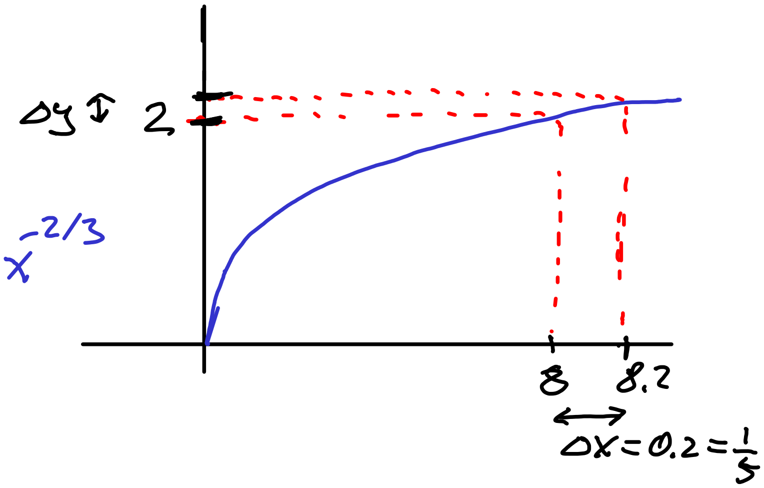
Rep. Lekt. 16



$$f(x) = x^{1/3}$$

$$f'(x) = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3} x^{-2/3}$$

$$= \frac{1}{3x^{2/3}}$$

$$8^{2/3} = \left(8^{1/3}\right)^2 = 2^2 = 4$$

$$\frac{d}{dx} x^a = a \cdot x^{a-1}$$

$$y = f(8.2) = f\left(8 + \frac{1}{5}\right)$$

$$= f(8) + \Delta y \approx 2 + dy = 2 + f'(8) \cdot \underbrace{\Delta x}_{1/5}$$

$$= 2 + \frac{1}{3\, 8^{2/3}} \cdot \frac{1}{5} = 2 + \frac{1}{3 \cdot 4 \cdot 5} = 2 + \frac{1}{60} \left(= \frac{121}{60} \right)$$

Approx: $2 + \frac{1}{60} \approx 2.0166667$ Exakt: 2.01653

Exempel

$$f(x) = 2 + 3x + 4x^2$$

$$f'(x) = 0 + 3 + 4 \cdot 2x = 3 + 8x$$

$$p(x) = \underbrace{x^2}_{f(x)} \cdot \underbrace{\sqrt{x}}_{g(x)} = x^{5/2} \qquad p'(x) = \frac{5}{2} \cdot x^{5/2-1} = \frac{5}{2} x^{3/2}$$

$$p'(x) = \left[f'(x) \cdot g(x) + f(x) \cdot g'(x)\right]$$

$$= 2x \cdot \sqrt{x} + \underbrace{x^2}_{= x\sqrt{x}\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = 2x\sqrt{x} + \frac{1}{2} x\sqrt{x}$$

$$= \frac{5}{2} x\sqrt{x}$$

Deriveringsregler

$$\frac{d}{dx} x^n = n \cdot x^{n-1}, \quad n \in \mathbb{Z} \quad (n \in \mathbb{R})$$

$$\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \left(f(x) + g(x)\right) = f'(x) + g'(x)$$

$$\frac{d}{dx} \left(k \cdot f(x)\right) = k \cdot f'(x)$$

$$\frac{d}{dx} \left(f(x) \cdot g(x)\right) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$$\frac{d}{dx} \sin x = \cos x \qquad \frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$$

Kjedjeregeln: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

Ex:

$$k(x) = \frac{2x+3}{5x^2-2x} \quad \begin{matrix} \}f(x) \\ \}g(x) \end{matrix}$$

$$k'(x) = \frac{f'(x)g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$= \frac{(2+0)(5x^2-2x) - (2x+3)\cdot(5\cdot 2x-2)}{(5x^2-2x)^2}$$

$$= \frac{10x^2-4x-(20x^2\underbrace{-4x+30x}_{+26x}-\underline{6})}{(5x^2-2x)^2}$$

$$= \frac{10x^2-4x-20x^2-26x+6}{(5x^2-2x)^2}$$

$$= \frac{-10x^2-30x+6}{(5x^2-2x)^2}$$

Derivata till sin(x)

$$\frac{d}{dx}\sin x = \lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0}\frac{\sin x\cdot\cos h+\cos x\cdot\sin h-\sin x}{h}$$

$$= \lim_{h\to 0}\frac{\sin x(\cos h-1)+\cos x\cdot\sin h}{h}$$

$$= \lim_{h\to 0}\left(\sin x\underbrace{\frac{\cos h-1}{h}}_{\to 0}+\cos x\cdot\underbrace{\frac{\sin h}{h}}_{\to 1}\right)$$

$$= \lim_{h\to 0}\left(-\sin x\underbrace{\frac{1-\cos h}{h}}_{\to 0}+\cos x\cdot\underbrace{\frac{\sin h}{h}}_{\to 1}\right) = \cos x$$

Ex: $\frac{d}{dx}\Big[\underbrace{(x^2+1)}_{f(x)}\ \underbrace{\sin x}_{g(x)}\Big] = (2x+0)\cdot\sin x + (x^2+1)\cdot\cos x$

$= 2x\sin x + (x^2+1)\cos x$

Ex: $\frac{d}{dx}\tan x = \frac{d}{dx}\frac{\sin x}{\cos x} = \frac{\cos x\cdot\cos x - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$ ①

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$ ②

Ex: $\frac{d}{dx}\sin(x^2) = \cos(x^2)\cdot\frac{d}{dx}(x^2) = \cos(x^2)\cdot 2x$

$f(x) = \sin(x^2)$ $\qquad f'(x) = \cos(x^2)2x$

Ex: $\frac{d}{dx}(x^2+3)^4 = 4(x^2+3)^3\cdot\frac{d}{dx}(x^2+3) = 4(x^2+3)^3\cdot 2x$

$= 8x(x^2+3)^3$

Ex: $\frac{d}{dx}\left(\frac{1}{\sqrt[3]{(1+x^5)^2}}\right) = \frac{d}{dx}\frac{1}{(1+x^5)^{2/3}} = \frac{d}{dx}(1+x^5)^{-2/3}$

$= -\frac{2}{3}(1+x^5)^{-\frac{2}{3}-1}\cdot\frac{d}{dx}(1+x^5)$

$= -\frac{2}{3}(1+x^5)^{-5/3}5x^4 = -\frac{10}{3}x^4(1+x^5)^{-5/3}$

Ex:

$$f(x) = \cos(3x)$$

$$f'(x) = -\sin(3x) \cdot \overbrace{\frac{d}{dx}(3 \cdot x)}^{3}$$

$$f(x) = 4 \cdot \sin(5x)$$

$$f'(x) = 4 \cdot \cos(5x) \cdot 5 = 20 \cos(5x)$$

$$f(x) = \cos^3 2x = (\cos 2x)^3 = 3(\cos 2 \cdot x)^2 \cdot \frac{d}{dx}(\cos 2 \cdot x)$$

$$= 3(\cos 2 \cdot x)^2(-\sin(2x)) \cdot 2$$