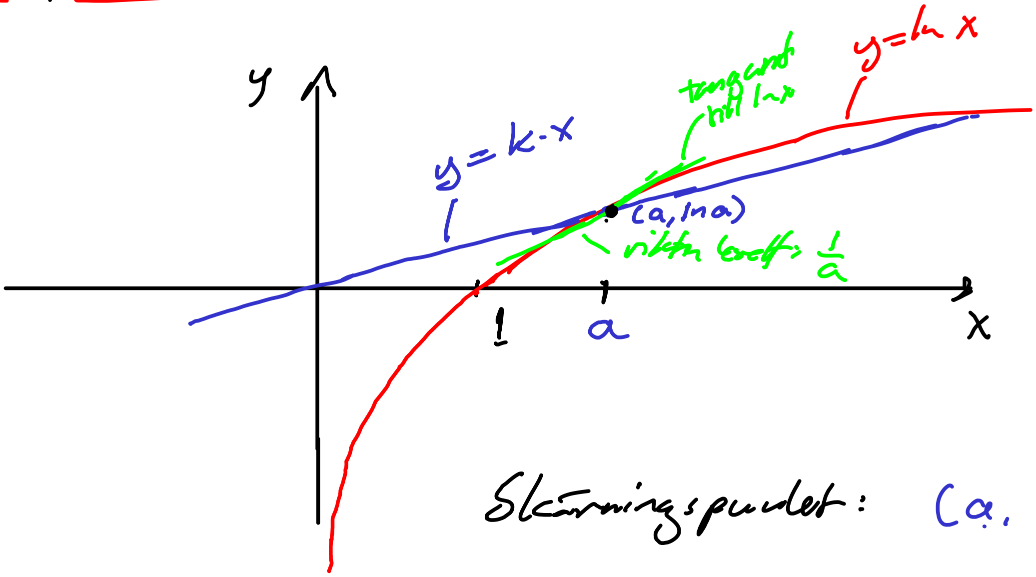
Rep. lekt. 18



Skärningspunkt: $(a, \ln a)$

ger riktningskoeff för linjen: $k = \frac{\ln a - 0}{a - 0} = \frac{\ln a}{a}$

Tangentens riktn.koeff: $\frac{d}{dx} \ln x \Big|_{x=a} = \frac{1}{x}\Big|_{x=a} = \frac{1}{a}$

Välj a så att $\frac{\ln a}{a} = \frac{1}{a}$

$$\ln a = 1$$

$$a = e^1$$

Ger linje $y = k \cdot x = \frac{\ln a}{a} x = \frac{\ln e^1}{e^1} x = \frac{1}{e} x = e^{-1} \cdot x$

Svar: $\boxed{y = e^{-1} \cdot x}$

Ex: $(x^2 + y^2 - 2x)^2 = 2x^2 + 2y^2$

Tangent och normal i $(x, y) = (2, 2)$

Derivera m.a.p. x, givet att $y = y(x)$

$$2(x^2 + y^2 - 2x) \cdot (2x + 2 \cdot y \cdot y' - 2) = 2 \cdot 2x + 2 \cdot 2y \cdot y'$$

$$2(2^2 + 2^2 - 2 \cdot 2) \cdot (2 \cdot 2 + 2 \cdot 2 \cdot y' - 2) = 2 \cdot 2 \cdot 2 + 2 \cdot 2 \cdot 2 \cdot y'$$

$$\overset{1}{\cancel{8}} \cdot (2 + 4y') = \overset{1}{\cancel{8}} + \overset{1}{\cancel{8}} \cdot y'$$

$$2 + 4y' = 1 + y'$$

$$3y' = -1$$

$$y' = -\frac{1}{3}$$

i $(2,2)$

Ger Tangent.

$$y-2 = -\frac{1}{3}(x-2)$$

$$y = -\frac{1}{3}x + \frac{2}{3} + 2$$

$$= -\frac{1}{3}x + \frac{8}{3}$$

och Normal, rikn. koeff $n = -\frac{1}{k} = -\frac{1}{-\frac{1}{3}} = 3$

$$y - 2 = 3\cdot(x-2)$$

$$y = 3x - 6 + 2$$

$$= 3x - 4$$

Beteckningar för derivata

$$y = f(x)$$

$$\frac{d}{dx}y = \frac{dy}{dx} = f'(x)$$

Andraderivata

$$\frac{d}{dx}\frac{d}{dx}y = \frac{d^2}{dx^2}y = \frac{d^2y}{dx^2} = f''(x)$$

Ex:

$$f(x) = x^3 - 12x + 1$$

strängt växande där $f'(x) > 0$

strängt avtagande där $f'(x) < 0$

$$f'(x) = 3x^2 - 12 = 3(x^2-4) = 3(x-2)(x+2)$$

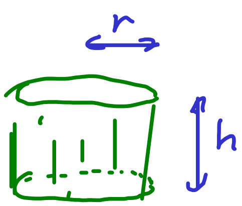
Teckenstudium

		-2		2	
$x-2$	$-$		$-$	0	$+$
$x+2$	$-$	0	$+$		$+$
$f'(x)=(x-2)(x+2)3$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$		$\searrow$		$\nearrow$

Strängt växande: $]-\infty, -2[\; \cup \;]2, \infty[$

strängt avtagande: $]-2, 2[$

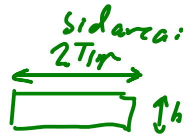
Ex: Cylindrisk burk med volym 1 dm^3 och minimal ytarea.



Höjd	h	[dm]
Bottenradie	r	[dm]
Ytarea	A	[dm²]
Volym	V	[dm³]

Volymen $V = 1 \text{ dm}^2$

$$V = \overbrace{\pi r^2}^{\text{Botten area}} \cdot h$$



$$A = \overbrace{2 \cdot \pi \cdot r^2}^{\text{Botten Topp}} + 2\pi \cdot r \cdot h = 2\pi r(r+h)$$

$V = 1$ ger $1 = \pi r^2 \cdot h$

$h = \frac{1}{\pi \cdot r^2}$ i formeln för A

ger

$$A = 2\pi \cdot r\left(r + \frac{1}{\pi \cdot r^2}\right) = 2\pi r^2 + \frac{2}{r}$$

$$A(r) = 2\pi r^2 + \frac{2}{r}, \quad r > 0$$

$$\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$$

$$\frac{d}{dx} x^{-1} = (-1)x^{-1-1} = -x^{-2}$$

Finn r som minimerar $A(r)$

$$A'(r) = 2\pi \cdot 2r - \frac{2}{r^2}$$

$$= \frac{4\pi r^3 - 2}{r^2}$$

$A'(r) = 0$ gives $\dfrac{4\pi r^3 - 2}{r^2} = 0$

$4\pi r^3 - 2 = 0$

$r^3 = \dfrac{2}{4\pi} = \dfrac{1}{2\pi}$

$r = \sqrt[3]{\dfrac{1}{2\pi}}$

		$\sqrt[3]{\frac{1}{2\pi}}$	
$A'(r)$	$-$	0	$+$
$A(r)$	$\searrow$		$\nearrow$

$\uparrow$ minimizing value

Min'g area at $r = \sqrt[3]{\dfrac{1}{2\pi}}$