Rep Lekt. 15

Aritmetisk talföljd: $a_n = a_1 + (n-1)\cdot d$

Vi vet:

$$\begin{cases} (1)\ \ 7 (= a_5) = a_1 + 4\cdot d \\ (2)\ \ \frac{1}{2} (= a_{31}) = a_1 + 30\cdot d \end{cases}$$

(2) − (1): $\frac{1}{2} - 7 = a_1 - a_1 + 30d - 4d$

$$-\frac{13}{2} = 26d$$

$$d = -\frac{\cancel{13}^{1}}{2\cdot\cancel{26}_{2}} = -\frac{1}{4}$$

sätt in $d = -\frac{1}{4}$ i (1)

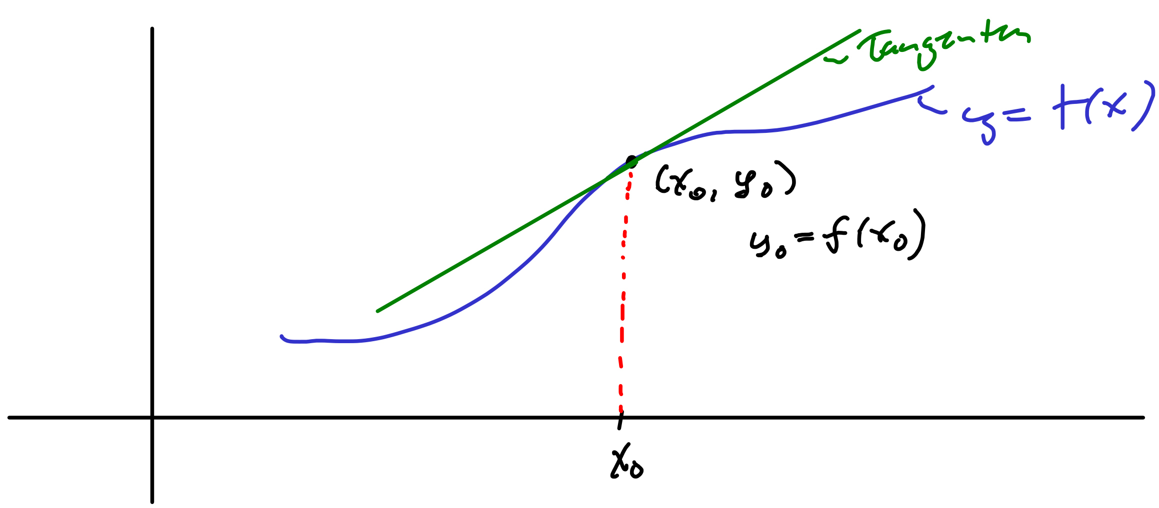
$$7 = a_1 + \underbrace{4\left(-\frac{1}{4}\right)}_{-1}$$

$$a_1 = 8$$

Svar: Differens $d = -\frac{1}{4}$

Första värde $a_1 = 8$

$8,\ 7+\frac{3}{4},\ 7+\frac{2}{4},\ 7+\frac{1}{4},\ 7,\ 6+\frac{3}{4},$



Ex: $f(x) = x^2 - 3x$

- Förändringskvoten

$$\frac{f(4+h) - f(4)}{h} = \frac{\overbrace{(4+h)^2 - 3(4+h)}^{f(4+h)} - \overbrace{(4^2 - 3\cdot 4)}^{f(4)}}{h}$$

$$= \frac{\cancel{16} + 8h + h^2 - \cancel{12} - 3h - \cancel{4}}{h} = \frac{\cancel{h}(5+h)}{\cancel{h}} = 5+h$$

- Låt $h \to 0$

$$\lim_{h\to 0} \frac{f(4+h) - f(4)}{2} = \lim_{h\to 0} 5+h = 5$$

- Tangent i $x = 4$, $y = f(4) = 4$

$$y - y_0 = k(x - x_0)$$

$$y - 4 = 5(x-4)$$

$$y = 5x - 16$$

Ex: $f(x) = \frac{x}{1+x}$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b\cdot c}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h}{1+x+h} - \frac{x}{1+x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)(1+x)}{(1+x+h)(1+x)} - \frac{(1+x+h)\cdot x}{(1+x+h)(1+x)}}{h} = \lim_{h\to 0} \frac{\frac{\cancel{x} + \cancel{x^2} + h + \cancel{hx} - \cancel{x} - \cancel{x^2} - \cancel{hx}}{(1+x+h)(1+x)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}^{\,1}}{\cancel{h}(1+x+h)(1+x)} = \lim_{h\to 0} \frac{1}{(1+x+h)(1+x)} = \frac{1}{(1+x+0)(1+x)}$$

$$= \frac{1}{(1+x)^2} = f'(x)$$

Ex: $f(x) = x^2 + 4x$

- Derivatans värde då $x = 1$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h\to 0} \frac{(1+h)^2 + 4(1+h) - (1^2 + 4\cdot 1)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{1} + 2h + h^2 + \cancel{4} + 4h - \cancel{5}}{h} = \lim_{h\to 0} \frac{\cancel{h}(6+h)}{\cancel{h}} = 6$$

- Tangenten $x_0 = 1$, $y_0 = f(1) = 1^2 + 4\cdot 1 = 5$

$$y - y_0 = k\cdot(x - x_0)$$
$$y - 5 = 6(x-1)$$
$$y = 6x - 1$$

Ex: $f(x) = x^2 + 2$

- Derivata i $x = 2$

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{(2+h)^2 + 2 - (2^2+2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{4} + 4h + h^2 + \cancel{2} - \cancel{6}}{h} = \lim_{h\to 0} 4 + h = 4$$

- Tangenten $x_0 = 2$, $y_0 = f(2) = 2^2 + 2 = 6$

$$y - 6 = 4(x-2)$$
$$y = 4x - 2$$

Normalen, samma punkt $(2, 6)$

Rikt.koeff $-\frac{1}{4}$: $y - 6 = -\frac{1}{4}(x-2)$

$$y = -\frac{1}{4}x + \frac{13}{2}$$

Ex: $f(x) = x^2$ $\quad x_0 = 1 \quad \Delta x = 0.01$

- $\Delta y = f(x_0 + \Delta x) - f(x_0) = f(1.01) - f(1)$

$$= 1.01^2 - 1^2 = 0.02010000\ldots$$

- $\Delta y \approx dy = f'(x_0) \cdot \Delta x = 2x_0 \cdot \Delta x = 2 \cdot 1 \cdot 0.01$

$$= 0.020000\ldots$$