

6.3:31

$$\int \frac{1+x^{1/2}}{1+x^{1/3}} dx = \left[\begin{array}{l} x = u^6 \\ \frac{dx}{du} = 6u^5 \\ dx = 6u^5 du \end{array} \right] = \int \frac{1+(u^6)^{1/2}}{1+(u^6)^{1/3}} 6u^5 du$$

$$= 6 \int \frac{1+u^3}{1+u^2} u^5 du = 6 \int \frac{u^8+u^5}{u^2+1} du$$

Polynomialdivision

$$\begin{array}{r} u^6 - u^4 + u^3 + u^2 - u - 1 \\ \hline u^2 + 1 \quad \left| \begin{array}{l} u^8 + u^5 \\ - (u^8 + u^6) \\ \hline -u^6 + u^5 \\ - (-u^6 - u^4) \\ \hline u^5 + u^4 \\ - (u^5 + u^3) \\ \hline u^4 - u^3 \\ (u^4 + u^2) \\ \hline -u^3 - u^2 \\ - (-u^3 + u) \\ \hline -u^2 + u \\ (-u^2 - 1) \\ \hline u + 1 \end{array} \right. \end{array}$$

so

$$\frac{u^8+u^5}{u^2+1} = u^6 - u^4 + u^3 + u^2 - u - 1 + \frac{u+1}{u^2+1}$$

ger integraten

$$6 \int (u^6 - u^4 + u^3 + u^2 - u - 1 + \frac{u}{u^2+1} + \frac{1}{u^2+1}) du$$

$$= 6 \left(\frac{1}{7} u^7 - \frac{1}{5} u^5 + \frac{1}{4} u^4 + \frac{1}{3} u^3 - \frac{1}{2} u^2 - u + \frac{1}{2} \ln(u^2+1) + \arctan u \right) + C$$

Återsubstituzia $x = u^6 \Leftrightarrow u = x^{1/6}$

$$= \frac{6}{7} x^{7/6} - \frac{6}{5} x^{5/6} + \frac{3}{2} x^{2/3} + 2 x^{1/2} - 3 x^{1/3} - 6 x^{1/6} + 3 \ln(x^{1/3} + 1) + 6 \arctan x^{1/6} + C$$