Rep. Lekt 22

$$f(x) = x - \arctan\left(\frac{x}{x+1}\right), \quad x > -1$$

$(D_f =]-1, \infty[\ )$

1. Lodrät asymptot?

Möjligen i $x = -1$

$$\lim_{x \to -1+} f(x) = \lim_{x \to -1+} \Big(\underset{\to -1}{x} - \underbrace{\arctan\Big(\frac{\overset{\to -1}{x}}{\underset{\to 0+}{x+1}} \Big)}_{\to -\frac{\pi}{2}} \Big) = \underbrace{-1 + \frac{\pi}{2}}_{\text{ändligt}}$$

$\underset{\to -\infty}{}$

Ingen lodrät asymptot!

Men $\lim_{x \to -1+} f(x) = \frac{\pi}{2} - 1$ är intressant info

2. ~~Lutande~~ Sned asymptot? $(\ y = k \cdot x + m\ )$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x - \arctan\left(\frac{x}{x+1}\right)}{x} = \lim_{x \to \infty} \left(1 - \frac{\arctan\left(\frac{\cancel{x}\,1}{\cancel{x}(1+\frac{1}{x})}\right)}{x} \right)$$

$$= \lim_{x \to \infty} \Big(1 - \underset{\to 0}{\frac{1}{x}} \cdot \arctan\Big(\frac{1}{1 + \underset{\to 0}{\frac{1}{x}}} \Big) \Big) = 1 - 0 \cdot \frac{\pi}{4} = 1$$

$$m = \lim_{x \to \infty} (f(x) - k \cdot x) = \lim_{x \to \infty} \left(\cancel{x} - \arctan\left(\frac{x}{x+1}\right) - \cancel{1 \cdot x} \right)$$

$$= \lim_{x \to \infty} -\arctan\Big(\frac{1}{1 + \underset{\to 0}{\frac{1}{x}}} \Big) = -\arctan 1 = -\frac{\pi}{4}$$

Lutande asymptot: $\boxed{y = x - \frac{\pi}{4}}$

3. Växande/avtagande

$f(x) = x - \arctan\left(\frac{x}{x+1}\right)$

$$f'(x) = 1 - \frac{1}{1+\frac{x^2}{(x+1)^2}} \cdot \frac{1(x+1) - x \cdot 1}{(x+1)^2} \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$= 1 - \frac{1}{(x+1)^2 + x^2} = \frac{(x+1)^2 + x^2}{(x+1)^2 + x^2} - \frac{1}{(x+1)^2 + x^2}$$

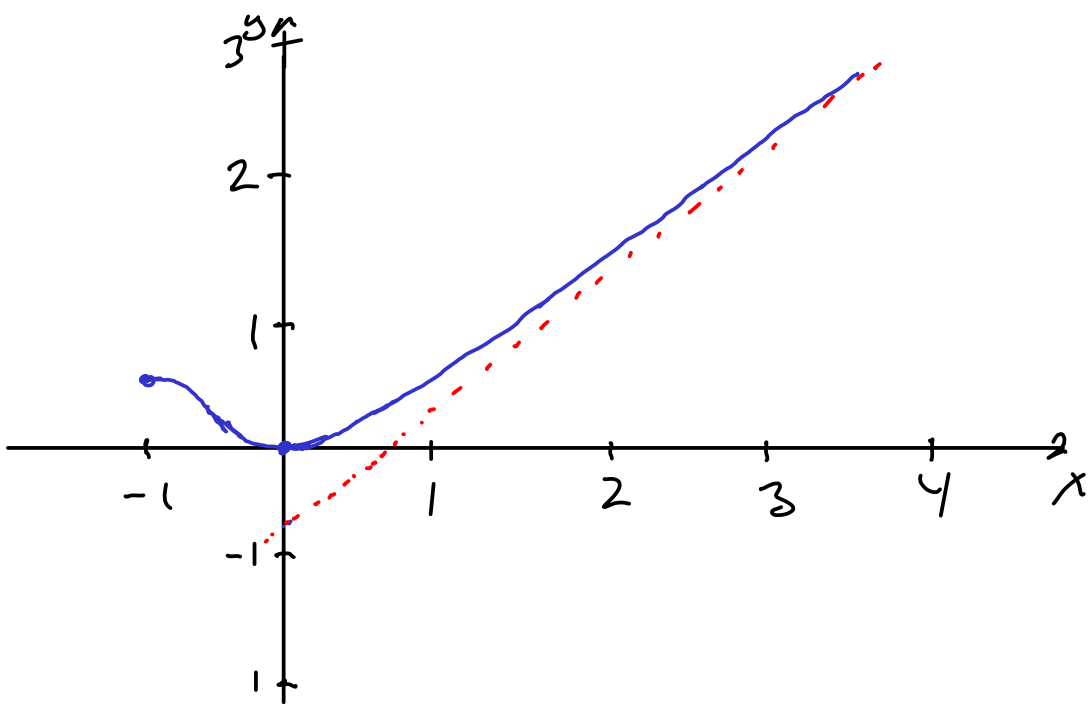
$$= \frac{x^2 + 2x + \cancel{1} + x^2 - \cancel{1}}{(x+1)^2 + x^2} = \frac{2x^2 + 2x}{(x+1)^2 + x^2} = \frac{2x(x+1)}{(x+1)^2 + x^2}$$

$f'(x) = 0$ då $x(x+1) = 0 \Leftrightarrow x = 0$ eller $x = -1$

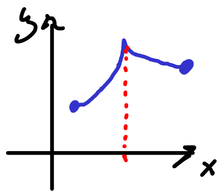
	-1		0		x
$f'(x)$	0	$-$	0	$+$	
$f(x)$		$\searrow$	lok. min	$\nearrow$	

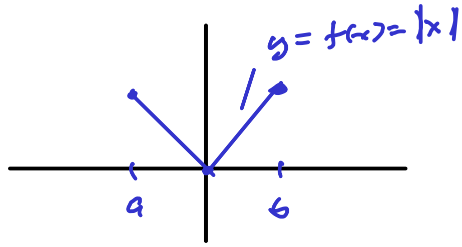
$f(x) \to \frac{\pi}{2} - 1$, $x \to -1^+$

$f(0) = 0 - \arctan 0 = 0$



Lokalt max/min där $f'(x)$ ej existerar





$$\frac{d}{dx}|x| = \operatorname{sgn}(x) = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

Ex $f(x) = -3x^4 - 4x^3 + 12x^2, \quad -0{,}5 \le x \le 2$

Största och minsta värde

Derivera

$$f'(x) = -12x^3 - 12x^2 + 24x$$

$$= -12x(x^2 + x - 2) = -12x(x+2)(x-1)$$

Stationära punkter $f'(x) = 0 \iff \boxed{x = 0}$ el. $\cancel{x = -2}$ el. $\boxed{x = 1}$

utanför intervallet

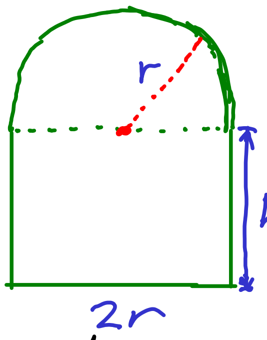
$f'(x)$ existerar i alla punkter

Ändpunkter $\boxed{x = -0{,}5}$ $\boxed{x = 2}$

Funktionsvärden

x	$f(x) = -3x^4 - 4x^3 + 12x^2$	
$-0{,}5$	$3{,}31250$	
0	0	
1	5	← största
2	-32	← minsta

Avslutande exempel



Area: $A = \underbrace{2r\cdot h}_{\text{rektangel}} + \underbrace{\tfrac{1}{2}\pi\cdot r^2}_{\text{halvcirkel}}$

Omkrets: $O = h + 2r + h + \underbrace{\pi r}_{\text{halv-cirkelbåge}}$

$= 2h + (2+\pi)r$

(de dimensioner på fönstret som ger)
Finn minimal omkrets då arean $A = 2$

- Villkoret ger $2 (= A) = 2\cdot r\cdot h + \frac{\pi}{2}\cdot r^2$

$$2\cdot r\cdot h = 2 - \frac{\pi}{2}\cdot r^2$$

$$\boxed{h = \frac{1}{r} - \frac{\pi}{4}\cdot r}$$

$$0 < r \le \frac{2}{\sqrt{\pi}}$$

$$0 = \frac{1}{r} - \frac{\pi}{4}\cdot r$$

$$\frac{\pi}{4}\cdot r^2 = 1$$

$$r = \frac{2}{\sqrt{\pi}}$$

- Omkretsen uttryckt i r

$$O(r) = 2h + (2+\pi)\cdot r = \frac{2}{r} - \frac{\pi}{2}\cdot r + (2+\pi)\cdot r$$

$$= \frac{2}{r} + \left(-\frac{\pi}{2} + 2 + \pi\right)r = \frac{2}{r} + \left(2 + \frac{\pi}{2}\right)r$$

- Stationära punkter, växande/avtagande

$$O'(r) = -\frac{2}{r^2} + \left(2 + \frac{\pi}{2}\right) = \frac{-2 + \left(2+\frac{\pi}{2}\right)r^2}{r^2}$$

$$O'(r) = 0 \qquad -2 + \left(2 + \frac{\pi}{2}\right)r^2 = 0$$

$$r^2 = \frac{2}{2 + \frac{\pi}{2}} = \frac{4}{4+\pi}, \qquad r = \pm\frac{2}{\sqrt{4+\pi}}$$

$O'(r)$ ej def. i $r=0$, men inte heller $O(r)$ och utanför def. mängd.

r	0		$\frac{2}{\sqrt{4+\pi}}$		$\frac{2}{\sqrt{\pi}}$
$O'(r)$		$-$	0	$+$	
$O(r)$		$\searrow$	lok. min och minsta värde	$\nearrow$	

Omkretsen minimeras då $r = \frac{2}{\sqrt{4+\pi}}$

$$h = \frac{1}{\frac{2}{\sqrt{4+\pi}}} - \frac{\pi}{4} \cdot \frac{2}{\sqrt{4+\pi}}$$

$$= \frac{\sqrt{4+\pi}}{2} - \frac{\pi}{2\sqrt{4+\pi}} = \frac{4 + \cancel{\pi} - \cancel{\pi}}{2\sqrt{4+\pi}}$$

$$= \frac{2}{\sqrt{4+\pi}}$$

dvs. $r = h$