

6.1:9 Alternativ Lösung

$$\int x \cdot \arcsin x \, dx = \left[\begin{array}{l} x = \sin \theta \\ \frac{dx}{d\theta} = \cos \theta \\ dx = \cos \theta \, d\theta \end{array} \right]_{-\frac{\pi}{2} < \theta < \frac{\pi}{2}}$$

$$= \int \sin \theta \cdot \overset{\arcsin(\sin \theta)}{\theta} \cdot \cos \theta \, d\theta = \int \theta \cdot \frac{1}{2} \sin 2\theta \, d\theta$$

$$= \theta \cdot \frac{1}{2} (-\cos 2\theta) \frac{1}{2} - \int \frac{1}{2} (-\cos 2\theta) \frac{1}{2} \, d\theta$$

$$= -\frac{1}{4} \theta \cos 2\theta + \frac{1}{4} \int \cos 2\theta \, d\theta$$

$$= -\frac{1}{4} \theta \cos 2\theta + \frac{1}{4} \sin 2\theta \cdot \frac{1}{2} + C$$

$$= -\frac{1}{4} \theta (1 - 2 \sin^2 \theta) + \frac{1}{8} \cdot 2 \sin \theta \overset{\geq 0 \text{ da } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}}{\cos \theta} + C$$

$$= -\frac{1}{4} \theta + \frac{1}{2} \theta \sin^2 \theta + \frac{1}{4} \sin \theta \sqrt{1 - \sin^2 \theta} + C$$

$$= -\frac{1}{4} \arcsin x + \frac{1}{2} x^2 \arcsin x + \frac{1}{4} x \sqrt{1 - x^2} + C$$