Rep. Lekt 8

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \tan\theta$

$\cos\theta = \frac{\sin\theta}{\cos\theta}$

$\cos^2\theta = \sin\theta$

$1 - \sin^2\theta = \sin\theta$

$\sin^2\theta + \sin\theta - 1 = 0$

$t = \sin\theta$

$t^2 + t - 1 = 0$

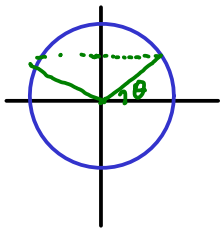
$t = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 1} = -\frac{1}{2} \pm \sqrt{\frac{5}{4}}$ ($\frac{1}{4} + 1 = \frac{4}{4}$)

$= -\frac{1}{2} \pm \frac{\sqrt{5}}{2}$

$t = -\frac{1}{2} + \frac{\sqrt{5}}{2}$ eller $t = -\frac{1}{2} - \frac{\sqrt{5}}{2}$

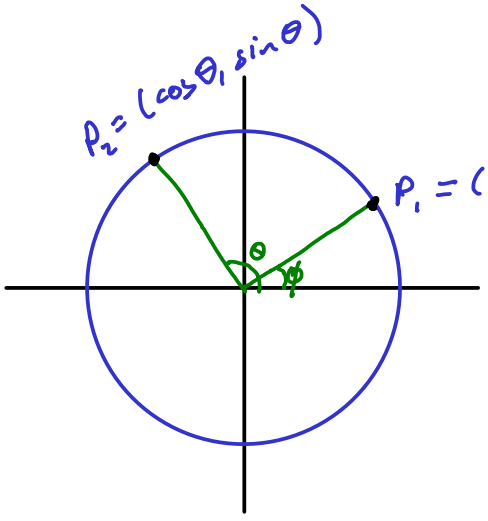
$\sin\theta = -\frac{1}{2} + \frac{\sqrt{5}}{2}$

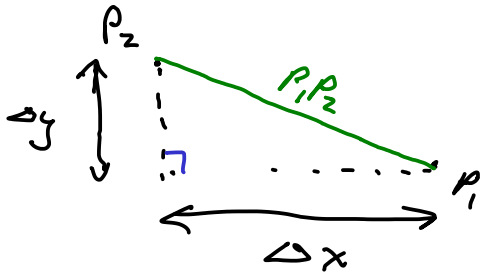
$\sin\theta = -\frac{1}{2} - \frac{\sqrt{5}}{2}$ (< -1)



$\sin\theta = \frac{\sqrt{5}-1}{2}$

$$\begin{cases} \theta = 0.66624 + n2\pi \\ \theta = \pi - 0.66624 + n2\pi \\ \quad = 2.4754 + n2\pi \end{cases}$$



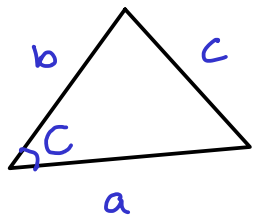


$\Delta x = \cos\theta - \cos\phi$

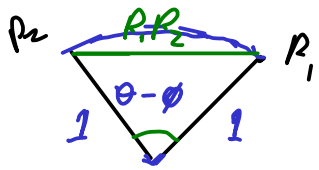
$\Delta y = \sin\theta - \sin\phi$

$P_1P_2^2 = \Delta x^2 + \Delta y^2$

Cosinussatsen



$$c^2 = a^2 + b^2 - 2ab\cos C$$



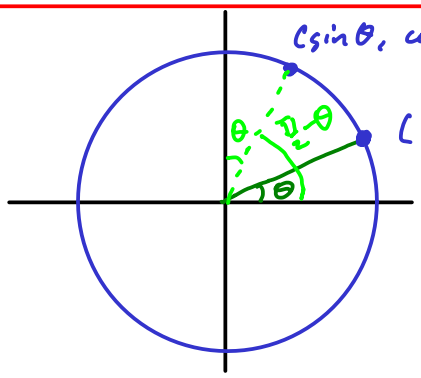
$$(P_1P_2)^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1\cdot\cos(\theta-\phi)$$

Lika:

$$(\cos\theta - \cos\phi)^2 + (\sin\theta - \sin\phi)^2 = 1^2 + 1^2 - 2\cdot 1\cdot 1\cdot\cos(\theta-\phi)$$

$$\cos^2\theta - 2\cos\theta\cos\phi + \cos^2\phi + \sin^2\theta - 2\sin\theta\sin\phi + \sin^2\phi = 2 - 2\cos(\theta-\phi)$$

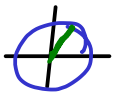
$$2 - 2\cos\theta\cos\phi - 2\sin\theta\sin\phi = 2 - 2\cos(\theta-\phi)$$



$(\sin\theta, \cos\theta) = (\cos(\frac{\pi}{2}-\theta), \sin(\frac{\pi}{2}-\theta))$

$(\cos\theta, \sin\theta)$

$$\frac{\pi}{3} + \frac{\pi}{4} = \frac{4\pi}{12} + \frac{3\pi}{12} = \frac{7\pi}{12}$$



$$\tan\frac{7\pi}{12} = \frac{\sin\frac{7\pi}{12}}{\cos\frac{7\pi}{12}} = \frac{\sin(\frac{\pi}{3}+\frac{\pi}{4})}{\cos(\frac{\pi}{3}+\frac{\pi}{4})} = \frac{\overbrace{\sin\frac{\pi}{3}}^{\frac{\sqrt{3}}{2}}\cdot\overbrace{\cos\frac{\pi}{4}}^{\frac{1}{\sqrt{2}}} + \overbrace{\cos\frac{\pi}{3}}^{\frac{1}{2}}\cdot\overbrace{\sin\frac{\pi}{4}}^{\frac{1}{\sqrt{2}}}}{\underbrace{\cos\frac{\pi}{3}}_{1/2}\cdot\underbrace{\cos\frac{\pi}{4}}_{1/\sqrt{2}} - \underbrace{\sin\frac{\pi}{3}}_{\sqrt{3}/2}\underbrace{\sin\frac{\pi}{4}}_{1/\sqrt{2}}}$$

$$= \frac{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\cdot\sqrt{2}}\right)\times 2\sqrt{2}}{\left(\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}\right)\times 2\sqrt{2}} \qquad \boxed{= \frac{\sqrt{3}+1}{1-\sqrt{3}}}$$

Klart!

$$\sin 2\theta = \sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\cdot\sin\theta = 2\cdot\sin\theta\cdot\cos\theta$$

$$\cos 2\theta = \cos(\theta+\theta) = \cos\theta\cdot\cos\theta - \sin\theta\cdot\sin\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - (1-\cos^2\theta)$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$= 2(1-\sin^2\theta) - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$\sin^2\theta + \cos^2\theta = 1$

- $\sin^2\theta = 1 - \cos^2\theta$
- $\cos^2\theta = 1 - \sin^2\theta$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

Ex: $\cos 3\theta = \cos(2\theta+\theta) = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

$$= (2\cos^2\theta - 1)\cdot\cos\theta - 2\sin\theta\cos\theta\sin\theta$$

$$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$$

$$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$$

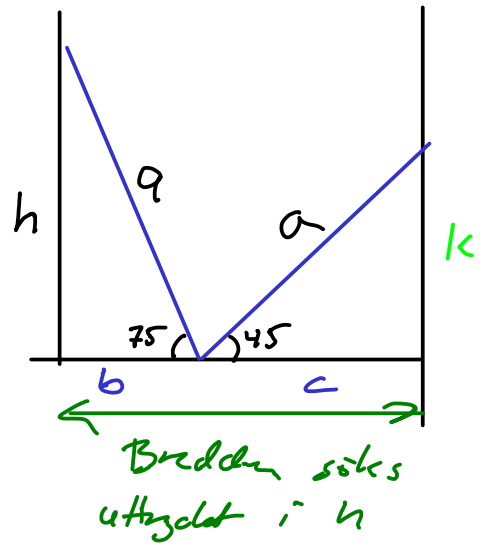
$$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$$

$$= 4\cos^3\theta - 3\cos\theta$$

Ex: $\dfrac{\cos x - \sin x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x - \sin x)}{(\cos x + \sin x)(\cos x - \sin x)} = \dfrac{\cos^2 x - 2\cos x\sin x + \sin^2 x}{\cos^2 x - \sin^2 x}$ ($\cos^2 x + \sin^2 x = 1$)

$$= \frac{1 - \sin 2x}{\cos 2x} = \frac{1}{\cos 2x} - \frac{\sin 2x}{\cos 2x} = \frac{1}{\cos 2x} - \tan 2x$$

Ex



$$\tan 75 = \frac{h}{b}$$

$$\boxed{b = \frac{h}{\tan 75}}$$

$$\sin 75 = \frac{h}{a}$$

$$a = \frac{h}{\sin 75}$$

$$\cos 45 = \frac{c}{a}$$

$$\boxed{c = a \cdot \cos 45 = \frac{h \cdot \cos 45}{\sin 75}}$$

Bredden: $b + c = \frac{h}{\tan 75} + \frac{h \cdot \cos 45}{\sin 75} = \left(\frac{\cos 75}{\sin 75} + \frac{\cos 45}{\sin 75}\right) h$

$$= \frac{\cos 75 + \cos 45}{\sin 75} \cdot h = \frac{\cos(45+30) + \cos 45}{\sin(45+30)} h$$

$$= \frac{\overbrace{\cos 45}^{\frac{1}{\sqrt{2}}} \cdot \overbrace{\cos 30}^{\frac{\sqrt{3}}{2}} - \overbrace{\sin 45}^{\frac{1}{\sqrt{2}}} \cdot \overbrace{\sin 30}^{\frac{1}{2}} + \overbrace{\cos 45}^{\frac{1}{\sqrt{2}}}}{\underbrace{\sin 45}_{\frac{1}{\sqrt{2}}} \underbrace{\cos 30}_{\frac{\sqrt{3}}{2}} + \underbrace{\cos 45}_{\frac{1}{\sqrt{2}}} \cdot \underbrace{\sin 30}_{\frac{1}{2}}} h$$

$$= \frac{\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\right) \times 2\sqrt{2}}{\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) \times 2\sqrt{2}} h = \frac{\sqrt{3} - 1 + 2}{\sqrt{3} + 1} h = 1 \cdot h$$