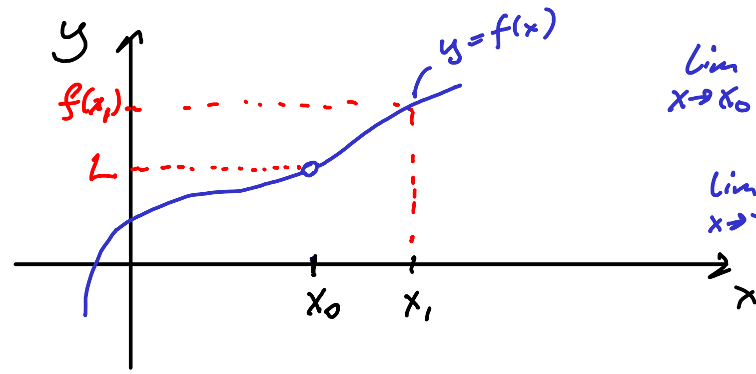
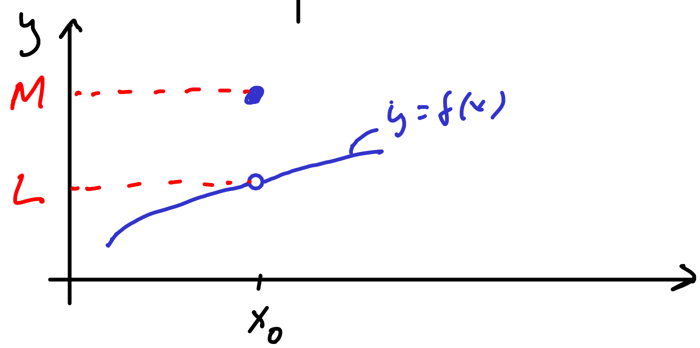
Gränsvärden



$$\lim_{x\to x_0} f(x) = L$$

$$\lim_{x\to x_1} f(x) = f(x_1)$$

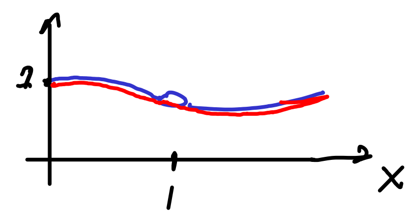


$$\lim_{x\to x_0} f(x) = L$$

$$f(x_0) = M$$

Ex:

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)}$$



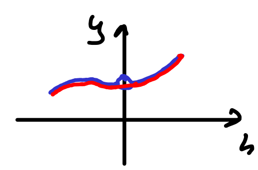
$$= \lim_{x\to 1} \frac{(\sqrt{x})^2-1^2}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1\cdot\cancel{(x-1)}}{\cancel{(x-1)}(\sqrt{x}+1)} = \lim_{x\to 1} \frac{1}{\sqrt{x}+1} = \frac{1}{\sqrt{1}+1} = \frac{1}{2}$$

Ex:

$$\lim_{x\to 1} \frac{x^2-1}{x-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}\cdot 1} = \lim_{x\to 1} (x+1) = 1+1 = 2$$

Ex:

$$\lim_{h\to 0} \frac{(3+h)^2-9}{h} = \left[\frac{0}{0}\right] = \lim_{h\to 0} \frac{\cancel{9}+2\cdot 3\cdot h+h^2-\cancel{9}}{h}$$



$$= \lim_{h\to 0} \frac{\overset{1}{\cancel{h}}(6+h)}{\underset{1}{\cancel{h}}} = \lim_{h\to 0} (6+h) = 6+0 = 6$$

Ex:

$$\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} = \left[\frac{0}{0}\right] = \lim_{t\to 0} \frac{(\sqrt{t^2+9}-3)(\sqrt{t^2+9}+3)}{t^2(\sqrt{t^2+9}+3)}$$

$$= \lim_{t\to 0} \frac{(\sqrt{t^2+9})^2-3^2}{t^2(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{\overset{1}{\cancel{t^2}}+\cancel{9}-\cancel{9}}{\underset{1}{\cancel{t^2}}(\sqrt{t^2+9}+3)} = \lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{\sqrt{9}+3}$$

$$= \frac{1}{6}$$

Ex:

$$\lim_{x\to\infty} \frac{x^2-1}{x^2+1} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\cancel{x^2}(1-\underbrace{\tfrac{1}{x^2}}_{\to 0})}{\cancel{x^2}(1+\underbrace{\tfrac{1}{x^2}}_{\to 0})} = \frac{1-0}{1+0} = 1$$

Ex:

$$\lim_{x\to\infty} \frac{3x^2-x-2}{5x^2+4x+1} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty} \frac{\cancel{x^2}(3-\underbrace{\tfrac{1}{x}}_{\to 0}-\underbrace{\tfrac{2}{x^2}}_{\to 0})}{\cancel{x^2}(5+\underbrace{\tfrac{4}{x}}_{\to 0}+\underbrace{\tfrac{1}{x^2}}_{\to 0})}$$

$$= \frac{3-0-0}{5+0+0} = \frac{3}{5}$$

$$\boxed{\sqrt{x^2} = |x|}$$

Ex:

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) = [\infty - \infty] = \lim_{x\to\infty} \frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to\infty} \frac{(\sqrt{x^2+1})^2 - x^2}{\sqrt{x^2+1}+x} = \lim_{x\to\infty} \frac{\cancel{x^2}+1-\cancel{x^2}}{\sqrt{x^2(1+\frac{1}{x^2})}+x} = \lim_{x\to\infty} \frac{1}{\underbrace{|x|}_{\substack{=x \\ \text{dé } x\to\infty}}\sqrt{1+\frac{1}{x^2}}+x}$$

$$= \lim_{x\to\infty} \frac{1}{x\left(\sqrt{1+\frac{1}{x^2}}+1\right)} = \lim_{x\to\infty} \underbrace{\frac{1}{x}}_{\to 0} \cdot \frac{1}{\sqrt{1+\underbrace{\frac{1}{x^2}}_{\to 0}}+1} = 0 \cdot \frac{1}{\sqrt{1+0}+1} = 0$$

Ex:

a)
$$\lim_{x\to 3} \frac{x^3-3x^2-x+3}{x^2-x-6} = \left[\frac{0}{0}\right] = \lim_{x\to 3} \frac{\cancel{(x-3)}(x^2-1)}{\cancel{(x-3)}(x+2)} = \frac{3^2-1}{3+2} = \frac{8}{5}$$

b)
$$\lim_{x\to -\infty} \left(\sqrt{x^2+x} - \sqrt{x^2+1}\right) = [\infty - \infty] = \lim_{x\to -\infty} \frac{(\sqrt{x^2+x}-\sqrt{x^2+1})(\sqrt{x^2+x}+\sqrt{x^2+1})}{(\sqrt{x^2+x}+\sqrt{x^2+1})}$$

$$= \lim_{x\to -\infty} \frac{x^2+x\overbrace{-(x^2+1)}^{-x^2-1}}{\sqrt{x^2(1+\frac{1}{x})}+\sqrt{x^2(1+\frac{1}{x^2})}} = \lim_{x\to -\infty} \frac{x-1}{\underbrace{|x|}_{=-x}\sqrt{1+\frac{1}{x}} + \underbrace{|x|}_{=-x}\sqrt{1+\frac{1}{x^2}}}$$

$x<0$

$$= \lim_{x\to -\infty} \frac{\cancel{x}(1-\overbrace{\tfrac{1}{x}}^{\to 0})}{\cancel{x}\left(-\sqrt{1+\underbrace{\tfrac{1}{x}}_{\to 0}} - \sqrt{1+\underbrace{\tfrac{1}{x^2}}_{\to 0}}\right)} = \frac{1-0}{-\sqrt{1+0}-\sqrt{1+0}} = \frac{1}{-2} = -\frac{1}{2}$$