

# Derivata

$$\frac{d}{dx} \frac{1}{x} = -\frac{1}{x^2}$$

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$\alpha \quad \frac{d}{dx} \left( 2x^3 - 5x^2 + \frac{3}{x} \right)$$

$$= \frac{d}{dx} \left( 2x^3 - 5x^2 + 3x^{-1} \right)$$

$$= 2 \cdot 3x^2 - 5 \cdot 2 \cdot x^1 + 3(-1) \cdot x^{-2}$$

$$= 6x^2 - 10x - \frac{3}{x^2}$$

$$\beta \quad \frac{d}{dx} x^3(x^2-1)^2 = \frac{d}{dx} x^3(x^4-2x^2+1)$$

$$= \frac{d}{dx} (x^7 - 2x^5 + x^3) = 7x^6 - 2 \cdot 5x^4 + 3x^2$$

$$= x^2(7x^4 - 10x^2 + 3)$$

$$\gamma \quad \frac{d}{dx} \frac{x}{1-x^2} = \frac{1(1-x^2) - x(-2x)}{(1-x^2)^2}$$

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$= \frac{1-x^2+2x^2}{(1-x^2)^2} = \frac{1+x^2}{(1-x^2)^2}$$

$$\delta \quad \frac{d}{dx} \frac{3x+1}{x^5} = \frac{d}{dx} \left( \frac{3x^1}{x^{5-4}} + \frac{1}{x^5} \right) = \frac{d}{dx} \left( 3 \cdot x^{-4} + x^{-5} \right)$$

$$= 3(-4) \cdot x^{-5} + (-5)x^{-6} = -\frac{12}{x^5} - \frac{5}{x^6} = -\left( \frac{12x}{x^6} + \frac{5}{x^6} \right)$$

$$= -\frac{12x+5}{x^6}$$

$$\epsilon \quad \frac{d}{dx} \sqrt[3]{x^7} = \frac{d}{dx} x^{7/3} = \frac{1}{3} x^{7/3-1} = \frac{1}{3} x^{4/3} = \frac{1}{3 \cdot x^{2/3}} = \frac{1}{3(x^2)^{1/3}} = \frac{1}{3\sqrt[3]{x^2}}$$

$$\begin{aligned} \xi \quad \frac{d}{dx} \sqrt{x \sqrt{x \sqrt{x}}} &= \frac{d}{dx} \left( x (x \cdot x^{1/2})^{1/2} \right)^{1/2} = \frac{d}{dx} \left( x \cdot (x^{3/2})^{1/2} \right)^{1/2} \\ &= \frac{d}{dx} (x \cdot x^{3/4})^{1/2} = \frac{d}{dx} (x^{7/4})^{1/2} = \frac{d}{dx} x^{7/8} = \frac{7}{8} x^{7/8-1} \\ &= \frac{7}{8} x^{-1/8} = \frac{7}{8 \cdot x^{1/8}} = \frac{7}{8 \sqrt[8]{x}} \end{aligned}$$

$a^b \cdot a^c = a^{b+c}$   
 $(a^b)^c = a^{b \cdot c}$

$$\begin{aligned} \eta \quad \frac{d}{dx} e^x (x^2 + 2x - 2) &= e^x (x^2 + 2x - 2) + e^x (2x + 2) \\ &= e^x (x^2 + 2x - 2 + 2x + 2) = e^x (x^2 + 4x) \end{aligned}$$

$\frac{d}{dx} f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$

$$\theta \quad \frac{d}{dx} x \cos x = 1 \cdot \cos x + x (-\sin x) = \cos x - x \cdot \sin x$$

$$\epsilon \quad \frac{d}{dx} (x \cdot \ln x - x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x + 1 - 1 = \ln x$$

$$\kappa \quad \frac{d}{dx} \frac{1}{\ln x} = \frac{d}{dx} (\ln x)^{-1} = (-1) (\ln x)^{-2} \cdot \frac{1}{x} = -\frac{1}{(\ln x)^2 \cdot x}$$

$\frac{d}{dx} \ln x = \frac{1}{x}$

$$\lambda \quad \frac{d}{dx} (ax + b)^n = n (ax + b)^{n-1} \cdot a = a \cdot n \cdot (ax + b)^{n-1}$$

$$\mu \quad \frac{d}{dx} \sin^3 x = \frac{d}{dx} (\sin x)^3 = 3 (\sin x)^2 \cdot \cos x$$

$$\nu \quad \frac{d}{dx} \sin(3x) = \cos(3x) \cdot 3 = 3 \cdot \cos(3x)$$

$$\xi \quad \frac{d}{dx} \ln(\sin x) = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x} = \cot x = \frac{1}{\tan x}$$

$$\boxed{0} \quad \frac{d}{dx} \frac{1}{(1+x^2)\sqrt{1+x^2}} = \frac{d}{dx} \frac{1}{(1+x^2)(1+x^2)^{1/2}}$$

$$= \frac{d}{dx} \frac{1}{(1+x^2)^{3/2}} = \frac{d}{dx} (1+x^2)^{-3/2} = -\frac{3}{2} (1+x^2)^{-5/2} \cdot 2x = -\frac{3x}{(1+x^2)^{5/2}}$$

$a^b \cdot a^c = a^{b+c}$

$$\boxed{\pi} \quad \frac{d}{dx} \sqrt{x+\sqrt{x}} = \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \left(1 + \frac{1}{2\sqrt{x}}\right) \quad \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x+\sqrt{x}}} \cdot \frac{2\sqrt{x}+1}{2\sqrt{x}} = \frac{2\sqrt{x}+1}{4\sqrt{(x+\sqrt{x})x}}$$

$$\boxed{e} \quad \frac{d}{dx} \ln \frac{1-x}{1+x} = \frac{d}{dx} (\ln(1-x) - \ln(1+x)) \quad \ln \frac{a}{b} = \ln a - \ln b$$

$$= \frac{1}{1-x} \cdot (-1) - \frac{1}{1+x} \cdot 1 = -\left(\frac{1}{1-x} + \frac{1}{1+x}\right) = -\frac{1+x+1-x}{(1-x)(1+x)}$$

$$= -\frac{2}{1-x^2} = \frac{2}{x^2-1}$$

$\frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = 1 + \tan^2 x$

$$\boxed{\sigma} \quad \frac{d}{dx} \ln \left( \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right) = \frac{1}{\tan \left( \frac{x}{2} + \frac{\pi}{4} \right)} \cdot \frac{1}{\cos^2 \left( \frac{x}{2} + \frac{\pi}{4} \right)} \cdot \frac{1}{2}$$

$\sin 2x = 2 \sin x \cos x$

$$= \frac{1}{2 \frac{\sin \left( \frac{x}{2} + \frac{\pi}{4} \right)}{\cos \left( \frac{x}{2} + \frac{\pi}{4} \right)} \cdot \cos^2 \left( \frac{x}{2} + \frac{\pi}{4} \right)} = \frac{1}{2 \sin \left( \frac{x}{2} + \frac{\pi}{4} \right) \cdot \cos \left( \frac{x}{2} + \frac{\pi}{4} \right)}$$

$$= \frac{1}{\sin \left( 2 \cdot \left( \frac{x}{2} + \frac{\pi}{4} \right) \right)} = \frac{1}{\sin \left( x + \frac{\pi}{2} \right)} = \frac{1}{\sin x \cdot \underbrace{\cos \frac{\pi}{2}}_{=0} + \cos x \cdot \underbrace{\sin \frac{\pi}{2}}_{=1}}$$

$$= \frac{1}{\cos x}$$

$$\boxed{\tau} \quad \frac{d}{dx} \arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \frac{1}{a} \arctan \frac{x}{a} = \frac{1}{a} \cdot \frac{1}{1+\left(\frac{x}{a}\right)^2} \cdot \frac{1}{a} = \frac{1}{a^2 \left(1+\frac{x^2}{a^2}\right)} = \frac{1}{a^2+x^2}$$

$$\begin{aligned}
 \text{v)} \quad \frac{d}{dx} \sqrt{2}^{\sqrt{1+x^2}} &= \frac{d}{dx} e^{\ln \sqrt{2}^{\sqrt{1+x^2}}} = \frac{d}{dx} e^{\sqrt{1+x^2} \cdot \ln \sqrt{2}} \\
 &= \frac{e^{\sqrt{1+x^2} \cdot \ln \sqrt{2}}}{\sqrt{2}^{\sqrt{1+x^2}}} \cdot \ln \sqrt{2} \cdot \frac{1}{2\sqrt{1+x^2}} \cdot 2x = \sqrt{2}^{\sqrt{1+x^2}} \cdot \frac{1}{2} \ln 2 \cdot \frac{x}{\sqrt{1+x^2}} \\
 &= \sqrt{2}^{\sqrt{1+x^2}} \frac{x \cdot \ln 2}{2\sqrt{1+x^2}}
 \end{aligned}$$

$$\text{y)} \quad \frac{d}{dx} e^{x \cdot \ln x} = e^{x \cdot \ln x} \cdot \left( 1 \cdot \ln x + x \cdot \frac{1}{x} \right) = e^{x \cdot \ln x} \cdot (\ln x + 1)$$

$$\begin{aligned}
 \text{x)} \quad \frac{d}{dx} x^x &= \frac{d}{dx} e^{\ln x^x} = \frac{d}{dx} e^{x \cdot \ln x} = e^{x \cdot \ln x} (\ln x + 1) \\
 &= x^x (\ln x + 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{y)} \quad \frac{d}{dx} x^{\sin x} &= \frac{d}{dx} e^{\ln x^{\sin x}} = \frac{d}{dx} e^{\sin x \cdot \ln x} \\
 &= \frac{e^{\sin x \cdot \ln x}}{x^{\sin x}} \left( \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} \right) = x^{\sin x} \left( \cos x \cdot \ln x + \frac{\sin x}{x} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{w)} \quad \frac{d}{dx} (x^3+x)^{\arctan x} &= \frac{d}{dx} e^{\ln (x^3+x)^{\arctan x}} = \frac{d}{dx} e^{\arctan x \cdot \ln (x^3+x)} \\
 &= \frac{e^{\arctan x \cdot \ln (x^3+x)}}{(x^3+x)^{\arctan x}} \left( \frac{1}{1+x^2} \cdot \ln (x^3+x) + \arctan x \cdot \frac{1}{x^3+x} (3x^2+1) \right) \\
 &= (x^3+x)^{\arctan x} \frac{x \cdot \ln (x^3+x) + (3x^2+1) \arctan x}{x(x^2+1)}
 \end{aligned}$$