

6.1:9

$$\int x \arcsin x \, dx = \frac{1}{2} x^2 \arcsin x - \frac{1}{2} \int x^2 \frac{1}{\sqrt{1-x^2}} \, dx$$

$$A = \int \frac{x^2}{\sqrt{1-x^2}} \, dx = - \int \frac{1-x^2-1}{\sqrt{1-x^2}} \, dx =$$

$$= - \int \frac{1-x^2}{\sqrt{1-x^2}} \, dx + \int \frac{1}{\sqrt{1-x^2}} \, dx = - \int \sqrt{1-x^2} \, dx + \arcsin x$$

$$= -x \sqrt{1-x^2} + \int x \frac{-2x}{2\sqrt{1-x^2}} \, dx + \arcsin x$$

$$= -x \sqrt{1-x^2} - \int \frac{x^2}{\sqrt{1-x^2}} \, dx + \arcsin x$$

so

$$2A = \arcsin x - x \sqrt{1-x^2} \quad (+C)$$

$$A = \frac{1}{2} \arcsin x - \frac{1}{2} x \sqrt{1-x^2} \quad (+D)$$

thus

$$\int x \arcsin x \, dx = \frac{1}{2} x^2 \arcsin x - \frac{1}{4} \arcsin x + \frac{1}{4} x \sqrt{1-x^2} + D$$

$$= \frac{1}{4} (2x^2 - 1) \arcsin x + \frac{1}{4} x \sqrt{1-x^2} + D$$