Gränsvärden

(a) $$\lim_{t\to 2}\frac{2t-t^2}{t^2+t-6}=\left[\frac{0}{0}\right]=\lim_{t\to 2}\frac{\cancel{(t-2)}(-t)}{\cancel{(t-2)}(t+3)}$$

$$=\lim_{t\to 2}\frac{-t}{t+3}=\frac{-2}{2+3}=-\frac{2}{5}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

(b) $$\lim_{x\to 0}\frac{\sin^2 x}{4x^2}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(\sin x)^2}{4x^2}=\lim_{x\to 0}\frac{1}{4}\cdot\left(\frac{\sin x}{x}\right)^2=\frac{1}{4}1^2=\frac{1}{4}$$

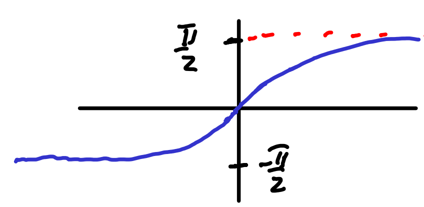
(c) $$\lim_{x\to\infty}\left(\sqrt{9x^2+5x}-3x\right)=\left[\infty-\infty\right]=\lim_{x\to\infty}\frac{\left(\sqrt{9x^2+5x}-3x\right)\left(\sqrt{9x^2+5x}+3x\right)}{\sqrt{9x^2+5x}+3x}$$

$$=\lim_{x\to\infty}\frac{\cancel{9x^2}+5x-\cancel{9x^2}}{\sqrt{x^2\left(9+\frac{5}{x}\right)}+3x}=\lim_{x\to\infty}\frac{5x}{\underbrace{|x|}_{=x}\sqrt{9+\frac{5}{x}}+3x}=\lim_{x\to\infty}\frac{5}{\sqrt{9+\underbrace{\tfrac{5}{x}}_{\to 0}}+3}$$

$$=\frac{5}{\sqrt{9+0}+3}=\frac{5}{6}$$

Kontinuitet

$$f(x)=\begin{cases}\arctan\left(\frac{1}{x}\right), & x>0\\ \frac{e^{ax}-1}{x}, & x<0\end{cases}$$



$$\lim_{x\to 0+}f(x)=\lim_{x\to 0+}\arctan\Big(\underbrace{\tfrac{1}{x}}_{\to\infty}\Big)=\frac{\pi}{2}$$

$$\lim_{h\to 0}\frac{e^h-1}{h}=1$$

$$\lim_{x\to 0-}f(x)=\lim_{x\to 0-}\frac{e^{ax}-1}{x}=\lim_{x\to 0-}a\cdot\frac{e^{a\cdot x}-1}{a\cdot x}=a\cdot 1=a$$

Gränsvärdet $\lim_{x\to 0} f(x)$ existerar om $\lim_{x\to 0-} f(x) = \lim_{x\to 0+} f(x)$ dvs

$$\boxed{a = \frac{\pi}{2}}$$

$$f(x) = \begin{cases} \arctan\left(\frac{1}{x}\right), & x > 0 \\ \frac{e^{\frac{\pi}{2}x} - 1}{x}, & x < 0 \end{cases}$$

Kontinuerlig utvidgning

$$\hat{f}(x) = \begin{cases} \arctan\left(\frac{1}{x}\right), & x > 0 \\ \frac{e^{\frac{\pi}{2}x} - 1}{x}, & x < 0 \\ \frac{\pi}{2} & x = 0 \end{cases}$$

Derivata

$$f(x) = \frac{x}{x-1}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)(x-1)}{(x+h-1)(x-1)} - \frac{(x+h-1)\cdot x}{(x+h-1)(x-1)}}{h}$$

$$\frac{\frac{a}{b}}{c} = \frac{a}{b\cdot c}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{x^2} \cancel{-x} \cancel{+hx} - h - \cancel{x^2} - \cancel{hx} \cancel{+x}}{(x+h-1)(x-1)}}{h} = \lim_{h\to 0} \frac{(-1)\cdot \cancel{h}}{\cancel{h}\cdot (x+h-1)(x-1)}$$

$$= \frac{-1}{(x+0-1)(x-1)} = -\frac{1}{(x-1)^2}$$

$$\frac{d}{dx} \frac{x}{x-1} = \frac{1\cdot(x-1) - x\cdot 1}{(x-1)^2} = \frac{\cancel{x} - 1 \cancel{-x}}{(x-1)^2}$$

$$= -\frac{1}{(x-1)^2}$$

Derivata

$f(x) = x^2 - 4x - 5$, Tangenten i $x = 2$.

och $y = f(2) = 2^2 - 4\cdot 2 - 5 = -9$

dvs. $(x, y) = (2, -9)$.

Riktning?

$f'(x) = 2x - 4$

$k = f'(2) = 2\cdot 2 - 4 = 0$

Ger tangent:

$y - (-9) = 0\cdot(x-2)$

$\boxed{y = -9}$

$F = k\cdot r^4$

ΔF – absolut förändring av flödet

$\frac{\Delta F}{F}$ – % ______ , ______

Δr – absolut förändring av radien

$\frac{\Delta r}{r}$ – % ______ , ______

$\Delta F \approx \frac{dF}{dr}\cdot \Delta r$

$\Delta F \approx k\cdot 4\cdot r^3 \cdot \Delta r$

$\frac{\Delta F}{F} \approx \frac{4kr^3\Delta r}{F} = \frac{4\cancel{k}\cdot \cancel{r^3}^{\,1}\cdot \Delta r}{\cancel{k}\cdot \cancel{r^4}_{\,r}} = 4\cdot\frac{\Delta r}{r}$

$\frac{\Delta F}{F} \approx 4\cdot \frac{\Delta r}{r}$ Om $\frac{\Delta F}{F} = 10\%$ så är $\frac{\Delta r}{r} \approx 2{,}5\%$

Räkneregler

$$f(x) = \frac{x^2-4}{(x-1)^2}$$

$$f'(x) = \frac{2x\cdot(x-1)^{\cancel{2}\,1} - (x^2-4)\,2\,\cancel{(x-1)}\cdot 1}{(x-1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{2x^2} - 2x - \cancel{2x^2} + 8}{(x-1)^3} = \frac{8-2x}{(x-1)^3}$$

$$g(t) = \tan(5-\sin 2t) \qquad \frac{d}{dx}\tan x = \frac{1}{\cos^2 x} = 1+\tan^2 x$$

$$g'(t) = \frac{1}{\cos^2(5-\sin 2t)}\cdot(-\cos(2t))\cdot 2 = -\frac{2\cos 2t}{\cos^2(5-\sin 2t)}$$

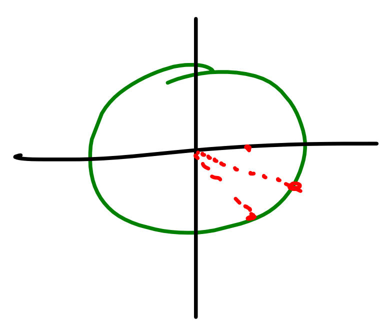
$$\frac{d}{dx}\,x(2x+1)^4 = 1\cdot(2x+1)^4 + x\cdot 4\cdot(2x+1)^3\cdot 2$$

$$= (2x+1)^3\,(2x+1+8x) = (2x+1)^3(10x+1)$$

Tillämpningar

$f(x) = \dfrac{1}{\cos x}$ normalen i $x = -\dfrac{\pi}{3}$

och $y = f\left(-\dfrac{\pi}{3}\right) = \dfrac{1}{\cos\left(-\frac{\pi}{3}\right)} = \dfrac{1}{\frac{1}{2}} = 2$



dvs $(x,y) = \left(-\dfrac{\pi}{3}, 2\right)$

Tangentens rikt.

$$f(x) = (\cos x)^{-1}$$

$$f'(x) = (-1)(\cos x)^{-2}\cdot(-\sin x) = \frac{\sin x}{\cos^2 x}$$

$$k = f'\left(-\frac{\pi}{3}\right) = \frac{\sin -\frac{\pi}{3}}{\cos^2 \frac{\pi}{3}} = \frac{-\frac{\sqrt{3}}{2}}{\left(\frac{1}{2}\right)^2} = -4\frac{\sqrt{3}}{2} = -2\sqrt{3}$$

Normalens rikt.

$$n = \frac{-1}{k} = \frac{-1}{-2\sqrt{3}} = \frac{1}{2\sqrt{3}}$$

Enpunktsformeln

$$y - 2 = \frac{1}{2\sqrt{3}}\left(x - \left(-\frac{\pi}{3}\right)\right)$$

$$y = \frac{1}{2\sqrt{3}}x + \frac{\pi}{6\sqrt{3}} + 2$$

$$x^2 + x \cdot y + y^2 = 3$$

bestäm tangenten som passerar $(x,y)=(1,1)$

derivera m.a.p. x givet $y=y(x)$

$$2x + 1\cdot y + x\cdot y' + 2y\cdot y' = 0$$

$$2\cdot 1 + 1\cdot 1 + 1\cdot y' + 2\cdot 1\cdot y' = 0$$

$$3 + 3y' = 0$$

$y' = -1$ då $(x,y) = (1,1)$

ger tangent

$$y - 1 = (-1)\cdot(x-1)$$

$$y = -x + 1 + 1 = -x + 2$$

$$f(x) = 2x + \cos x$$

Sökes $(f^{-1})'(1)$

Lösning

$$y = f^{-1}(x)$$

$$f(y) = x$$

$$2y + \cos y = x$$

Givet $x = 1$ är $y = 0$

Deriveren naar x geeft $y = y(x)$

$$2y' + (-\sin y) \cdot y' = 1$$

subst in $(x, y) = (1, 0)$

$$2y' - \underbrace{(\sin 0)}_{=0} \cdot y' = 1$$

$$\boxed{y' = \frac{1}{2}}$$ dus $$\boxed{(f^{-1})'(1) = \frac{1}{2}}$$