Rep. L 24

$$f(x) = e^x + \cos 2x - \sin x$$

Lok. max el. min i $x=0$?

$$f'(x) = e^x - 2\cdot\sin 2x - \cos x$$

Testa $x=0$

$$f'(0) = e^0 - 2\cdot\sin 0 - \cos 0 = 1 - 2\cdot 0 - 1 = 0$$

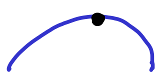
dvs $x=0$ är en stationär punkt.

Andraderivatatestet

$$f''(x) = e^x - 4\cdot\cos 2x + \sin x$$

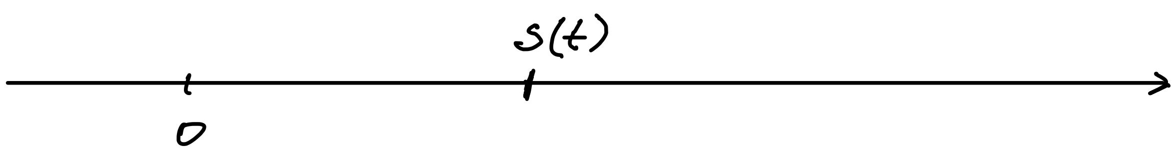
sätt in $x=0$

$$f''(0) = e^0 - 4\cos 0 + \sin 0 = 1 - 4\cdot 1 + 0 = -3 < 0$$



Dvs lokalt maximum

Fysikalisk tolkning



Sträcka: $s(t)$

Hastighet: $v(t) = s'(t)$

Acceleration: $a(t) = v'(t) = s''(t)$

Fart: $|v(t)|$

Newtons metod

Vi söker ett x^* som gör att

$$f(x) = 0$$

Newtons metod gör det genom att bilda en talföljd som närmar sig x^* så att

$$\lim_{k \to \infty} x_k = x^*$$

Talföljden bildas av

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}}$$

Exempel:

Lös ekvationen

$$e^x = x + 2$$

$$e^x - x - 2 = 0$$

Bilda

$$f(x) = e^x - x - 2$$

$$f'(x) = e^x - 1$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{e^{x_k} - x_k - 2}{e^{x_k} - 1}$$

$$x_0 = 2$$

$$x_1 = 2 - \frac{e^2 \ldots}{\ldots}$$

$x_0 = 2$

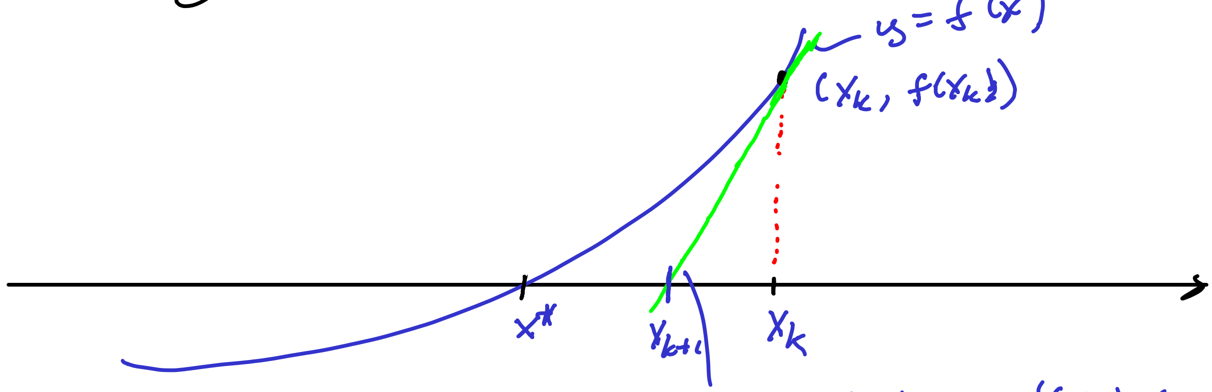
$x_1 = 1{,}4695$

$x_2 = 1{,}2073$

$x_3 = 1{,}1488$

$x_4 = 1{,}1462$

Hur fungerar Newtons metod?



$$y - f(x_k) = f'(x_k)(x - x_k)$$

Sätt $y = 0$

$$0 - f(x_k) = f'(x_k)(x - x_k)$$

$$-\frac{f(x_k)}{f'(x_k)} = x - x_k$$

$$\underbrace{x}_{x_{k+1}} = x_k - \frac{f(x_k)}{f'(x_k)}$$

Ex:

Lös $\sin x = \dfrac{x^2}{4}$

$$\sin x - \frac{x^2}{4} = 0$$

Bilda

$$f(x) = \sin x - \frac{x^2}{4}$$

$$f'(x) = \cos x - \frac{x}{2}$$

ger

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{\sin x_k - \frac{x_k^2}{4}}{\cos x_k - \frac{x_k}{2}}$$

$x_0 = 2$
$x_1 = 1.93595$
$x_2 = 1.93376$
$x_3 = 1.93375$
$x_4 = 1.93375$