Rep. Lekt. 17

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$\frac{d}{dt}\left(\frac{t^2-1}{t^2+1}\right)\Bigg|_{t=-2} = \frac{2t\cdot(t^2+1)-(t^2-1)2t}{(t^2+1)^2}\Bigg|_{t=-2}$$

$$= \frac{2(-2)\cdot 5 - 3\cdot 2(-2)}{5^2} = \frac{-8}{25} = -\frac{8}{25}$$

Ex: Implicit derivering

$$\cos(x-y) = (2x+1)^3\cdot y$$

Sökes $y'(x)$. Metod: Derivera map. x, glöm ej $y=y(x)$

dvs. $\frac{d}{dx}y = y'$

$$-\sin(x-y)\cdot\frac{d}{dx}(x-y) = 3(2x+1)^2\cdot 2\cdot y + (2x+1)^3\cdot y'$$

$$-\sin(x-y)\cdot(1-y') = 6\cdot y(2x+1)^2 + (2x+1)^3\cdot y'$$

$$-\sin(x-y) + y'\sin(x-y) = 6\cdot y(2x+1)^2 + (2x+1)^3\cdot y'$$

$$\underbrace{y'\sin(x-y) - (2x+1)^3 y'}_{(\sin(x-y)-(2x+1)^3)y'} = 6\cdot y(2x+1)^2 + \sin(x-y)$$

$$y' = \frac{6\cdot y(2x+1)^2 + \sin(x-y)}{\sin(x-y) - (2x+1)^3}$$

Ex Tangent till $x+2y+1 = \frac{y^2}{x-1}$ i $(x,y)=(2,-1)$

$$(x+2y+1)(x-1) = y^2$$

Derivera map. x $\quad (y = y(x))$

$$(1+2y')(x-1) + (x+2y+1)\cdot 1 = 2\cdot y\cdot y'$$

sätt in $(x,y)=(2,-1)$

$$(1+2y')(2-1) + (2+2(-1)+1) = 2(-1)\cdot y'$$

$$1+2y'+1 = -2y'$$

$$4y' = -2$$

$$y' = -\frac{2}{4} = -\frac{1}{2}$$

Ger tangent

$$y-(-1) = -\frac{1}{2}(x-2)$$

$$\boxed{y = -\frac{1}{2}x}$$

Ex: Inversens derivata

$$f(x) = x^3+3x \qquad \text{Bestäm: } (f^{-1})'(4)$$

$$y = f^{-1}(x) \qquad x=4;$$

$$f(y) = x$$

$$y^3+3y = x \qquad y^3+3y=4$$

$$y=1$$

Bestäm y' då $(x,y)=(4,1)$

Derivera map. x

$$3y^2\cdot y' + 3y' = 1$$

$$3\cdot 1^2\cdot y' + 3y' = 1$$

$$y' = \frac{1}{6}$$

sätt in $(x,y)=(4,1)$

Svar: $(f^{-1})'(4) = \frac{1}{6}$

Arctan, derivata

$$y = \arctan x$$

$$x = \tan y, \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Derivera m.a.p x

$$1 = (1 + \tan^2 y)\, y'$$

$$y' = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$$

Ex:

- $$\frac{d}{dx} \arcsin(2x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

- $$\frac{d}{dx} \arctan\left(\sqrt{x^2-1}\right) = \frac{1}{1 + \underbrace{\left(\sqrt{x^2-1}\right)^2}_{x^2-1}} \cdot \frac{1}{\cancel{2}\sqrt{x^2-1}} \cdot \cancel{2}x$$

$$= \frac{1}{x\sqrt{x^2-1}}$$