^{\pi/2} \overbrace{a\cos\theta}^{|\cos\theta|}\, a\cos\theta\,d\theta = 4\pi b a^2 \int_{-\pi/2}^{\pi/2} \cos^2\theta\,d\theta$$

$$= 4\pi b a^2 \int_{-\pi/2}^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\,d\theta = 2\pi b a^2 \left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\pi/2}^{\pi/2}$$

$$= 2\pi b a^2 \left(\frac{\pi}{2} + \overbrace{\frac{1}{2}\sin\pi}^{=0} - \left(-\frac{\pi}{2}\right) - \overbrace{\frac{1}{2}\sin(-\pi)}^{=0}\right) = 2\pi^2 a^2 b \quad \text{v.s.v.}$$