Rep. Lekt 12

(a) $$\lim_{x\to 4}\frac{x^2+x-20}{x-4}=\left[\frac{0}{0}\right]=\lim_{x\to 4}\frac{\cancel{(x-4)}(x+5)}{\cancel{(x-4)}\cdot 1}$$

$$=\lim_{x\to 4} x+5 = 4+5=9$$

$$\sqrt{x^2}=|x|$$

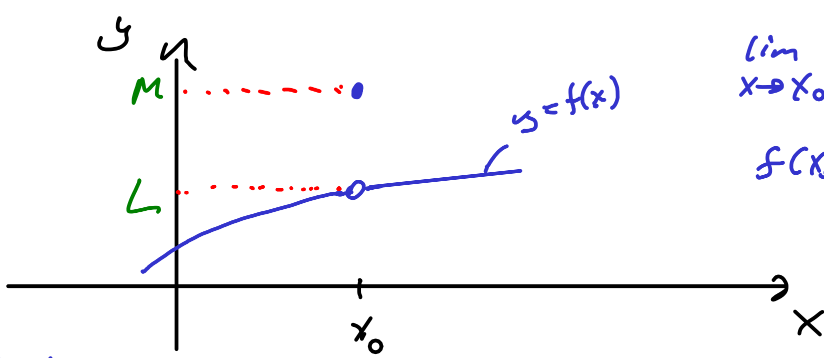
(b) $$\lim_{x\to\infty}\left(x-\sqrt{x^2-1}\right)=[\infty-\infty]=\lim_{x\to\infty}\frac{(x-\sqrt{x^2-1})(x+\sqrt{x^2-1})}{(x+\sqrt{x^2-1})}$$

$$=\lim_{x\to\infty}\frac{x^2-(\sqrt{x^2-1})^2}{x+\sqrt{x^2-1}}=\lim_{x\to\infty}\frac{x^2-(x^2-1)}{x+\sqrt{x^2(1-\frac{1}{x^2})}}=\lim_{x\to\infty}\frac{x^2-x^2+1}{x+\sqrt{x^2}\sqrt{1-\frac{1}{x^2}}}$$

$$=\lim_{x\to\infty}\frac{1}{x+\underbrace{|x|}_{\substack{=x\\ \text{ty } x>0}}\sqrt{1-\frac{1}{x^2}}}=\lim_{x\to\infty}\frac{1}{x\left(1+\sqrt{1-\frac{1}{x^2}}\right)}=\lim_{x\to\infty}\underbrace{\frac{1}{x}}_{\to 0}\cdot\frac{1}{1+\sqrt{1-\underbrace{\frac{1}{x^2}}_{\to 0}}}$$

$$=0\cdot\frac{1}{1+\sqrt{1-0}}=0$$

Kontinuitet

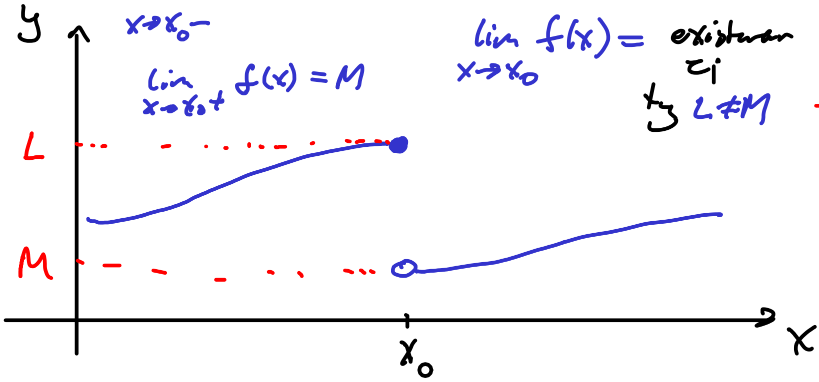


$$\lim_{x\to x_0} f(x)=L$$

$$f(x_0)=M$$

$$\lim_{x\to x_0} f(x)\neq f(x_0)$$

EJ Kontinuerlig

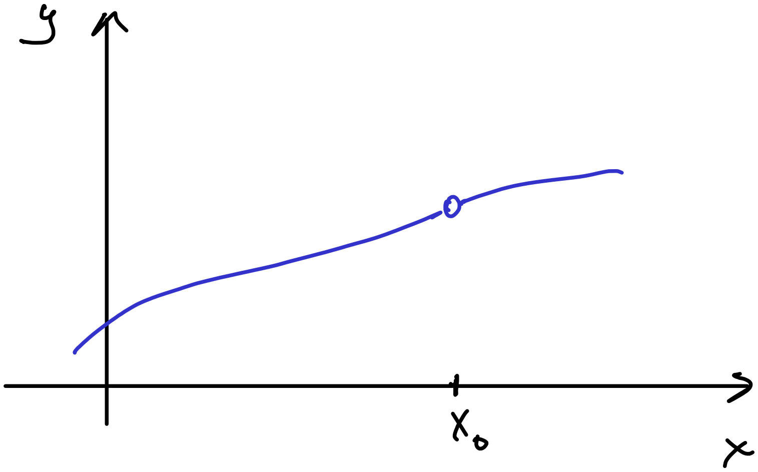


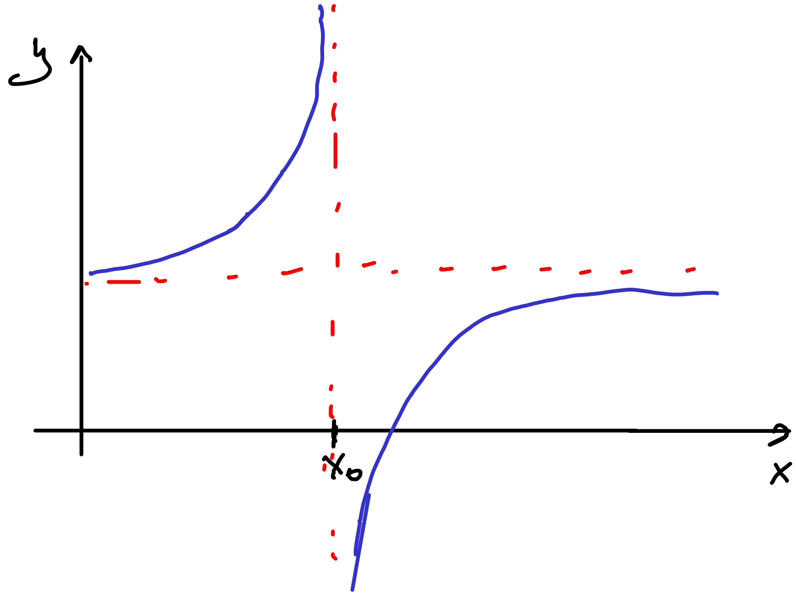
$$\lim_{x\to x_0^-} f(x)=L$$

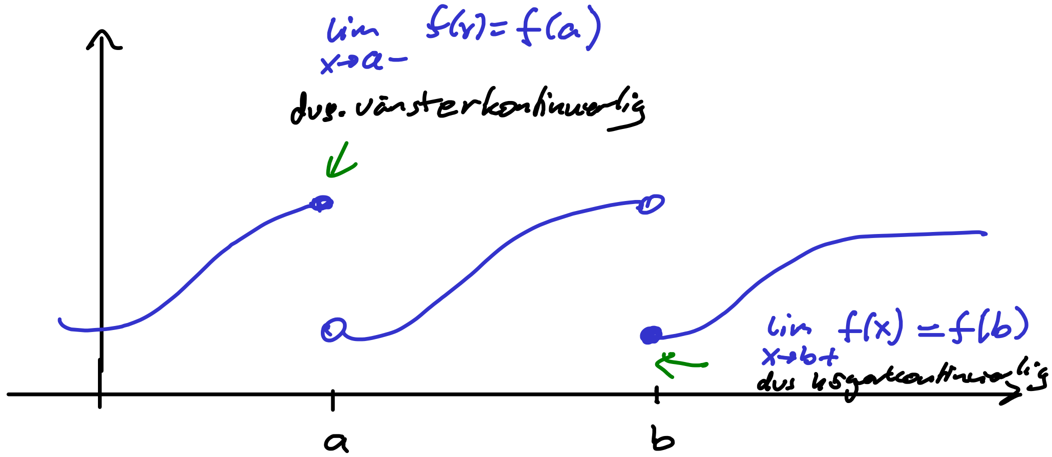
$$\lim_{x\to x_0^+} f(x)=M$$

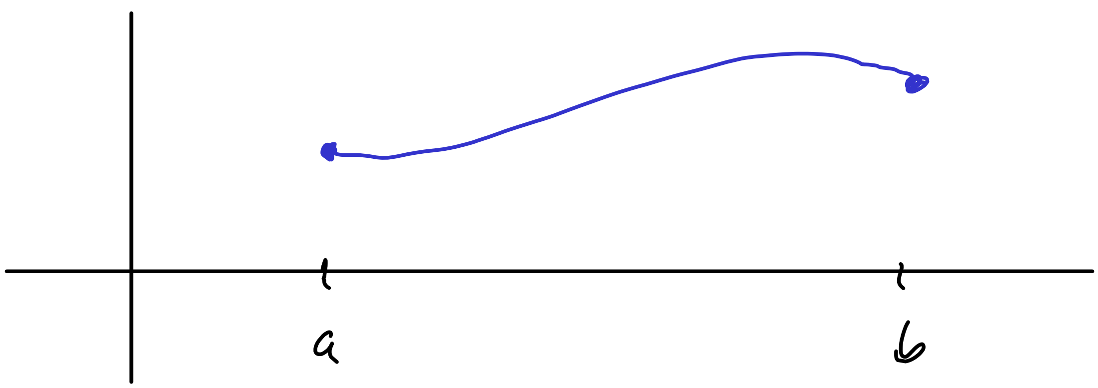
$\lim_{x\to x_0} f(x)=$ existerar ej ty $L\neq M$

$$\lim_{x\to x_0} f(x)=\underbrace{f(x_0)}^{=L}$$


y
x_0
x


y
x_0
x

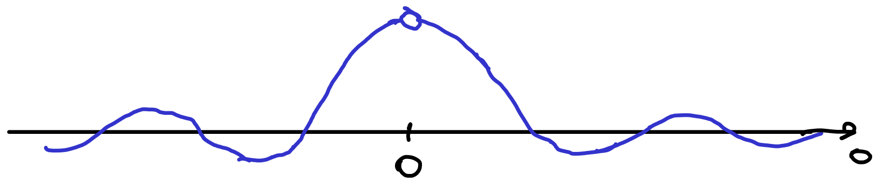

$\lim_{x \to a-} f(x) = f(a)$
dvs. vänsterkontinuerlig
$\lim_{x \to b+} f(x) = f(b)$
dvs högerkontinuerlig
a
b


a
b

Ex: Hävbar diskontinuitet

$$f(x) = \frac{\sin x}{x}$$

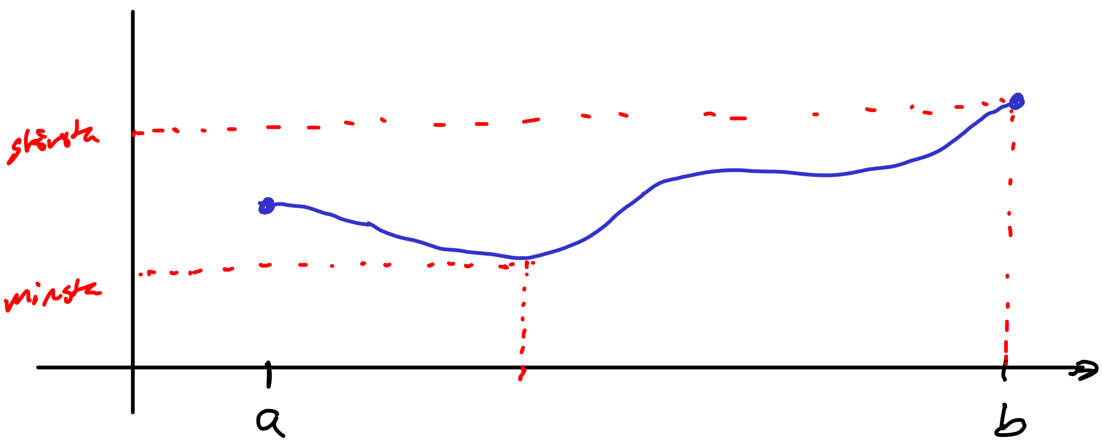
$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



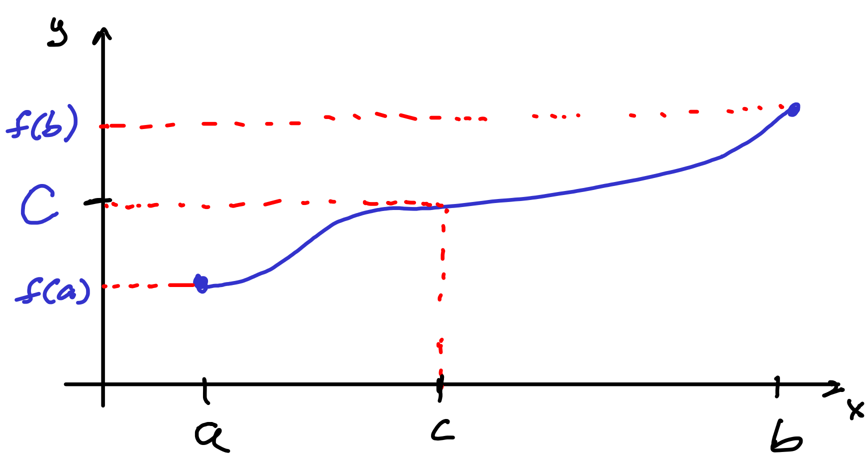
Häv diskontinuitet

$$\operatorname{sinc} x = \begin{cases} \dfrac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Satsen om största och minsta värde

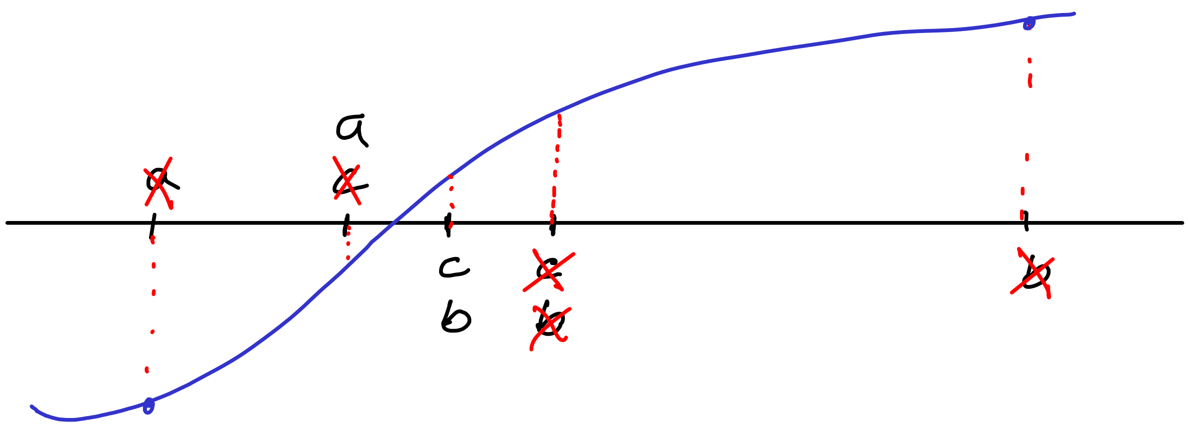


Satsen om mellanliggande värden



$$f(c) = C$$

Hitta nollställe (intervallhalvering)



Ex:

$$\lim_{x\to 1} \frac{6x^2-5ax+a^2}{x-1} = \left[\frac{6-5a+a^2}{0}\right] \leftarrow$$ Måste $=0$ om gränsvärde ska existera

$$a^2-5a+6=0$$
$$(a-2)(a-3)=0$$
$a=2$ eller $a=3$

Fall 1: $\boxed{a=2}$

$$\lim_{x\to 1} \frac{6x^2-10x+4}{x-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{\cancel{(x-1)}(6x-4)}{\cancel{(x-1)}\cdot 1} =$$

$$= \lim_{x\to 1} 6x-4 = 6-4 = 2$$

Kontinuerlig om $\boxed{2=b}$

Kont om
$$\lim_{x\to 1} f(x) = f(1)$$
$$f(1) = b$$

Fall 2: $\boxed{a=3}$

$$\lim_{x\to 1} \frac{6x^2-15x+9}{x-1} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{\cancel{(x-1)}(6x-9)}{\cancel{(x-1)}\,1}$$

$$= \lim_{x\to 1} 6x-9 = 6-9 = -3$$

Kontinuerlig om $\boxed{b=-3}$