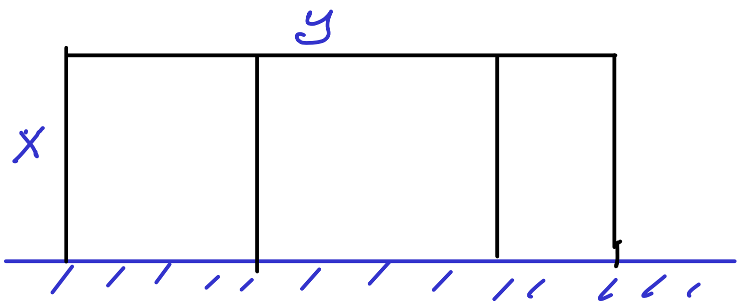
Rep. Lekt 23



Sidor	x, y	$[m]$
Area	A	$[m^2]$
Staket längd	S	$[m]$

$$A = x \cdot y$$

$$S = y + 4x$$

Villkoret $S = 400\,m$ ger

$$400 = y + 4x$$

$$\boxed{y = 400 - 4x}$$

$y \geq 0$

$$0 \leq x \leq 100$$

Insatt i A ger

$$A(x) = x \cdot y = x(400 - 4x) = 400x - 4x^2$$

- □ Stationära punkter

$$A'(x) = 400 - 8x$$

$A'(x) = 0 \qquad 400 - 8x = 0$

$$\boxed{x = \frac{400}{8} = 50}$$

- □ Punkter där $A'(x)$ ej existerar. Finns inga sådana.
- □ Ändpunkter $\boxed{x = 0}$ $\boxed{x = 100}$

ger

$A(x) = x(400 - 4x)$

x	$A(x)$
0	0
50	10000
100	0

← Max då $x = 50$, $y = 200$

Ex:

$$y = \tan(kx)$$

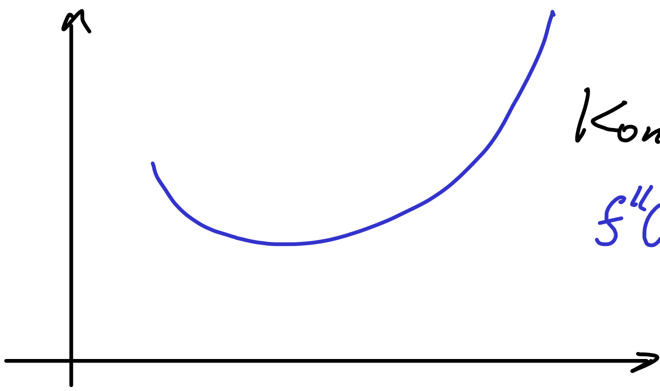
$$\frac{d}{dx}\tan x = \frac{1}{\cos^2 x} = 1+\tan^2 x$$

$$\frac{dy}{dx} = \left(1+\tan^2(kx)\right)\cdot k = k + k\left(\tan(k\cdot x)\right)^2$$

$$\frac{d^2y}{dx^2} = 0 + k\cdot 2\left(\tan(k\cdot x)\right)\cdot\left(1+\tan^2(k\cdot x)\right)\cdot k$$

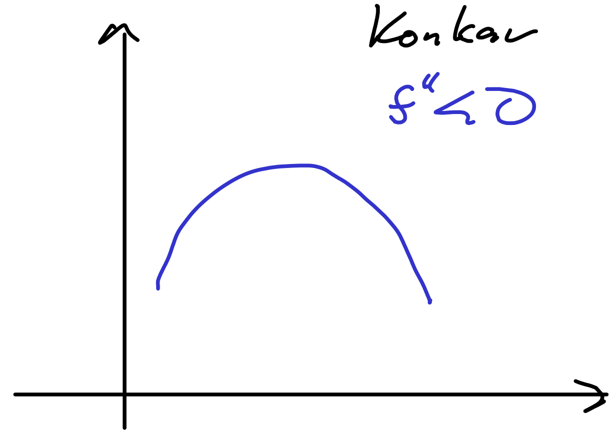
$$= 2\cdot k^2\cdot \tan(k\cdot x)\left(1+\left(\tan(k\cdot x)\right)^2\right)$$

$$= 2\cdot k^2\cdot y\,(1+y^2)$$



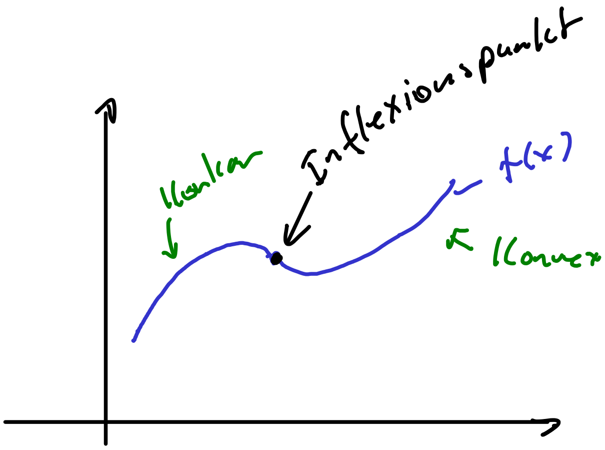
Konvex

$f''(x) > 0$

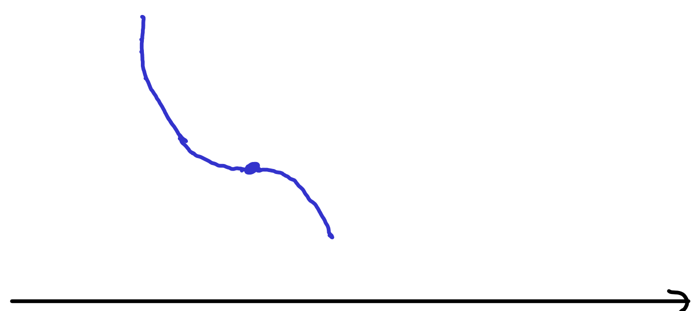


Konkav

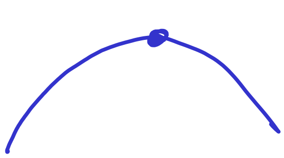
$f'' < 0$



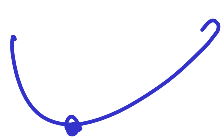
Terasspunkt v.s. Inflexionspunkt



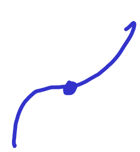
Andraderivatatest



$f''(x) < 0$
$\Rightarrow$ lok. max



$f''(x) > 0$
$\Rightarrow$ lok. min



$f''(x) = 0$
$\Rightarrow$ terasspunkt

Ex: $f(x) = \dfrac{2x^2 - x + 2}{x^2+1}$

1. Asymptoter

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{2x^2-x+2}{x^2+1} = \lim_{x\to\infty} \frac{\cancel{x^2}(2 - \overset{\to 0}{\frac{1}{x}} + \overset{\to 0}{\frac{2}{x^2}})}{\cancel{x^2}(1 + \underset{\to 0}{\frac{1}{x^2}})} = \frac{2-0+0}{1+0} = 2$$

$$\lim_{x\to-\infty} f(x) \overset{\text{se ovan}}{=} \dots = 2$$

Vågrät asymptot: $\boxed{y=2}$

Nämnaren $x^2+1 \geq 0+1 > 0$ så lodrät asymptot finns ej!

2. Växande / avtagande

$f(x) = \dfrac{2x^2-x+2}{x^2+1}$

$$f'(x) = \frac{(4x-1)\cdot(x^2+1) - (2x^2-x+2)2x}{(x^2+1)^2}$$

$$= \frac{\cancel{4x^3} + \cancel{4x} - x^2 - 1 - \cancel{4x^3} + 2x^2 - \cancel{4x}}{(x^2+1)^2} = \underline{\frac{x^2-1}{(x^2+1)^2}}$$

$$= \underline{\frac{(x-1)(x+1)}{(x^2+1)^2}}$$

$f'(x) = 0 \Leftrightarrow x = 1$ eller $x = -1$

x		-1		1	
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	lok. max	$\searrow$	lok. min	$\nearrow$

$f(-1) = \frac{5}{2}$ $\qquad$ $f(1) = \frac{3}{2}$

3. Konkavitet

$$f''(x) = \frac{2x(x^2+1)^{\cancel{2}\,1} - (x^2-1)\cdot 2\,\overset{1}{\cancel{(x^2+1)}}\,2x}{(x^2+1)^{\cancel{4}\,3}}$$

$$= \frac{2x^3 + 2x - 4x^3 + 4x}{(x^2+1)^3} = \frac{6x - 2x^3}{(x^2+1)^3}$$

$$= \frac{2x(3 - x^2)}{(x^2+1)^3} = \frac{2x(\sqrt{3}-x)(\sqrt{3}+x)}{(x^2+1)^3}$$

$f''(x) = 0 \Leftrightarrow x = 0$ el. $x = \sqrt{3}$ el. $x = -\sqrt{3}$

		$-\sqrt{3}$		0		$\sqrt{3}$	
x	$-$		$-$	0	$+$		$+$
$\sqrt{3}-x$	$+$		$+$		$+$	0	$-$
$\sqrt{3}+x$	$-$	0	$+$		$+$		$+$
$f''(x)$	$+$	0	$-$	0	$+$	0	$-$
$f(x)$	$\cup$	infl.	$\cap$	infl.	$\cup$	infl.	$\cap$

$f(-\sqrt{3}) = \frac{8+\sqrt{3}}{4}$ $\quad$ $f(0) = 2$ $\quad$ $f(\sqrt{3}) = \frac{8-\sqrt{3}}{4}$

Skizze