Exempel

$$f(x) = x^2 - 5x + 3$$

$$f'(x) = 2x - 5$$

$$f'(x) = 0 \iff 2x - 5 = 0$$

$$2x = 5$$

$$\boxed{x = \frac{5}{2}}$$ stationär punkt

x		$\frac{5}{2}$	
$f'(x)$	$-$	0	$+$
$f(x)$	$\searrow$	lok. min & globalt minimum	$\nearrow$

Minsta värde för $f(x)$ antas i $x = \frac{5}{2}$ där

$$f\left(\frac{5}{2}\right) = \left(\frac{5}{2}\right)^2 - 5\,\frac{5}{2} + 3 = -\frac{13}{4}$$

Ex:

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3 = 3(x-1)(x+1)$$

$$f'(x) = 0 \iff 3x^2 - 3 = 0$$

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$x = 1$ eller $x = -1$

Stationära punkter

x		-1		1	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	lok. max	$\searrow$	lok. min	$\nearrow$

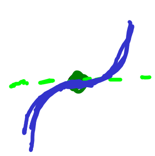
lok max i $x = -1$

$f(-1) = 2$

lok min i $x = 1$

$f(1) = -2$

Terasspunkt:



Ex: Analysera och rita

$$y = f(x) = \frac{x^3+3x^2-4}{x^2+2x+1} = \frac{x^3+3x^2-4}{(x+1)^2}$$

Växande / avtagande

$$f'(x) = \frac{(3x^2+6x)(x+1)^{\cancel{2}\,1} - (x^3+3x^2-4)\cdot 2\cdot \cancel{(x+1)}\cdot 1}{(x+1)^{\cancel{4}\,3}}$$

$$= \frac{3x^3+6x^2+3x^2+6x-2x^3-6x^2+8}{(x+1)^3}$$

$$= \frac{x^3+3x^2+6x+8}{(x+1)^3} = \frac{(x+2)\overbrace{(x^2+x+4)}^{>0}}{(x+1)^3}$$

Stationära pkt $\quad f'(x)=0 \qquad \dfrac{(x+2)(x^2+x+4)}{(x+1)^3}=0$

$$\boxed{x=-2}$$

$f'(x)$ ej def. i $\boxed{x=-1}$

x		-2		-1	
$x+2$	$-$	0	$+$		$+$
x^2+x+4	$+$		$+$		$+$
$(x+1)^3$	$-$		$-$	0	$+$
$f'(x)=\dfrac{(x+2)(x^2+x+4)}{(x+1)^3}$	$+$	0	$-$	ej. def.	$+$
$f(x)$	$\nearrow$	lok. max	$\searrow$	ej. def.	$\nearrow$

lok max i $\quad x=-2 \qquad f(-2)=0$

Lodrät asymptot

$$\lim_{x \to -1+} \frac{x^3+3x^2-4}{(x+1)^2} = \left[\frac{-2}{0+}\right] = -\infty$$

$$\lim_{x \to -1-} \frac{x^3+3x^2-4}{(x+1)^2} = \left[\frac{-2}{0+}\right] = -\infty$$

Lodrät asymptot : $\boxed{x=-1}$

Lutande asymptot

$$y = f(x) = \frac{x^3+3x^2-4}{x^2+2x+1} = x+1+\frac{-3x-5}{x^2+2x+1}$$

$\to 0$ då $x \to \infty$, $x \to -\infty$

$$\begin{array}{r|l} & \boxed{x+1} \text{ Kvot} \\ \hline x^2+2x+1 & x^3+3x^2 \quad -4 \\ & -(x^3+2x^2+x) \\ \hline & x^2-x-4 \\ & -(x^2+2x+1) \\ \hline & \boxed{-3x-5} \text{ rest} \end{array}$$

Lutande asymptot:

$\boxed{y=x+1}$

Graf: