Lektion 11b - genomgång av duggan

1a

$$3x^2-3x+1 = 3\left(x^2-x+\frac{1}{3}\right)$$

$$= 3\left(\left(x-\frac{1}{2}\right)^2-\frac{1}{4}+\frac{1}{3}\right) = 3\left(\left(x-\frac{1}{2}\right)^2-\frac{3}{12}+\frac{4}{12}\right)$$

$x^2-x+\frac{1}{4}$

$$= 3\left(\left(x-\frac{1}{2}\right)^2+\frac{1}{12}\right) = 3\left(x-\frac{1}{2}\right)^2+\frac{1}{4} \leftarrow \text{Svar}$$

1b

$$\frac{x^4-x^3+2}{x^2-3}$$ bestäm kvot och rest

$$\begin{array}{r|l} & x^2-x+3 \\ x^2-3 & x^4-x^3+2 \\ & -(x^4-3x^2) \\ & -x^3+3x^2+2 \\ & -(-x^3+3x) \\ & 3x^2-3x+2 \\ & -(3x^2-9) \\ & -3x+11 \end{array}$$

Kvot: x^2-x+3

Rest: $-3x+11$

2a

$y_1 = 3x+2$ Finn x så avståndet < 1

$y_2 = 5-x$

$|z|<b$

$-b<z<b$

$|y_1-y_2|<1$

$|3x+2-(5-x)|<1$

$|3x+2-5+x|<1$

$|4x-3|<1$

$-1<4x-3<1$

$3-1<4x<1+3$

$2<4x<4$

$\frac{2}{4}<x<1$

$\boxed{\frac{1}{2}<x<1}$ Svar!!

2b

$$3\ln x + \ln(x-1) = \ln(2-x) + 2\ln x$$

Subtraherer $2\ln x$

$$\ln x + \ln(x-1) = \ln(2-x)$$

$$\ln(x(x-1)) = \ln(2-x)$$

$$x(x-1) = 2-x$$

$$x^2 - x = 2 - x$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$\boxed{x=\sqrt{2}}$ eller $\cancel{x=-\sqrt{2}}$

3a

$$\frac{1}{x-3} < \frac{1}{1-x}$$

$$\frac{1}{x-3} - \frac{1}{1-x} < 0$$

$$\frac{1\cdot(1-x)}{(x-3)(1-x)} - \frac{1\cdot(x-3)}{(1-x)(x-3)} < 0$$

$$\frac{1-x-\overbrace{(x-3)}^{-x+3}}{(x-3)(1-x)} < 0$$

$$\frac{4-2x}{(x-3)(1-x)} < 0$$

x		1		2		3	
$4-2x$	+		+	0	−		−
$x-3$	−		−		−	0	+
$1-x$	+	0	−		−		−
$\frac{4-2x}{(x-3)(1-x)}$	−	ej. def.	+	0	−	ej. def.	+

$x<1$ eller $2<x<3$

$$x \in \,]-\infty, 1[\, \cup \,]2, 3[$$

$\boxed{3b}$

$$|x-1| + |x-3| = x$$

$$|x-1| = \begin{cases} x-1 & x-1 \geq 0 \\ -(x-1) & x-1 < 0 \end{cases}$$

$1 \qquad 3$

$$-(x-1)-(x-3) = x$$
$$-x+1-x+3 = x$$
$$4 = 3x$$
$$\cancel{x = \frac{4}{3}}$$

ej giltig >1

$$(x-1)-(x-3) = x$$
$$\cancel{x}-1-\cancel{x}+3 = x$$
$$\boxed{x = 2}$$

$$(x-1)+(x-3) = x$$
$$x-1+x-3 = x$$
$$\boxed{x = 4}$$

Svar: