Jag sitter i rum 605.

Rep. Lekt 14

$a^{b+c} = a^b \cdot a^c$

$\ln(a \cdot b) = \ln a + \ln b$

(a) $$\lim_{x\to 0} \frac{\cos x \cdot \tan x}{x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{\cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}}}{x} = \lim_{x\to 0} \frac{\sin x}{x} = 1$$

(b) $$\lim_{x\to\infty} \frac{x + \ln(e^{2x} + x)}{\sqrt{x} + e^{\ln x + 1}} = \lim_{x\to\infty} \frac{x + \ln\left(e^{2x}\left(1 + \frac{x}{e^{2x}}\right)\right)}{\sqrt{x} + e^{\ln x} \cdot e^1}$$

$$= \lim_{x\to\infty} \frac{x + \overbrace{\ln e^{2x}}^{2x} + \ln\left(1 + \frac{x}{e^{2x}}\right)}{\sqrt{x} + x \cdot e^1} = \lim_{x\to\infty} \frac{3x + \ln\left(1 + \frac{x}{e^{2x}}\right)}{\sqrt{x} + e^1 \cdot x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\left(3 + \frac{\ln\left(1 + \frac{x}{e^{2x}}\right)}{x}\right)}{\cancel{x}\left(\frac{\sqrt{x}}{\underset{=\sqrt{x}\cdot\sqrt{x}}{x}} + e^1\right)} = \lim_{x\to\infty} \frac{3 + \underbrace{\frac{\ln\left(1 + \frac{x}{e^{2x}}\right)}{x}}_{\to 0}}{\underbrace{\frac{1}{\sqrt{x}}}_{\to 0} + e^1}$$

$$= \frac{3 + 0}{0 + e^1} = \frac{3}{e}$$

Ex: Aritmetisk talföljd

$$11,\ 9,\ 7,\ 5,\ 3,\ 1,\ -1,\ -3$$
$$a_1\quad a_2\quad a_3\quad a_4\quad a_5\quad a_6\quad a_7$$

$$a_n = 11 - 2 \cdot (n - 1)$$

Ex: Geometrisk talföljd

$$27 \xrightarrow{\times\frac{1}{3}} 9 \xrightarrow{\times\frac{1}{3}} 3, \quad 1, \quad \frac{1}{3}, \quad \frac{1}{9}, \quad \frac{1}{27},$$

$$a_1 \quad a_2 \quad a_3 \quad a_4$$

$$a_n = 27 \cdot \frac{1}{3}^{n-1}$$

Ex:

$$a_n = 9 + (n-1)(-2)$$

$a_n = -17$, Vad är n ?

$$-17 = 9 + (n-1)(-2)$$

$$-26 = (n-1)(-2)$$

$$n-1 = \frac{-26}{-2} = 13$$

$$\boxed{n = 14}$$

Ex:

$a_n = 4 \cdot 3^{n-1}$ Söks: a_6

$$\boxed{a_6 = 4 \cdot 3^{6-1} = 4 \cdot 3^5 = 972}$$

Ex:

$$a_n = \frac{3n^2 - 5n + 7}{4 - 6n^2} \qquad n = 1, \ldots, \infty$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3n^2 - 5n + 7}{4 - 6n^2} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(3 - \underbrace{\frac{5}{n}}_{\to 0} + \underbrace{\frac{7}{n^2}}_{\to 0}\right)}{\cancel{n^2}\left(\underbrace{\frac{4}{n^2}}_{\to 0} - 6\right)}$$

$$= \frac{3 - 0 + 0}{0 - 6} = -\frac{1}{2}$$

Ex: **Fixpunktsiteration**

$$x = \cos x$$

Startgissning: $x_1 = 1$

$$x_n = \cos(x_{n-1})$$

n	x_n
1	
2	
3	
4	
5	
6	