

Ex: Fördubbling av ljudtrycket påverkar decibellattet hur?

$$L_1 = 10 \lg \frac{P_1}{P_0}$$

Vad blir decibellattet för  $2 \cdot P_1$  (ljudtryck)

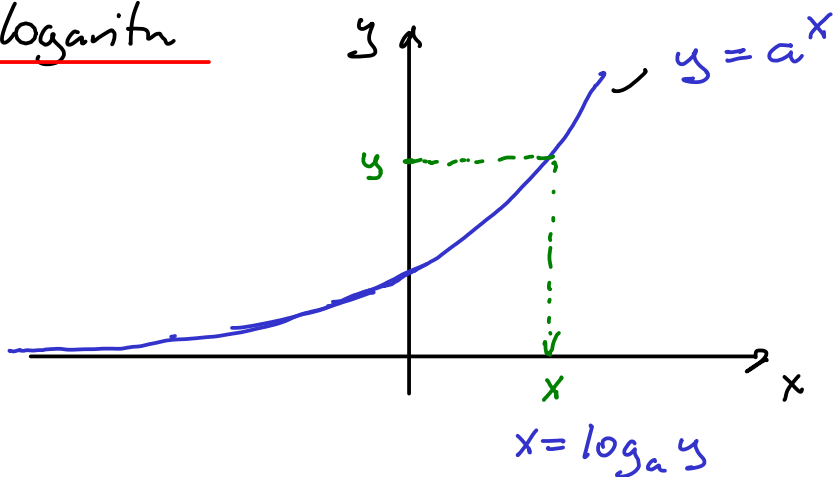
$$10 \cdot \lg \frac{2 \cdot P_1}{P_0} = 10 \cdot \lg \left( 2 \cdot \frac{P_1}{P_0} \right)$$

$$= 10 \cdot \left( \lg 2 + \lg \frac{P_1}{P_0} \right)$$

$$= 10 \cdot \lg 2 + \underbrace{10 \cdot \lg \frac{P_1}{P_0}}_{= L_1}$$

$$= \underbrace{10 \cdot \lg 2}_{3,0103} + L_1$$

a-logaritm



$$y = a^x$$
$$x = \log_a y$$

$$\left\{ \begin{array}{l} y = a^{\log_a y} \\ x = \log_a a^x \end{array} \right.$$

$$\log_a x + \log_a y = \log_a x \cdot y$$

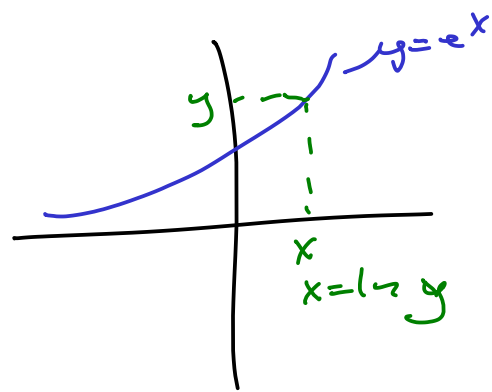
$$\log_a x^n = n \log_a x$$

⋮

$$\log_a 1 = 0$$

Exponentialfunktioner:  $e^x$

$$\frac{d}{dx} e^x = e^x$$



Naturlig logaritmer

$$\ln x = \log_e x$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$\ln a + \ln b = \ln a \cdot b$$

$$\ln a - \ln b = \ln \frac{a}{b}$$

$$\ln a^c = c \cdot \ln a$$

$$\ln 1 = 0$$

Ex:  $\log_7 49 = \frac{\ln 49}{\ln 7} = 2$

Ex:  $\log_3 2 \cdot \log_2 3 = \frac{\ln 2}{\ln 3} \cdot \frac{\ln 3}{\ln 2} = 1$

Ex: Matrinn -22 halveringstid 2.6 år

$N(t)$  mängd vid tid  $t$

$t$  tid [år]

$N_0$  mängd då  $t=0$

$k$  - sönderfallskonstant  
(på något sätt)

$$N(t) = N_0 e^{-k \cdot t}$$

$$N_0 / 2 = N(2.6)$$

~~$N_0 / 2 = N_0 e^{-k \cdot 2.6}$~~

$$\frac{1}{2} = e^{-k \cdot 2.6}$$

$$\ln \frac{1}{2} = \ln e^{-k \cdot 2.6}$$

$$\ln 1 - \ln 2 = -k \cdot 2.6$$

$$k = \frac{-\ln 2}{-2.6} = \frac{\ln 2}{2.6}$$

sa<sup>o</sup>

$$N(t) = N_0 e^{-\frac{\ln 2}{2.6} \cdot t}$$

□ Procentuell minskning på ett år

$$N(1) = N_0 \cdot \underbrace{e^{-\frac{\ln 2}{2.6} \cdot 1}}_{0.76598}$$

Minskning:  $1 - 0.76598 = 0.23402$

dvs. 23.4%

□ Tid till att  $N(t) = N_0/10$

$$\cancel{N_0} \cdot e^{-\frac{\ln 2}{2.6} \cdot t} = \cancel{N_0}/10$$

$$e^{-\frac{\ln 2}{2.6} \cdot t} = \frac{1}{10}$$

$$\ln e^{-\frac{\ln 2}{2.6} \cdot t} = \ln \frac{1}{10}$$

$$-\frac{\ln 2}{2.6} \cdot t = -\ln 10$$

$$t = \frac{-\ln 10}{-\frac{\ln 2}{2.6}}$$

$$= \frac{2.6 \cdot \ln 10}{\ln 2}$$

$$\approx 8.637 \text{ år}$$

Ex:  $\ln(x-1) + \ln(x+5) = \ln(2x)$

$$\ln((x-1)(x+5)) = \ln(2x)$$

$$(x-1)(x+5) = 2x$$

$$x^2 + 4x - 5 = 2x$$

$$x^2 + 2x - 5 = 0$$

$$x = -1 \pm \sqrt{1+5} = -1 \pm \sqrt{6}$$

$$x = -1 + \sqrt{6}$$

el.  ~~$x = -1 - \sqrt{6} < 0$~~   
Fälgk rot

Ex

$$\ln(x+3) - \ln(x+1) = \ln 2$$

$$\ln \frac{x+3}{x+1} = \ln 2$$

$$\frac{x+3}{x+1} = 2$$

$$x+3 = 2(x+1)$$

$$x+3 = 2x+2$$

$$\boxed{1 = x}$$

Ex:

$$e^{3x} - 2e^{2x} - e^x = -2$$

$$(a^b)^c = a^{b \cdot c}$$

$$(e^x)^3 - 2(e^x)^2 - e^x + 2 = 0$$

$$t = e^x$$

$$t^3 - 2t^2 - t + 2 = 0$$

Devisen an: 1, -1, 2, -2

$\boxed{t=1}$  ein vort

$$\begin{array}{r} t^2 - t - 2 \\ t-1 \overline{) t^3 - 2t^2 - t + 2} \\ \underline{-(t^3 - t^2)} \phantom{+ 2} \\ -t^2 - t + 2 \\ \underline{-(-t^2 + t)} \phantom{+ 2} \\ -2t + 2 \\ \underline{-(-2t + 2)} \\ 0 \end{array}$$

$$t^2 - t - 2 = 0$$

$$(t+1)(t-2) = 0$$

$$\boxed{t=-1} \text{ el. } \boxed{t=2}$$

$$e^x = 1$$

$$\cancel{e^x = -1}$$

$$e^x = 2$$

$$\ln e^x = \ln 1$$

$$\boxed{x = 0}$$

$$\ln e^x = \ln 2$$

$$\boxed{x = \ln 2}$$

Ex:

$$\log_2 x + \log_2 (x-1) = 1$$

$$\log_2 x(x-1) = 1$$

$$2^{\log_2 x(x-1)} = 2^1$$

$$x(x-1) = 2$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \text{ el. } x = 2$$

Falsch

Ex:

$$(2x)^{\lg 2} = (3x)^{\lg 3} \quad x > 0$$

$$\lg(2x)^{\lg 2} = \lg(3x)^{\lg 3}$$

$$\lg 2 \cdot \lg(2x) = \lg 3 \cdot \lg(3x)$$

$$\lg 2 (\lg 2 + \lg x) = \lg 3 (\lg 3 + \lg x)$$

$$(\lg 2)^2 + \lg 2 \cdot \lg x = (\lg 3)^2 + \lg 3 \cdot \lg x$$

$$(\lg 2)^2 - (\lg 3)^2 = \lg 3 \cdot \lg x - \lg 2 \cdot \lg x$$

$$(\lg 2 - \lg 3)(\lg 2 + \lg 3) = (\lg 3 - \lg 2) \cdot \lg x$$

$$-(\lg 3 - \lg 2)$$

$$- \lg 2 \cdot 3 = \lg x$$

$$\lg 6^{-1} = \lg x$$

$$x = 6^{-1} = \frac{1}{6}$$