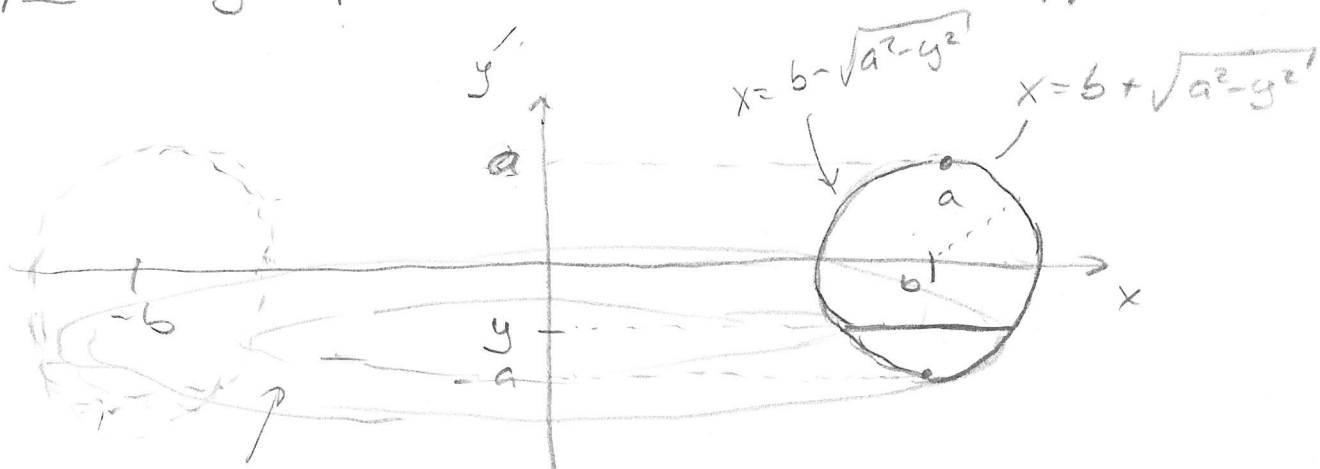


Exempel: Volym för "musk" med rotationskropp



$$A(y) = \pi (b + \sqrt{a^2 - y^2})^2 - \pi (b - \sqrt{a^2 - y^2})^2$$

$$V = \int_{-a}^a A(y) \cdot dy = \int_{-a}^a (\pi (b + \sqrt{a^2 - y^2})^2 - \pi (b - \sqrt{a^2 - y^2})^2) dy$$

$$= \pi \int_{-a}^a (b^2 + 2b\sqrt{a^2 - y^2} + a^2 - y^2 - (b^2 - 2b\sqrt{a^2 - y^2} - a^2 + y^2)) dy$$

$$= \pi 4b \int_{-a}^a \sqrt{a^2 - y^2} dy = \left[\begin{array}{l} y = a \sin \theta \\ dy = a \cdot \cos \theta d\theta \end{array} \right. \left. \begin{array}{l} y = -1 \\ \theta = -\frac{\pi}{2} \\ y = 1 \\ \theta = \frac{\pi}{2} \end{array} \right]$$

$$= \pi 4b \int_{-\pi/2}^{\pi/2} \sqrt{a^2 - a^2 \sin^2 \theta} \cdot a \cos \theta d\theta = \pi 4b \int_{-\pi/2}^{\pi/2} a \cdot \cos \theta \cdot a \cdot \cos \theta d\theta$$

$$= \pi 4b a^2 \int_{-\pi/2}^{\pi/2} \cos^2 \theta d\theta = \pi 4b a^2 \int_{-\pi/2}^{\pi/2} \frac{1}{2} (1 + \cos 2\theta) d\theta$$

$$= 2\pi b a^2 \left[\theta + \frac{1}{2} \sin 2\theta \right]_{-\pi/2}^{\pi/2} = 2\pi b a^2 \left(\frac{\pi}{2} + \frac{1}{2} \sin \pi - (-\frac{\pi}{2}) - \frac{1}{2} \sin(-\pi) \right)$$

$$= 2\pi^2 a^2 b \quad \text{v.e.}$$