$\boxed{\text{081220 - Problem 1}}$

(a)

$$z = \frac{4-3i}{a+i} = \frac{(4-3i)(a-i)}{(a+i)(a-i)} = \frac{4a-4i-3ai+3\overset{-1}{i^2}}{a^2+1^2}$$

$$= \frac{(4a-3)+i(-4-3a)}{a^2+1}$$

så

$$\frac{1}{2} = \operatorname{Im} z = \frac{-4-3a}{a^2+1}$$

$$a^2+1 = -8-6a$$

$$a^2+6a+9=0$$

$$(a+3)^2=0$$

Svar: $\boxed{a=-3}$

(b) Låt $z = x+iy$

Villkoret

$$|z+i|^2 = |z+2|^2$$

$$|x+iy+i|^2 = |x+i\cdot y+2|^2$$

$$|x+i(y+1)|^2 = |(x+2)+i\cdot y|^2$$

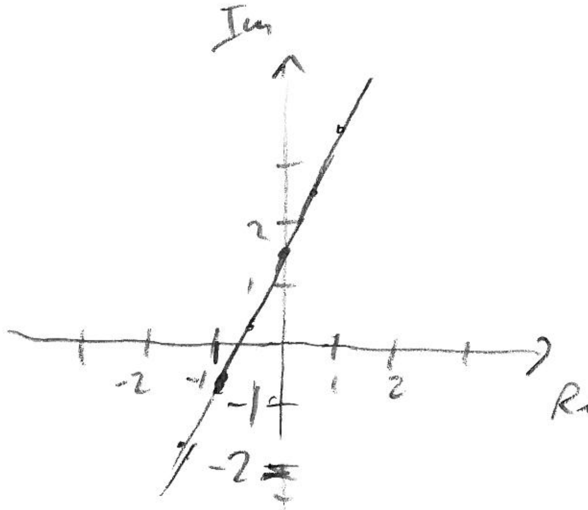
$$x^2+(y+1)^2 = (x+2)^2+y^2$$

$$\cancel{x^2}+\cancel{y^2}+2y+1 = \cancel{x^2}+4x+4+\cancel{y^2}$$

$$2y = 4x+3$$

$$\boxed{y = 2x+\frac{3}{2}}$$

Linjens ekvation



$\boxed{\text{110518 - Problem 2}}$

(b) $$\left|\frac{(1-2i)\cdot(1-i)^{11}}{(3+i)^3}\right| = \frac{|1-2i|\cdot|(1-i)^{11}|}{|(3+i)^3|} = \frac{|1-2i|\cdot|1-i|^{11}}{|3+i|^3}$$

$$= \frac{\sqrt{1^2+(-2)^2}\cdot\left(\sqrt{1^2+(-1)^2}\right)^{11}}{\left(\sqrt{3^2+1^2}\right)^3} = \frac{\sqrt{5}\cdot(\sqrt{2})^{11}}{(\sqrt{\underbrace{10}_{2\cdot 5}})^3} = \frac{5^{1/2}\cdot 2^{11/2}}{2^{3/2}\cdot 5^{3/2}}$$

$$= 5^{1/2-3/2}\cdot 2^{11/2-3/2} = 5^{-1}\cdot 2^4 = \frac{16}{5}$$

$\boxed{110322 - \text{Problem } 1}$

$$i\cdot z^2 + (-i-1)z - 4 = 0 \qquad \text{mult. m. } i$$

$$\underbrace{i^2}_{-1} z^2 + (-\underbrace{i^2}_{-1} - i)z - 4i = 0$$

$$z^2 + (-1+i)z + 4i = 0$$

$$\left(z + \frac{-1+i}{2}\right)^2 - \frac{(-1+i)^2}{4} + 4i = 0$$

$$\left(z - \frac{1}{2} + i\frac{1}{2}\right)^2 = \frac{\cancel{1} - 2i \cancel{-1}}{4} - 4i$$

$$\left(z - \frac{1}{2} + i\frac{1}{2}\right)^2 = -\frac{9}{2}i$$

$\boxed{w = z - \frac{1}{2} + i\frac{1}{2}}$ $\quad$ $\boxed{z = w + \frac{1}{2} - i\frac{1}{2}}$ $\qquad$ $\left(w^2 = -\frac{9}{2}i\right)$

$w = a + ib \qquad w^2 = a^2 - b^2 + i2ab$

$$a^2 - b^2 + i2ab = \frac{7}{2}i$$

① $\begin{cases} a^2 - b^2 = 0 \\ 2ab = -\frac{9}{2} \end{cases}$ ②

② ger $b = -\frac{9}{4a}$ i ①

$$a^2 - \frac{81}{16a^2} = 0$$

$$(a^2)^2 = \frac{81}{16}$$

$$a^2 = \overset{+}{(-)}\frac{9}{4}$$

$a = \frac{3}{2}$ $\qquad$ eller $\qquad$ $a = -\frac{3}{2}$

$\boxed{b = -\frac{9}{4a}}$

$b = -\frac{9}{4\cdot\frac{3}{2}} = -\frac{3}{2}$ $\qquad$ $b = \frac{3}{2}$

$w = \frac{3}{2} - i\frac{3}{2}$ $\qquad$ $w = -\frac{3}{2} + i\frac{3}{2}$

$\boxed{z = w + \frac{1}{2} - i\frac{1}{2}}$

$\boxed{z = 2 - i2}$ $\qquad$ $\boxed{z = -1 + i}$

071031 – Problem 1

(a) $z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$r = |z| = \sqrt{\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$

$\varphi = \arg z = \frac{\pi}{3}$

Polar form

$z = 1 \cdot e^{i\frac{\pi}{3}}$

och

$z^{75} = \left(1 \cdot e^{i\frac{\pi}{3}}\right)^{75} = 1^{75} \cdot \left(e^{i\frac{\pi}{2}}\right)^{75} = 1 \cdot e^{i\frac{75\pi}{3}}$

$= e^{i25\pi} = e^{i(\pi + 12 \cdot 2\pi)} = e^{i\pi} = -1$

(b) Rot $\boxed{z_1 = -2+i}$

Reella koefficienter ger då rot $\boxed{z_2 = -2-i}$

Delbar med

$(z-z_1)(z-z_2) = (z+2-i)(z+2+i) = z^2 + (2-i+2+i)z + (2-i)(2+i)$

$= z^2 + 4z + 5$

$$\begin{array}{r|l} & z^2 \quad + 9 \\ \hline z^2+4z+5 & z^4+4z^3+14z^2+36z+45 \\ & -(z^4+4z^3+5z^2) \\ \hline & 9z^2+36z+45 \\ & -(9z^2+36z+45) \\ \hline & 0 \end{array}$$

$z^2 + 9 = 0$

ger

$\boxed{z_3 = 3i}$ $\boxed{z_4 = -3i}$

# 111025 – Problem 1

(a)

$z_1 = \sqrt{3} - i \qquad |z_1| = 2 \qquad \arg z_1 = -\frac{\pi}{6}$

$z_1 = 2\, e^{i\left(-\frac{\pi}{6}\right)}$

$z_2 = 2 + 2i \qquad |z_2| = 2\sqrt{2} \qquad \arg z_2 = \frac{\pi}{4}$

$z_2 = 2\sqrt{2}\, e^{i\frac{\pi}{4}}$

$$z = \frac{z_1}{z_2} = \frac{\cancel{2}^{1} \cdot e^{i\left(-\frac{\pi}{6}\right)}}{\cancel{2}\sqrt{2} \cdot e^{i\frac{\pi}{4}}} = \frac{1}{\sqrt{2}} \cdot e^{i\left(\overset{-\frac{2\pi}{12}}{-\frac{\pi}{6}} - \overset{\frac{3\pi}{12}}{\frac{\pi}{4}}\right)} = \frac{1}{\sqrt{2}}\, e^{i\left(-\frac{5\pi}{12}\right)}$$

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(b)

$z = r \cdot e^{i\varphi} \qquad i = 1 \cdot e^{i\frac{\pi}{2}}$

I $z^3 = i$ gir

$r^3 \cdot e^{i3\varphi} = 1 \cdot e^{i\frac{\pi}{2}}$

så

$$\begin{cases} r^3 = 1 \\ 3\varphi = \frac{\pi}{2} + n2\pi \end{cases}$$

$$\begin{cases} r = 1 \\ \varphi = \frac{\pi}{6} + n\frac{2\pi}{3} \end{cases}$$

$z = 1 \cdot e^{i\left(\frac{\pi}{6} + n\frac{2\pi}{3}\right)}$

$z = 1\left(\cos\left(\frac{\pi}{6} + n\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{6} + n\frac{2\pi}{3}\right)\right)$

$\boxed{n = 0}$

$z_1 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\frac{1}{2}$

$\boxed{n = 1}$

$z_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} = -\frac{\sqrt{3}}{2} + i\frac{1}{2}$

$\boxed{n = 2}$

$z_3 = \cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2} = 0 + i(-1)$

$= -i$